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Determination of Natural Frequency of Liquid Sloshing in Tanks of Various Shapes

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Abstract

Carriage of liquid cargo is essential to national economy and hence the annual volumes of liquid cargo are fairly large. In many cases this involves carrying toxic and fire-hazardous substances. Spillage thereof in case of emergency may cause environmental damage and even human losses. So when carrying such substances the main stress is on safety.

The paper dwells on defining sloshing frequency for a liquid in tanks of different shapes similar to transport tanks. Such sloshing causes extra dynamic loads on vehicles. A method for calculation of sloshing will help to determine conditions, under which sloshing starts, and hence to take respective measures for elimination thereof.

A tank of the simplest shape – a rectangular prism– is taken as an example, and manner of liquid motion at its free surface sloshing is experimentally studied. Depth of tank filling, within which impact of the tank's lower bottom manifests itself, is determined. Expression obtained permits determination of sloshing frequencies in tanks of arbitrary size.

Liquid sloshing is considered for cylinder-shaped tanks at various orientation of a tank in space: vertical – sloshing across the tank, and horizontal – sloshing across and along the tank. Effective filling depth is determined. Based on experimental data for all cases considered, coefficients have been obtained, which are necessary for determination of liquid sloshing frequencies.

To prove broad applicability of the proposed calculation method the paper studies liquid sloshing between concentric cylinders. Based on experimental data, necessary coefficients have been determined and compared with results of numerical calculations given in the references. Good agreement of results is demonstrated at a far simpler calculation technique.

Content

Introduction	4
State of the art. Analysis of reference sources.	5
Research task	6
1 Manner of Liquid Motion in a Rectangular-Shaped Tank	7
2 Model of Liquid Motion in a Rectangular-Shaped Tank	10
3 Liquid Motion in a Cylinder-Shaped Tank	11
4 Liquid Motion Between Two Cylinders	13
5 Conclusion	15

Determination of Natural Frequency of Liquid Sloshing in Tanks of Various Shapes

Introduction

Most energy sources (crude oil, petrochemicals), feedstock and chemical products are carried in liquid state using liquid cargo carriers (tankers), rail tank cars and tank trucks. Consumption of resources and hence the volume of cargo traffic grow. Number of carriage-related accidents grows as well [1-4]. Due to high toxicity of carried cargo environmental pressure increases. In Ukraine major part of liquid cargo is carried by rail, due to its geographical position. That is why analysis of causes of such accidents and development of preventive measures are important. Situation is further exacerbated by the fact that major part of rail tank cars manufactured in Ukraine is exported (namely, to Russia). Some accidents involving such tank cars are caused by failure of supporting frames and are attributed to their low quality. It causes significant damage to the image of Ukraine.

When designing engineering systems, it is sufficient to do estimative calculations in some cases, without resorting to any accurate and costly solution. At the same time it is necessary to estimate margin of error at such approach.

One can assume that loads affecting structural elements of vehicles that carry liquid with free surface depend on many factors, such as:

- Weight, or density of liquid, at a given tank capacity;
- Level of tank fullness;
- Value of external forces;
- Interval coincidence of external effects and liquid sloshing frequency (resonance) and other.

As for external effect interval, it may also depend on many factors:

- Condition of a traffic route (road humps, railway junctions and condition thereof);
- Track curvature (force of inertia);
- Vehicle speed and other.

There are many factors, and liquid motion is complex. So it is important not only to take them into account, but do it quickly and cost-effectively. In a rapidly changing traffic situation specialists, not necessarily highly qualified, should be able to do such calculations and assessments. And in many cases determination of liquid sloshing frequency in a tank is a major task.

Objective of the paper is to obtain a simple relation allowing analytical (algebraic) calculation of liquid sloshing frequency, preferably in variously shaped tanks.

State of the art. Analysis of reference sources.

While carrying partially filled tank, dynamic qualities of a liquid-carrying vehicle change due to comparability of unladen weight to carried liquid mass, and hence they are very different from dynamic qualities of other vehicles. Due to large travels of the centre of cargo mass in a reservoir extra loads appear and affect the tank structure and its supporting frame and decrease steadiness and handling of the tank truck. Global statistics shows a high percentage of accidents when carrying hazardous cargo not only by rail, but by road as well. And level of threat to human life and environment grows twelvefold compared to other types of transport [5].

Research was stimulated by problems occurring when carrying liquids in tanks of certain shapes. Yet, similar problems occur with liquids in other types of tanks, and not only while being conveyed.

Tasks relating to determination of natural sloshing frequencies in a liquid with free surface in tanks of various shapes have remained critical for a long time. One of the first to study the problem of standing waves in the limited volume of liquid with free surface inside a still basin was M.V. Ostrogradsky [6], who presented his research paper to the Paris Academy of Sciences in 1826. Krylov A.N. [7] delivered a report “Frahm Tanks Used For Stabilizing Motions of a Vessel” before the Russian Physics and Chemistry Society in 1913, and his research became the cornerstone of the theory of ship stabilizing gear. Solution of a number of other tasks was published with reference to other matters of the ship theory and hydraulic engineering [8, 9]. Problems relating to motion of liquid with free surface have been covered while solving tasks of liquid cargo carriage [10-12], and effect of seismic loads on tanks [13-15]. Special attention was paid to such problems when

designing aviation and missile equipment [16-23]. Analytic, numerical analytic and numerical experiment-based solution techniques have been applied. Research dwelt on studying liquid motion in tanks of various shapes, such as rectangular parallelepiped, cylinder, cone, torus-like, with centre bodies, in rotating tanks of arbitrary shape, etc. Interest to these problems is still alive, which is obvious in publication of articles [12], [23-25] and books [20], and theses defense [26-28].

Such close and sustained attention proves significance of the studied phenomena. And abundance of published research confirms complexity of the tasks and failure to find solution to a number of issues of practical importance. Many papers dwell exclusively on the method for solution of the task at hand. When there is need in results of such solution, the consumer (researcher, designer) is forced to solve the problem on their own applying complex techniques. If the paper does provide solution, its results merely illustrate potential of the method, or there is no comparison of results obtained to experimental data. In some studies calculation results and experimental data are compared at a quality level without reference to quantitative evaluation of their accuracy.

Results of the experiment-based paper [29] are a rare exception, containing description of the experiment method and a relation for processed experimental data. Regrettably, this paper studied only one type of tank shape located in one place.

Research task

To determine peculiarities of liquid motion at free surface sloshing in tanks of various shapes. To determine general patterns of such motion. To obtain simple relations allowing determination of sloshing frequency of free surface for a liquid in tanks of various sizes and shapes.

1. Manner of Liquid Motion in a Square-Shaped Tank

On the first stage the task was to determine manner of liquid motion at sloshing: what parts of liquid are involved in motion at different levels of tank fullness. As mentioned above, liquids can be carried in tanks of various shapes. However, it is easier to start research of liquid motion in a tank of the simplest shape – a square.



Photo 1

A flat square-shaped tank (photo 2) was fabricated, with bottom and side walls made of wood and front and rear walls made of glass. Wood was chosen as the easiest material to work with. Joints between wooden elements have been treated with silicone sealant (used in plumbing). Glass walls have been installed using rubber gaskets and sealant. Size of the fabricated tank was determined by the available glass shelf: $250 \times 500 \times 100$

Manner of liquid motion at sloshing of its free surface is determined through revealing (visualizing) manner of liquid motion within its mass. To achieve this, liquid was coloured along the tank midline, and then smearing of this line at liquid sloshing was inspected by sight check method. Colouring was done through dipping a rod made of porous material soaked in washable dye in a sloshing liquid (photo 3, 4). The porous rod used is a welding electrode (or rather electrode covering). The dye used is fountain pen ink. Liquid sloshing was generated by lateral impact (hand push).

Experiments showed (photo 5-10) that dye washing occurs mainly in the bounded area (V_{ef}), its approximate boundaries being within a semicircle with a radius equal to a half of the tank width (l_2) (fig. 1), which is equivalent to depth (h_{ef}). The assumption was made that liquid motion occurs mainly in this area and that filling depth, in case it is larger than this area, has no effect on sloshing frequency of the free surface.



Photo 2



Photo 3



Photo 4



Photo 5



Photo 6



Photo 7



Photo 8



Photo 9

To confirm this assumption a number of tests were performed. Sloshing interval for the free surface of the liquid was determined at a given tank width ($2 \cdot l_2 = 250$ mm) and various filling levels (liquid depth in a tank h).

Experimental algorithm is as follows:

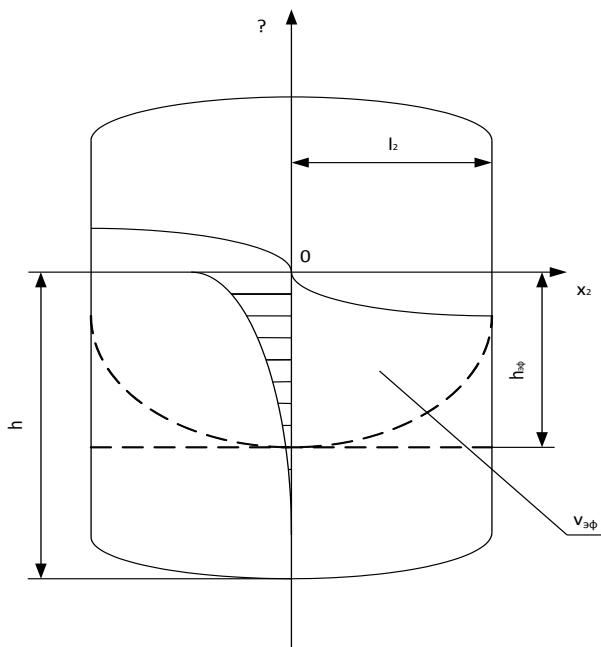


Fig. 1

- To average out, an interval of 10 sloshes of the free surface of the liquid (T_{10}) was determined, and a sloshing interval (T) was calculated at a given filling depth h ;
- For further averaging-out the experiment was done thrice, and mean period of the sloshing (T_{mean}) and the frequency of sloshing ($\omega = 1/T_{mean}$) for the free surface of the liquid was determined.

Filling lev. h , mm	40			60			80			100			120		
T_{10} , sec	8.10	8.14	8.11	6,96	7,07	7,04	6,34	6,27	6,29	6,08	6,07	6,06	5,86	5,87	5,87
T , sec	0,81	0,81	0,81	0,70	0,71	0,70	0,63	0,63	0,63	0,61	0,61	0,61	0,59	0,59	0,59
T_{mean} , sec	0,81			0,70			0,63			0,61			0,59		
v , 1/sec	1,24			1,43			1,59			1,64			1,7		

Experimental results confirmed the assumptions with fractional errors. Sloshing interval depends largely on filling depth up to $h=l_2=125$ mm.

2. Model of Liquid Motion in a Square-Shaped Tank

Similar study was performed for tanks of varying width (by changing width of the fabricated tank through application of various insertions). $2 \cdot l_2 = 230, 210, 190, 170$ mm tanks were tested. Results were identical.

The assumption was made that at high filling levels liquid motion of bounded and constant volume resembles pendulum motion. Pendulum swing frequency depends on free fall acceleration rate and suspension length (1):

$$\omega = \sqrt{\frac{g}{l}} . \quad (1)$$

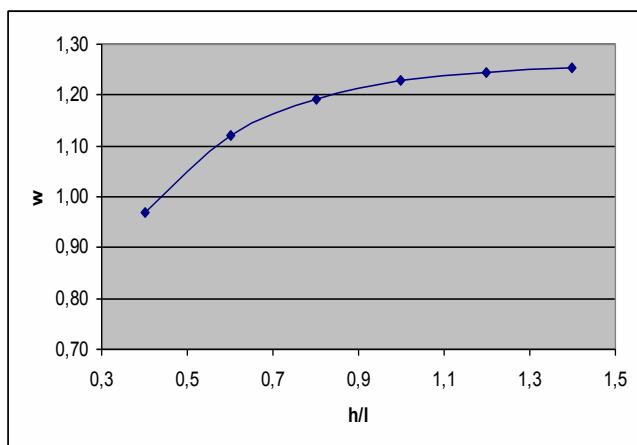
In our case suspension length may be taken as $l_2 = h_{ef}$ – a half of the tank width. All other factors that may affect liquid motion shall be allowed for by a coefficient. Expression of closed form shall be as follows:

$$\omega = \sqrt{\frac{g}{l_2}} \cdot k_\omega \quad (2)$$

In scientific research obtained results are represented in non-dimensionalized form for generalization purposes. To achieve this, right-hand and left-hand sides of the expression (2) are to be divided by $\sqrt{\frac{g}{l_2}}$.

$$\varpi = \frac{\omega}{\sqrt{g/l_2}} = k_\omega \quad (3)$$

Upon processing experimental data in such a manner for tanks of various sizes at high filling levels, interesting results have been obtained. It was found that the value $\varpi = k_\omega$ is the same for all tanks, or rather almost the same. One can assume that any deviations can be attributed to experimental error. It turned out to be $k_\omega = \varpi \approx 1.26$, deviation not exceeding 0.05 for given tank sizes. Consistency of ϖ is observed only at high filling levels ($h \geq l_2$). As h is reduced, value ϖ decreases as well, and is different for tanks of different sizes.



Impact limit of filling depth on sloshing frequency is value ($h_{ef}/l_2 = 1$). Due to this an attempt was made to consider a relation of ϖ not to h , but to $h/l_2 < 1$, when $h < l_2$. Such an approach proved to be sound. For all given tanks values ϖ in relation to h/l_2 agreed. As the final result, general character of the relation ϖ to h/l_2 is demonstrated on the graph (w – corresponds to ϖ , and h/l – corresponds to h/l_2).

Such curve can be described through an expression

$$\varpi = k_\omega \cdot (1 - e^{-k\bar{h}}) \quad (4)$$

where $\bar{h} = h/l_2$;

$$k_\omega = 1,26.$$

Coefficient k must be matched. It was found that when $k = 3,67$ values obtained through the expression (4) are almost the same as experimental data. To be more accurate, deviation of calculated values ϖ differ from respective experimental data by less than 0.05 for all values.

As the final result, expression for determining liquid sloshing frequency might be as follows:

$$\omega = \sqrt{\frac{g}{l_2}} \cdot k_\omega \cdot (1 - e^{-k\bar{h}}) \quad (5)$$

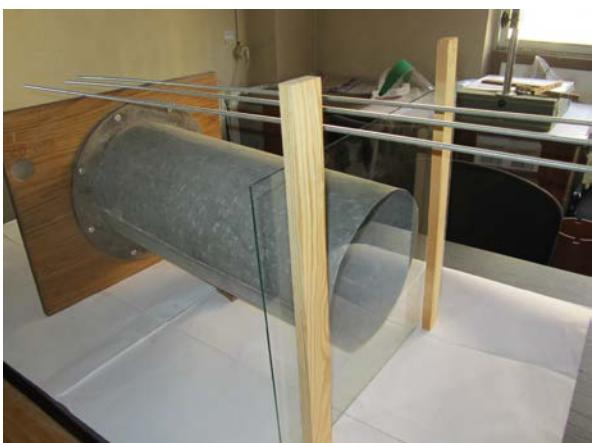
Tests performed in this part of the research brought the following results:

1. manner of liquid motion in a square-shaped tank is determined based on visualization;
2. mathematical relation (4) enabling to determine liquid motion frequencies in a square-shaped tank of various sizes and at different filling levels is found;
3. two coefficients k_ω and k obtained based on experimental studies are used in the proposed expression (5).

3. Liquid Motion in a Cylinder-Shaped Tank

Most tanks, in which liquids are stored or carried, are cylinder-shaped. They may have varying spatial arrangement. For instance, in oil storages tanks stand erect (vertical position of longitudinal axis), where liquid can slosh in transverse direction (e.g. at lateral impact of forces during earthquake). In railway cars and road vehicles tanks are placed horizontally. Liquid may slosh both along longitudinal axis (when railway cars or motor cars brake) and in transverse direction (when vehicles turn).

Thus, in case of cylinder-shaped tanks there are three possible types of liquid sloshing.



A tank was fabricated for the test using a piece of tin cylinder-shaped flanged air pipe 227 mm in diameter. A wooden panel was used as one of the bottoms (for ease of fabrication). This tank was used to study liquid motion when placed vertically. To study liquid motion in a horizontally spaced tank a second bottom was made of glass. It was inserted using a rubber

Photo 10

gasket and was tightened with another bottom using pins and wooden bars (photo 11).

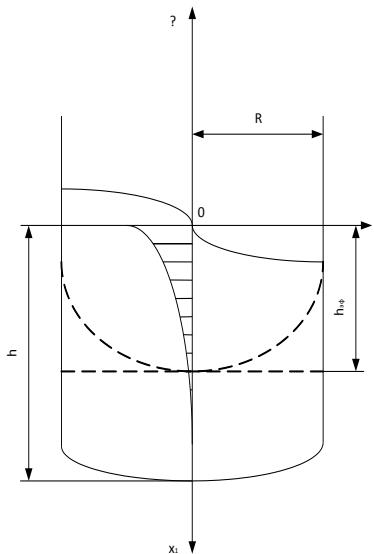


Fig.2

This tank structure did not offer easy visualization of liquid motion through colouring. An assumption was made that liquid motion in a cylinder-shaped tank is the same as in a square-shaped one. What was necessary is to determine depth, at which the lower bottom ceases to affect sloshing frequency (effective depth).

At first liquid motion in a vertically spaced tank was studied. Experimental algorithm was the same as in par. 1 for a square-shaped tank. It was found that there was an effective depth and the tank radius might be taken as its value. In this case liquid sloshing frequency is determined based on correlation similar to (5):

$$\omega = \sqrt{\frac{g}{R}} \cdot k_{\omega} \cdot (1 - e^{-k\bar{h}}). \quad (6)$$

Instead of the half of the tank width the tank radius R shall be used. Coefficients k_{ω} & k take on the following values:

$$k_{\omega} = 1.36; \quad k = 4.29 \quad (7)$$

Coefficient values differ from respective coefficient values for a square-shaped tank, yet they are very similar.

Similar approach was applied when studying liquid motion in a horizontally-spaced cylinder-shaped tank. Thus, when liquid sloshes across the axis,

$$k_{\omega} = 1.29; \quad k = 2.43, \quad (8)$$

And when it sloshes along the axis

$$k_{\omega} = 1.6; \quad k = 3.5. \quad (9)$$

At the same time in an expression for sloshing frequency

$$\omega = \sqrt{\frac{g}{l}} \cdot k_{\omega} \cdot (1 - e^{-k\bar{h}}) \quad (10)$$

characteristic dimension l is taken as a half of the surface width in the sloshing direction.

Tests performed in this part of the research brought the following results:

1. it is proved that manner of liquid motion in a cylinder-shaped tank placed in different locations is similar to liquid motion in a square-shaped tank. It is possible to determine effective depth of the tank filling. When filling depth is more than the effective depth, the tank bottom ceases to affect value of the free surface sloshing frequency;

2. expression (10) is determined for calculation of liquid sloshing frequency in a cylinder-shaped tank. It proved to be identical to expression (5) for a square-shaped tank;
3. coefficients k_ω and k (7), (8), (9) used in expression (10) at varying location of a cylinder-shaped tank are determined.

4. Liquid Motion Between Two Cylinders

Proposed calculation method for sloshing frequency of the liquid surface proved to be applicable to both square-shaped and cylinder-shaped tanks. It is interesting to study its applicability to tanks of more complex shapes. For better accuracy one should use not only data of their own experiments, but also results of other researchers.

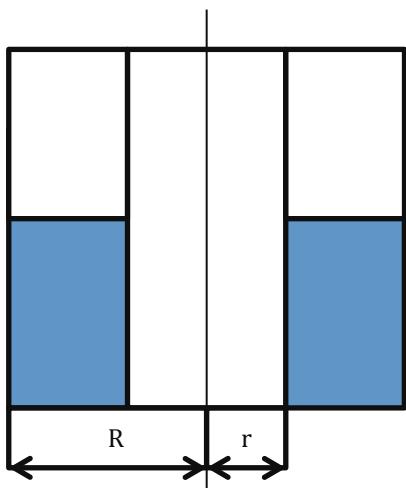
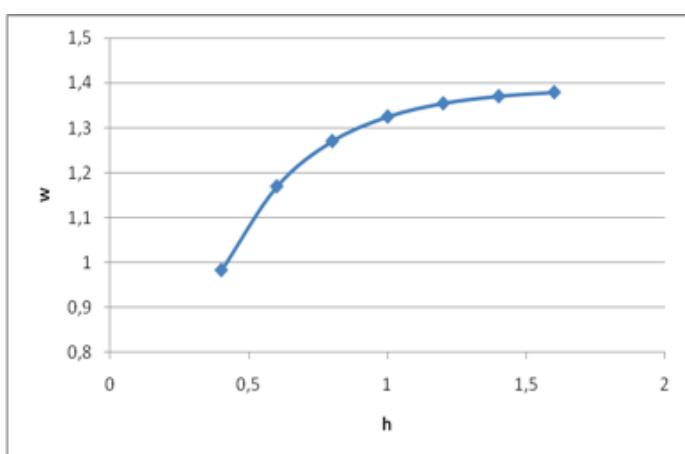


Fig. 3

Source [20] offers calculation results obtained by complex (variational) numerical method for liquid sloshing between concentric cylinders. Source [20] does not sum up any patterns of such motion, and the result is provided for only one dimension ratio (Fig. 3) – radii of inner and outer cylinders ($R=1.0$ m., $r=0.4$ m.). However, the result provided (Fig. 4) enables to assess accuracy and generality of our method in all its simplicity. In the source [20], which the graph (Fig. 4) originates from, tank filling depth (h) is plotted on the abscissa, and the value corresponding to sloshing frequency is plotted on the ordinate. As one can see on the graph, when filling depth is

$h \approx 1.4$ m = $R + r$ liquid sloshing frequency stops changing, i.e. this depth is effective. In all formulae of type (6) or (10) above characteristic dimension equaled to this depth. So in the given case we shall determine liquid sloshing frequency based on correlation similar to the above:



$$\omega = \sqrt{\frac{g}{R+r}} \cdot k_\omega \cdot (1 - e^{-k\bar{h}}), \quad (11)$$

and values of coefficients k_ω and k shall be determined based on experimental data.



Photo 11



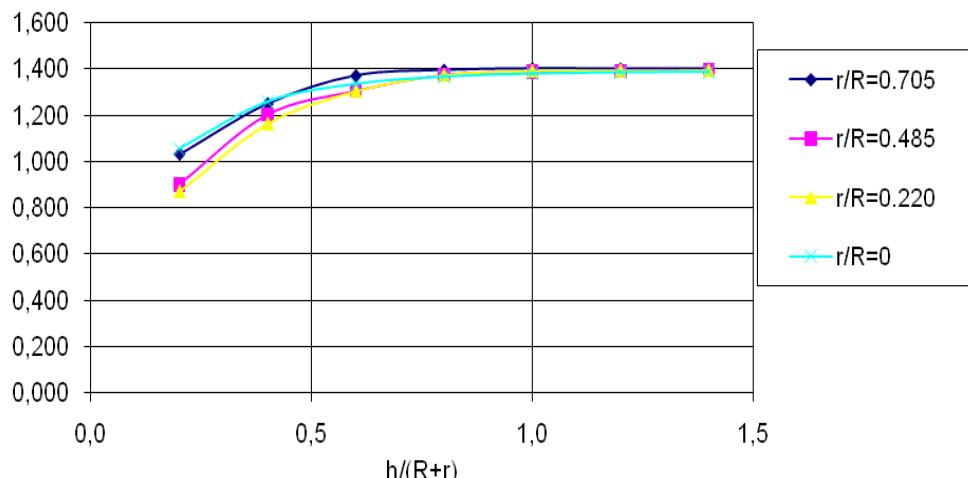
Photo 12

To achieve this, another experimental unit was fabricated. Cylinder-shaped tank from the previous unit was used as the outer cylinder. Inner cylinders were made of pieces of plastic sewage pipes 50, 110 and 160 mm in diameter (photo 13). Cylinder alignment was done using a wooden crosspiece (photo 12). Experimental algorithm is similar to the one described in par. 1 herein. Experimental results have been processed based on the scheme described in par. 2 using expression (11). Results of processed experimental data are shown on the graph (Fig. 5). Relative filling levels ($h/(R+r)$) are plotted on the abscissa. Non-dimensionalized liquid sloshing frequencies, obtained as a result of dividing

expression (11) by the value $\omega = \sqrt{\frac{g}{R+r}}$, are plotted on the ordinate. Coefficients

$$k_\omega = 1.39 \text{ and } k = 4.3 \quad (12)$$

have been determined.



As shown on the graphs, the assumption that effective filling depth equals to the sum of the tank radii has proved to be correct, and we succeeded in describing all experimental

data in one relation. Moreover, expression (6) for a vertical circular cylinder is a special case of the expression (11), when $r=0$. This is confirmed by close values of the coefficients k_ω and k for both cases (expressions (7) and (12)).

1. Expression for determining liquid sloshing frequency between concentric cylinders was obtained;
2. generality of the method proposed for determining liquid sloshing frequencies in tanks of various shapes is confirmed;
3. high accuracy of the proposed simple calculation method is proved compared to complex numerical method described in the reference source [20].

5. Conclusion

- 1 manner of liquid motion in a rectangular-shaped tank is determined based on visualization;
- 2 based on experimental determination of effective filling depth it is shown that manner of liquid motion at sloshing of its surface in tanks of other shapes is identical;
4. a simple universal relation enabling to determine liquid sloshing frequencies in tanks of various shapes is found;
5. based on experimental studies, coefficients enabling to use obtained relation in practical calculations are determined. These coefficients are summarized in a table.

Coefficient values for calculating
the frequency of liquid sloshing in tanks

Nº	Tank shape	Orientation	Direction of sloshing	k	k_ω
1	Rectangular prism	vertical	along the lateral edge	3,67	1,26
2	Cylinder	vertical	across the axis	4,29	1,36
3	Cylinder	horizontal	along the axis	3,5	1,6
4	Cylinder	horizontal	across the axis	2,43	1,29
5	Coaxial cylinders	Horizontal(axis)	across the axis $\bar{h} \geq 0.4$	4,3	1,39

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REFEREE REPORT

on a research paper

Determination of Natural Frequency of Liquid Sloshing in Tanks of Various Shapes

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Carriage of liquid cargo is essential to national economy and hence the annual volumes of liquid cargo are fairly large. In many cases this is about carrying toxic and fire-hazardous substances. Spillage thereof in case of rolling stock accidents may cause environmental damage or even human losses. So when carrying such substances the main stress is on safety.

The paper dwells on defining sloshing frequency for a liquid in tanks of different shapes similar to transport tanks. Such sloshing causes extra dynamic loads on vehicles. A method for calculation of sloshing will help to determine conditions, under which sloshing starts, and hence to take respective measures for elimination thereof.

The paper is characterized by applied orientation and significance of problems to be solved. All results obtained are based on experimental data, which is a reasonable ground for believing in theoretical results provided. Expressions obtained for calculation of liquid sloshing frequencies are simple and uniform, so personnel of any qualification will be able to use them while determining a safe route for vehicles.

The paper's wording is stylistically correct. 29 references sources are used. The research may be further studied and continued in future.

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