# On the Limitations of Wilson's Renormalization Group Program

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*Abstract*

This informal report surveys several lesser-known limitations of Wilson's Renormalization Group program. The account is not intended to be either rigorous or complete as our sole purpose is to stimulate further discussions and research.

**Key words**: Perturbative Renormalization Group, Renormalization Group flow, Effective Field Theory, Standard Model, Fixed Points, Fractal Spacetime, Minimal Fractal Manifold.

It is widely recognized that a key program built in the structure of the Standard Model of high-energy physics (SM) is the Renormalization Group, whose function is to preserve self-consistency and describe how parameters of the theory evolve with the energy scale. The Wilson treatment of critical phenomena using the perturbative *Renormalization Group* program (RG) [1, 11] develops from the premise that quantum fields present in the theory $(\Phi_\mu)$ depend on the running scale $\mu$ and can be segregated into a pair of un-coupled components

$$\Phi_\mu^{(l)} : 0 \leq \mu \leq \frac{\Lambda_{UV}}{s} \tag{1}$$

$$\Phi_\mu^{(s)} : \frac{\Lambda_{UV}}{s} \leq \mu \leq \Lambda_{UV} \tag{2}$$

Here, $\Lambda_{UV}$ stands for the cutoff scale in the ultraviolet sector, the parameter "$s$" is an arbitrary scaling factor ($s > 1$), $\Phi_{\mu}^{(l)}$ and $\Phi_{\mu}^{(s)}$ are the long and short wavelength excitations and correspond, respectively, to the light and heavy particles carried by $\Phi_{\mu}$. Starting with an effective field theory (EFT) defined at $\Lambda_{UV}$, the core idea of Wilson's approach is to integrate out all heavy particles contained in the "momentum shell" (2) and form a new EFT with the remaining fields below the separation scale $\Lambda_{UV}/s$. Since $\mu$ is considered a running parameter, iterating this process yields a flow of EFT's from $\Lambda_{UV}$ toward their low-energy limit. It is customary to refer to this iterative process as a *RG flow* (or *RG trajectory*). A key property of local EFT's is that the low-energy endpoint of the RG flow must describe phenomena that are fully decoupled from physical processes occurring near the high-energy limit $\Lambda_{UV}$. This property conveys the basic idea behind the concept of *scale invariance* [10].

Despite being accepted as a paradigm for securing consistency of the SM, Wilson's renormalization model presents several lesser-known aspects and limitations which are often neglected in standard textbooks. Namely,

- The RG flow is *not laminar* in general; turbulent behavior is a possibility that cannot be excluded [1-2, 4-5].
- The hypothesis of *local* (next-neighbor) coupling of fields across the RG flow may be violated in the presence of un-damped excitations which perturb the "smooth" evolution of trajectories towards fixed points.
- The RG flow may display complex structure and non-trivial dynamics near fixed points, including *ergodic* behavior, *limit cycles* and *strange attractors* [1, 5-7, 12].

- Decoupling of long and short wavelength excitations (1) and (2) may no longer be possible when *out-of-equilibrium* conditions start to develop. Typical examples are RG flows with temporal memory (non-Markovian flows) or long-range spatial interactions that may surface in the mid to the deep TeV region of high-energy physics [13-14].
- The RG flow is neither *linear* nor *perturbative* in general [2].
- Under sizable deviations from four dimensions $\varepsilon = 4 - D$, $\varepsilon$ ~ O(1), the epsilon expansion advocated by Wilson's model leads to the emergence of *negative norm states* [3]. Likewise, Lorentz symmetry turns out to be ill-defined as a result of the intrinsic non-differentiability of fractal trajectories [9-10]. The condition $\varepsilon$ << 1, defined within the framework of the so-called *minimal fractal manifold* (MFM), is the only sensible setting where fractal geometry asymptotically approaches all consistency requirements mandated by the EFT and the SM [8-10].

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