Consecutive, Reversed, Mirror, and Symmetric Smarandache Sequences of Triangular Numbers

Delfim F. M. Torres

delfim@mat.ua.pt
Department of Mathematics
University of Aveiro, Portugal

Viorica Teca†
viorica_teca@yahoo.com
Faculty of Mathematics-Informatics
University of Craiova, Romania

07/June/2004

Abstract

We use the Maple system to check the investigations of S. S. Gupta regarding the Smarandache consecutive and the reversed Smarandache sequences of triangular numbers [Smarandache Notions Journal, Vol. 14, 2004, pp. 366–368]. Furthermore, we extend previous investigations to the mirror and symmetric Smarandache sequences of triangular numbers.

Mathematics Subject Classification 2000. 11B83, 11-04, 11A41.

The $n$th triangular number $t_n$, $n \in \mathbb{N}$, is defined by $t_n = \sum_{i=1}^{n} i = n(n+1)/2$. These numbers were first studied by the Pythagoreans.

The first $k$ terms of the triangular sequence $\{t_n\}_{n=1}^{\infty}$ are easily obtained in Maple:

```maple
> t := n -> n*(n+1)/2:
> first := k -> seq(t(n), n=1..k):
```

For example:

```maple
> first(20);  
1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210
```

In this short note we are interested in studying Smarandache sequences of triangular numbers with the help of the Maple system.

To define the Smarandache sequences, it is convenient to introduce first the concatenation operation. Given two positive integer numbers $n$ and $m$, the concatenation operation $\text{conc}$ is defined in Maple by the following function:

```maple
> conc := (n, m) -> n*10^\text{length}(m)+m:
```

†MSc student of Informatics at the University of Craiova, Romania. Student at the University of Aveiro under the Socrates/Erasmus European programme, 2004.
For example,

\[ > \text{conc}(12,345); \]

12345

Given a positive integer sequence \( \{u_n\}_{n=1}^\infty \), we define the corresponding Smarandache Consecutive Sequence \( \{scs_n\}_{n=1}^\infty \) recursively:

\[
\begin{align*}
scs_1 &= u_1, \\
scs_n &= \text{conc}(scs_{n-1}, u_n).
\end{align*}
\]

In Maple we define:

\[
\begin{align*}
> \text{scs}_n := (u,n) -> \text{if } n=1 \text{ then } u(1) \text{ else } \text{conc}(\text{scs}_n(u,n-1), u(n)) \text{ fi}; \\
> \text{scs} := (u,n) -> \text{seq}(\text{scs}_n(u,i), i=1..n);
\end{align*}
\]

The standard Smarandache consecutive sequence, introduced by the Romanian mathematician Florentin Smarandache, is obtained when one chooses \( u_n = n, \forall n \in \mathbb{N} \). The first 10 terms are:

\[
> \text{scs}(n->n,10);
\]

1, 12, 123, 1234, 12345, 123456, 1234567, 12345678, 123456789, 12345678910

Another example of a Smarandache consecutive sequence is the Smarandache consecutive sequence of triangular numbers. With our Maple definitions, the first 10 terms of such sequence are obtained with the following command:

\[
> \text{scs}(t,10);
\]

1, 13, 136, 13610, 1361015, 136101521, 13610152128, 1361015212836, 136101521283645, 13610152128364555

Sometimes, it is preferred to display Smarandache sequences in “triangular form”.

\[
> \text{show} := L -> \text{map}(i->\text{print}(i),L); \\
> \text{show}([\text{scs}(t,10)]):
\]

1
13
136
13610
1361015
136101521
13610152128
1361015212836
136101521283645
13610152128364555
The Reversed Smarandache Sequence \((rss)\) associated with a given sequence \(\{u_n\}_n^{\infty}\), is defined recursively by

\[
\begin{align*}
rss_1 &= u_1, \\
rss_n &= \text{conc}(u_n, rss_{n-1}).
\end{align*}
\]

In Maple we propose the following definitions:

\[
> \text{rss}_n := (u, n) -> \text{if } n=1 \text{ then } u(1) \text{ else conc}(u(n), \text{rss}_n(u, n-1)) \text{ fi}; \\
> \text{rss} := (u, n) -> \text{seq}(\text{rss}_n(u, i), i=1..n);
\]

The first terms of the reversed Smarandache sequence of triangular numbers are now easily obtained:

\[
> \text{rss}(t, 10);
\]

1, 31, 631, 10631, 1510631, 211510631, 28211510631, 3628211510631, 453628211510631, 55453628211510631

We define the Smarandache Mirror Sequence \((sms)\) as follows:

\[
\begin{align*}
sms_1 &= u_1, \\
sms_n &= \text{conc}(\text{conc}(u_n, sms_{n-1}), u_n)
\end{align*}
\]

\[
> \text{sms}_n := (u, n) -> \text{if } n=1 \text{ then } u(1) \text{ else conc(conc}(u(n), sms_n(u, n-1)), u(n)) \text{ fi}; \\
> \text{sms} := (u, n) -> \text{seq}(\text{sms}_n(u, i), i=1..n);
\]

The first 10 terms of the Smarandache mirror sequence introduced by Smarandache are:

\[
> \text{sms}(n->n, 10);
\]

1, 121, 32123, 4321234, 543212345, 65432123456, 7654321234567, 876543212345678, 98765432123456789, 109876543212345678910

We are interested in the Smarandache mirror sequence of triangular numbers. The first 10 terms are:

\[
> \text{sms}(t, 10);
\]

1, 31, 631, 10631, 1510631, 211510631, 28211510631, 3628211510631, 453628211510631, 55453628211510631

Finally, we define the Smarandache Symmetric Sequence \((sss)\). For that we introduce the function “But Last Digit” \((\text{bld})\):
> bld := n -> iquo(n,10):
> bld(123);

12

If the integer number is a one-digit number, then function bld returns zero:

> bld(3);

0

This is important: with our conc function, the concatenation of zero with a positive integer n gives n.

> conc(bld(1),3);

3

The Smarandache Symmetric Sequence (sss) is now easily defined, appealing to the Smarandache consecutive, and reversed Smarandache sequences:

\[
\begin{align*}
sss_{2n-1} &= \text{conc}(\text{bld}(\text{scs}_{2n-1}), \text{rss}_{2n-1}), \\
sss_{2n} &= \text{conc}(\text{scs}_{2n}, \text{rss}_{2n}),
\end{align*}
\]

\(n \in \mathbb{N}\). In Maple, we give the following definitions:

> sss_n := (u,n) -> if type(n,odd) then
> conc(bld(sc(n(u,(n+1)/2)),rss_n(u,(n+1)/2))
> else
> conc(sc(n(u,n/2)),rss_n(u,n/2))
> fi:
> sss := (u,n) -> seq(sss_n(u,i),i=1..n):

The first terms of Smarandache’s symmetric sequence are

> sss(n->n,10);

1, 11, 121, 1221, 12321, 123321, 1234321, 12344321, 123454321

while the first 10 terms of the Smarandache symmetric sequence of triangular numbers are

> sss(t,10);

1, 11, 131, 1331, 13631, 136631, 136110631, 1361011510631, 1361015151510631

One interesting question is to find prime numbers in the above defined Smarandache sequences of triangular numbers. We will restrict our search to the first 1000 terms of each sequence. All computations were done with Maple 9 running on a 2.00Ghz Pentium 4 with 256Mb RAM. We begin by collecting four lists with the first 1000 terms of the consecutive, reversed, mirror, and symmetric Smarandache sequences of triangular numbers:

> st:=time(): Lscs1000:=[scs(t,1000)]: printf("%a seconds",round(time()-st));

20 seconds

> st:=time(): Lrss1000:=[rss(t,1000)]: printf("%a seconds",round(time()-st));

75 seconds
We note that $scs_{1000}$ and $rss_{1000}$ are positive integer numbers with 5354 digits;

$$\text{length}(Lscs1000[1000]), \text{length}(Lrss1000[1000])$$

5354, 5354

while $sms_{1000}$ and $sss_{1000}$ have, respectively, 10707 and 4708 digits.

$$\text{length}(Lsms1000[1000]), \text{length}(Lsss1000[1000])$$

10707, 4708

There exist two primes (13 and 136101521) among the first 1000 terms of the Smarandache consecutive sequence of triangular numbers;

$$\text{select(isprime,Lscs1000)};$$

$$\text{printf(\"%a minutes\",round((time()-st)/60))};$$

[13, 136101521]

9 minutes

six primes among the first 1000 terms of the reversed Smarandache sequence of triangular numbers;

$$\text{select(isprime,Lrss1000)};$$

$$\text{printf(\"%a minutes\",round((time()-st)/60))};$$

[31, 631, 10631, 55453628211510631, 786655453628211510631, 10591786655453628211510631]

31 minutes

only one prime (313) among the first 600 terms of the Smarandache mirror sequence of triangular numbers;\(^1\)

$$\text{length}(Lsms1000[600]); \# sms_{600} \text{ is a number with 5907 digits}$$

5907

$$\text{select(isprime,Lsms1000[1..600])};$$

$$\text{printf(\"%a minutes\",round((time()-st)/60))};$$

[313]

---

\(^1\)Our computer runs low in memory when one tries the first 1000 terms of the Smarandache mirror sequence of triangular numbers. For this reason, we have considered here only the first 600 terms of the sequence.
and five primes among the first 1000 terms of the Smarandache symmetric sequence of triangular numbers (the fifth prime is an integer with 336 digits).

```
> st := time():
> select(isprime,Lss1000);
> printf("%a minutes",round((time()-st)/60));

[11, 131, 136110631, 136101521283645556678911051201201059178665453628211510631,
  1361015212836455566789110512013615317119021023125327630032535137840643546549652856159
  5630666703741780820861903946990103510811128117612251275132613781431148515401596165316
  5315961540148514311378132612751225117611281081103599094690386182078074170366663059556
  15284964654354063783513253002762532312101901711531361201059178665453628211510631]

19 minutes

> length(%[5]);

336
```

How many primes are there in the above defined Smarandache sequences of triangular numbers? This seems to be an open question.

Another interesting question is to find triangular numbers in the Smarandache sequences of triangular numbers. We begin by defining in Maple the boolean function `istriangular`.

```
> istriangular := n -> evalb(nops(select(i->evalb(whattype(i)=integer),
  [solve(t(k)=n)])) > 0):
```

There exist two triangular numbers (1 and 136) among the first 1000 terms of the Smarandache consecutive sequence of triangular numbers;

```
> st := time():
> select(istriangular,Lscs1000);
> printf("%a seconds",round(time()-st));

[1, 136]
```

6 seconds

while the other Smarandache sequences of triangular numbers only show, among the first 1000 terms, the trivial triangular number 1:

```
> st := time():
> select(istriangular,Lrss1000);
> printf("%a seconds",round(time()-st));

[1]
```

6 seconds
> st := time():
> select(istriangular,Lsms1000);
> printf("%a seconds",round(time()-st));

10 seconds

> st := time():
> select(istriangular,Lss1000);
> printf("%a seconds",round(time()-st));

6 seconds

Does exist more triangular numbers in the Smarandache sequences of triangular numbers? This is, to the best of our knowledge, an open question needing further investigations. Since checking if a number is triangular is much faster than to check if a number is prime, we invite the readers to continue our search of triangular numbers for besides the 1000th term of the Smarandache sequences of triangular numbers. We look forward to readers discoveries.

References