A useful criterion to identify candidate twin primes

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Abstract: In a previous paper we derived that if \( p, p+2 \) are twin-primes then \( 2^{p-2} \) is of the form \( (pz+y) \) where \( z, y \) must have unique solutions. We extend this result to derive a single criterion that we believe is novel that may be useful to screen for candidate twin primes.

Results:

If \( p, p+2 \) are large twin primes then as shown in a previous paper

\[
2^{p-2} = pz + y \quad \text{(where \( z \) and \( y \) are unique solutions for any pair of twin primes)}
\]

Multiplying by 4,

\[
2^p = 4pz + 4y
\]

Subtracting 2 from both sides,

\[
2^p - 2 = 4pz + 4y - 2
\]

Since \( p \) is prime, therefore it follows from Fermat's little theorem that \( 2^{p-2} \) is divisible by \( p \) therefore

\[
4y - 2 = p^*u \quad \text{(where \( u \) is an even integer, since \( p \) is a large prime and therefore odd)}
\]

Therefore \( y = (p^*u + 2)/4 \quad \text{......(I)} \)

Since \( p+2 \) is also prime, therefore it follows Fermat’s little theorem, \( 2^{p+2-2} \) is divisible by \( p+2 \).

\[
2^{p+2-2} = (p+2)b \quad \text{where \( b=6y \), (as shown in previous paper)}
\]

we can rewrite \( b = 6y = (2^{p+2-2})/(p+2) \)

Therefore \( y = ((2^{p+2-2})/(6(p+2)) \)

Or \( y = ((2^{p+1-1})/(3(p+2)) \) .............(II)

From (I) and (II) it follows that

\[
(p^*u + 2)/4 = ((2^{p+1-1})/(3(p+2)) \)
\[(p^*u+2) = \{(4)(2^{p+1}-1)\}/\{3(p+2)\}\]

Therefore

\[p^*u = \{(4)(2^{p+1}-1)\}/\{3(p+2)\} - 2\]

\[p^*u = \{(2^{p+3}-4)-(2)(3)(p+2)\}/\{3(p+2)\}\]

\[p^*u = \{2(2^{p+2} - 2p - 6)\}/\{3(p+2)\}\]

\[p^*u = \{2(2^{p+2} - 3p - 8)\}/\{3(p+2)\}\]

\[u = \{2(2^{p+2} - 3p - 8)\}/\{3p(p+2)\}\]

Therefore if \(p, p+2\) represent large twin-primes then the expression \((2^{p+2} - 3p - 8)\) is perfectly divisible by thrice the product of the twin primes. This single criterion may be directly applied to screen for large candidate twin primes.

References:

1. An elementary approach to explore possible constraints on the infinite nature of twin primes
[816] viXra:1410.0112 (Number Theory)