URREF Self-Confidence in Information Fusion Trust

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Abstract — The Uncertainty Representation and Reasoning Evaluation Framework (URREF) includes an ontology that represents concepts and criteria needed to evaluate the uncertainty management aspects of a fusion system. The URREF ontology defines self-confidence as a measure of the information credibility as evaluated by the sensor itself. The concept of confidence, which is not explicitly defined in the ontology at URREF, has been extensively explored in the literature about evaluation in information fusion systems (IFS). In this paper, we provide a discussion on confidence as it relates to the evaluation of IFS, and compare it with the existing concepts in the URREF ontology. Our goal is two-fold, since we address both the distinctions between confidence and self-confidence, as well as the implications of these differences when evaluating the impact of uncertainty to the decision-making processes supported by the IFS. We illustrate the discussion with an example of decision making that involves signal detection theory, confusion matrix fusion, subjective logic, and proportional conflict redistribution. We argue that uncertainty can be minimized through confidence (information evidence) and self-confidence (source agent) processing. The results here seek to enrich the ongoing discussion at the ISIF’s Evaluation of Techniques for Uncertainty Representation Working Group (ETURWG) on self-confidence and trust in information fusion systems design.

Keywords: Self-Confidence, Confidence, Trust, Level 5 Fusion, High-Level Information Fusion, PCR5/6, Subjective Logic

I. INTRODUCTION

Information fusion aims to achieve uncertainty reduction through combining information from multiple complementary sources. The International Society of Information Fusion (ISIF) Evaluation of Techniques of Uncertainty Reasoning Working Group (ETURWG) was chartered to address the problem of evaluating fusion systems’ approaches to representing and reasoning with uncertainty. The working group developed the Uncertainty Representation and Reasoning Evaluation Framework (URREF) [1]. Discussions during 2013 explored the notions of credibility and reliability [2]. One recent issue is the difference between confidence and self-confidence as related to the data, source, and processing. While agreement is not complete among the ETURWG, this paper seeks to provide one possible approach to relate the mathematical, semantic, and theoretical challenges of confidence analysis.

The key position of the paper is to analyze the practical differences in evaluating the two concepts. More specifically, self-confidence is mostly relevant to HUMINT, which makes its evaluation a primarily subjective; whereas confidence can be easily traced to machine data analysis, allowing for the use of objective metrics in its evaluation. That is, a machine can process large amounts of data to represent the state of the world, and the evaluation of how well uncertainty is captured in these processes can be traced to various objective metrics. In contrast, for a human to assess its own confidence on the credibility of his “data collection process” (i.e. self-confidence), he or she has to make a judgment on limited choices. Objective assessment is determined from the credibility of the reports, processing, and decisions. Typical approaches include artificial intelligence (AI) methods (e.g., Neural Networks), pattern recognition (e.g., Bayesian, wavelets), and automatic target exploitation (i.e., over sensor, target, and environment operating conditions [3]). Subjective analysis is a report quality opinion that factors in analysis (e.g., completeness; accuracy, and veracity), knowledge (e.g., representation, uncertainty, and reasoning), and judgment (e.g., intuition, experience, decision making) [4]. In terms of IFS support for decision-making, numerous methods have been explored, mainly from Bayesian reasoning, Dempster-Shafer Theory [5], Subjective Logic [6], DSmT [7], fuzzy logic and possibility theory; although it also includes research on approximating belief functions to subjective probability measures (BetP [8], DSmP [9]).

Figure 1 provides a framework for our discussion. The world contains some truth $T$, of which data is provided from different sources ($A, B$). Source $A$ analysis goes to a machine agent for information fusion processing while source $B$ goes to a human agent. Either the machine or the human can generate beliefs about the state of the world (either using qualitative or quantitative semantics). The combination of $A$ and $B$ is a subject of Level 5 Fusion (user refinement) [10, 11].

![Figure 1 – Information Fusion System assessment of confidence (machine) and self-confidence (sensor or human agent).]
On one hand, confidence is typically related to machine processing such as signal detection theory where self-confidence is associated with sensors (and humans) assessing their own capability. On the other hand, the manipulation of the data requires understanding of the source, and self-confidence is applicable for the cases in which the user can provide a self-assessment on how confident he is on its data. It is important to emphasize that are not dealing (at least not directly) with information veracity: even if the sensor (e.g., a human reporting on an event or providing assessment on a situation) considers the information as possible, and he trusts it, it could be false at the end (e.g., even in summer time we can have a cloudy day). That is, self-confidence assesses how much the author trust the information, but not necessarily that this information is false or true [1]. For this paper, we take the URREF ontology definition of self-confidence as implied in Figure 1. The rationale for this choice is that self-confidence and uncertainty are typically associated with humans whereas confidence has been typically used in signal detection. Fusion of beliefs ultimately relates to states of the world with a reported confidence that can be compared to a truth state. Debating on the overlaps in terminology would be welcomed to clarify these positions for the ETURWG and the information community as a whole.

From Figure 1, we note the importance of confidence as related from a decision to the estimated states. Self-confidence is within the human agent assessing their understanding (e.g. experience) that can also be combined with the computer agent. The issue at hand for a user is whether or not the machine analysis (or their own) state decision represents reality. The notion of reality comes from the fact that currently, there are many technical products that perceive the world for the user (e.g., video) from which the user must map a mental model to a physical model of what is being represented. Some cases are easy such as video of cars moving on a road [12]; however, others are complex such as cyber networks [13]. The example used through the rest of the paper requires High-Level Information Fusion (HLIF) of target detection from a machine and human [14, 15].

In designing computer-aided detection machines, it is desirable to provide intelligence amplification (IA) [16] where Qualia motivates subjective analysis as a relation between the human consciousness/self-awareness to external stimuli. Qualia is the internal perception of the subjective aspect of the human’s perception of the stimuli. Knowing oneself can then be utilized to understand/evaluate the use of meaningful and relevant data in decision-making. The more that a sensor understands its Qualia [17], the better it will be in providing an assessment of its self-confidence in a report or on a decision. Qualia then encompasses an important component to uncertainty reasoning associated with subjective beliefs, trust, and self-confidence in decision making as a sense of intuition. Not surprisingly, these are natural discussion topics in Level 5 fusion (‘user refinement’), which includes operator-machine collaboration [18], situation awareness/assessment displays [19], and trust [20]. In order to explore self-confidence on these issues, we need to look at the psychology literature on trust as it relates to self-confidence.

From data available on the web (e.g., twitter, documents), intelligent users need the capability to rapidly monitor and analyze event information over massive amounts of unstructured textual data [21]. Text from human sources is subjected to opinions, beliefs, and misperceptions, generating various forms of self-assumed self-confidence. In contrast, computer sensed data can be stochastic or deterministic, from which we have to coordinate the agent information. For example, with Gaussian observations generates stochastic probability analyses (e.g., Kalman Filter). However, structural information in the sensor models and sensitivities for a given state condition (which come from a deterministic ontology) could be used to improve the estimate [22]. This combination of both stochastic and deterministic decisions with uncertainty elements is usual in modeling and system deployment, and understanding its key aspects is a fertile area for producing better decision support from IFS.

A related example from the analysis of uncertainty is evidence assessment from opinion makers. Dempster-Shafer theory has been used in connection with Bayesian analysis for decision making [5]. Likewise, Josang [23] demonstrated how subjective analysis within Dempster-Shafer theory could be used to determine the weight of opinions. Ontologies such as the one used in the URREF must be able to account for the uncertainty of data and to model it qualitatively, semantically, and quantitatively [24]. Metrics such as quality of service (QoS) and quality or information (IQ) are example of tools that can support and enhance a modeling capability between ontologies and uncertainty analysis [25]. The rest of this paper includes Sect. II as an overview of self-confidence. Sect. III discusses the mathematical analysis. Sect. IV highlights subjective logic for opinion making. Sect. V is an example and Sect. VI provides conclusions.

II. URREF NOTIONS OF SELF-CONFIDENCE

The ETURWG has explored many topics as related to a systems analysis of information fusion, which includes characteristics of uncertainty with many unknowns [26, 27]. In this paper, we categorize the characteristics of uncertainty into four areas, shown in Table 1. Assuming that the flow of information first goes from an agent to evidence beliefs, and subsequently to fusion with knowledge representation, then these areas help understand the terminology. Note that the defined information fusion quality of service (QoS) parameters are in blue {timeliness, accuracy, throughput, and confidence}. These could also be measures of performance [28]. For measures of effectiveness [25], one needs to understand system robustness (e.g., consistency, completeness, correctness, integrity). Here we focus on the red terms as related to self-confidence and confidence.

Knowledge representation in IFS [29, 30] includes applying decision-making semantics to support the structuring of extracted information. One example is the use of well defined concepts (e.g. confirmed, probable, possible, doubtful, and improbable) to support information extraction with natural language processing (NLP) algorithms. As related to confidence and self-confidence, there is the notion of integrity.
Integrity for human agents is associated with their subjective accountability and consistency in making judgments. Integrity for a machine could be objective in the faithful representation and validity on the data [31].

Algorithm performance focus on the information fusion method. URREF criteria for evaluating it relates to how the uncertainty model performs operations with information. An example of related metrics is to assess uncertainty reduction by weighting good data over bad given conflicting data.

Evidence: From [2], we explored the weight of evidence (WOE) as a function of reliability, credibility, relevance, and completeness. In URREF, WOE assesses how well an uncertainty representation technique captures the impact of an input affecting the processing and output of the IFS.

Source: Self-confidence, while yet to have a clear definition in the engineering literature, is typically associated with trust.

A. Trust
Closely associated with subjective analysis is trust [32]. Trust includes many attributes for man-machine systems such as dependability (machine), competence (user), and application [33]. Trust is then related to machine processing (confidence) and human assessment (self-confidence). Trust in automation is a key attribute associated with machine-driven solutions. Human trust in automation determines a user’s reliance on automation. In [32], they explored self-confidence defined as the user anticipatory (or post) performance with machines which impacts with trust in policy application.

Measuring trust as related to uncertainty is an open topic [34]. As a focus of discussion, we have a machine agent and a human agent of which a measure of trust comes from the uncertainty associated between the man-machine interactions. Reliability trust could be between human agents of which subjective probability is useful [35]. Decision trust could be between human agents or between a human and a machine and takes into account the risk associated with situation-dependent attitudes, attention, and workload of a human agent. The distinction between reliability and decision trust is important as related to self-confidence and confidence. This can be seen in Table 2, which depicts the main aspects for each of the six potential interactions between sensors..

Table 1: Characteristics of Uncertainty

<table>
<thead>
<tr>
<th>Agent Source</th>
<th>Evidence Information</th>
<th>Algorithm Fusion</th>
<th>Representation Knowledge Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Scalability</td>
</tr>
<tr>
<td>Objectivity</td>
<td>Relevance</td>
<td>Computational Cost</td>
<td>Knowledge Handling</td>
</tr>
<tr>
<td>Observational Sensitivity</td>
<td>Conclusiveness</td>
<td>Adaptability</td>
<td>Simplicity</td>
</tr>
<tr>
<td>Veracity (truthfulness)</td>
<td>Veracity (truth)</td>
<td>Traceability (pedigree)</td>
<td>Expressiveness</td>
</tr>
<tr>
<td>Secure</td>
<td>Ambiguity</td>
<td>Stability</td>
<td>Polarity</td>
</tr>
<tr>
<td>Resilient</td>
<td>Precision</td>
<td>Throughput</td>
<td>Genericity</td>
</tr>
<tr>
<td>Trust</td>
<td>Accuracy</td>
<td>Timeliness</td>
<td>Tense</td>
</tr>
<tr>
<td>Reliability</td>
<td>Credibility</td>
<td>Correctness</td>
<td>Completeness</td>
</tr>
<tr>
<td>Self-Confidence</td>
<td>Confidence</td>
<td>Consistency</td>
<td>Integrity</td>
</tr>
</tbody>
</table>

This table is presented to the ETURWG in this paper to support ongoing discussions on the categorization of types of uncertainty.

Table 2: Trust Aspects in Sensor Interactions

<table>
<thead>
<tr>
<th>Human</th>
<th>Others</th>
<th>Machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-confidence</td>
<td>Reliability</td>
<td>Trustworthy</td>
</tr>
<tr>
<td>Trust</td>
<td>Credibility</td>
<td>Confidence</td>
</tr>
</tbody>
</table>

- **Human**: Individuals must provide introspection on their own analysis and interaction with a machine. Here we distinguish between self-confidence and trust. In this case, human agents must have self-confidence in themselves as well as trust in the machine.
- **Others**: With the explosion of the Internet, recent work has explored the uncertainty of human sensing, such as Twitter reports in social networks, showing humans as less calibrated and reliable in their sensing. Wang et al. [36, 37] developed an estimation approach for truth discovery in this domain. Another recent example explored the decision-making trust between humans interfacing through a machine. The user interface was shown to have a strong impact on trust, cooperation, and situation awareness [38]. As an interesting result, credibility resulted as the computer interaction afforded complete and incomplete information towards understanding both the machine and the user analysis.
- **Machine**: A large body of literature is devoted to network trust. Examples include the hardware, cyber networks [39], protocols and policies. Given the large amount of cyber attacks written by hackers, it comes down to a trustworthy network of confidentiality, integrity, and availability. For machine-machine processing without user-created malware, network engineering analysis is mostly one of confidence. Machine trust is also important to enterprise systems [40].

Since we seek to understand self-confidence as a URREF criterion, explorations included human processing and the human as a data source as shown in Figure 3.

![Figure 2 – Methods of Trust.](image-url)
In the figure, multiple forms of trust are shown and related to the processing steps. Starting from the real world, data is placed on the network from which a machine (or sensor) processes the data. With context, a detection assessment is made for such domains as image, text, or cyber processing. That detection is the fused with the context information. For example, a detection of an object in an image is fused with contextual road information. The detection confidence is assessed and made available to the user with the context. The green line is the human-machine trust boundary as the human can look at the machine results or process the data themselves for given a level of confidence and render a decision. The dotted line then is a human assessment of whether or not the information presented represents reality and could be trusted.

Note, if the human is the only sensor source, then he/she is looking at data and making a decision. Their self-confidence could be based on the machine results from which they factor in many types of trust. For example, context, as related to the real world (see Figure 1), provides a validation of the machine (network to algorithm trust as a measure of confidence), while at the same time understanding the situation to determine if the information fusion analysis is providing meaningful and useful information towards the application of interest. Together a trusted decision is rendered based on the many factors.

Included in Figure 3 are many forms of trust in the analysis all of which can lead to confidence in the decision:

<table>
<thead>
<tr>
<th>Trust</th>
<th>Processing</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Network</td>
<td>Data put on a network</td>
<td>Assessment of data timeliness and lost packets</td>
</tr>
<tr>
<td>Machine</td>
<td>Sensor transformation</td>
<td>Calibration of cameras for image content</td>
</tr>
<tr>
<td>Software</td>
<td>Information management</td>
<td>Getting the correct data from a data base (e.g., a priori data)</td>
</tr>
<tr>
<td>Algorithm</td>
<td>Fusion method</td>
<td>Target tracking and classification results</td>
</tr>
<tr>
<td>Modeling</td>
<td>State models</td>
<td>Kinematic and target recognition models (e.g., training data)</td>
</tr>
<tr>
<td>Application</td>
<td>Situation of interest</td>
<td>Analysis over the correct area (e.g., target moving on a road)</td>
</tr>
<tr>
<td>User</td>
<td>Situation awareness</td>
<td>Use of cultural and behavior (e.g., assume big cars move on roads)</td>
</tr>
</tbody>
</table>

The self-confidence of the user analysis includes working with data, networks, and machines. The URREF ontology must account for trust over human-machine decisions for confidence analysis. To further explore how URREF is aligned with trust, we must look at self-confidence.

B. Self-Confidence

Statistically speaking, the machine decision-making accuracy is based on the data available, the model chosen, and the estimation uncertainty associated with the measured data. Given the above analysis, we could start to derive self-confidence for machine fusion operations based on the literature in human self-confidence.

Self-confidence is the socio-psychological concept related to self-assuredness in one's personal judgment and ability. As an example, researchers are often called to review papers and after their review asked to give a quality rating of their own review based on their understanding of the subject, expertise, and experience. In another example, a person might be asked to identify an object in an image with a certain rating {unlikely, possible, probable, confirm} from which they could determine self-confidence based on their answer. Thus, there is a need to assess “self-confidence” in relation to “confidence”, which is linked to uncertainty measures of trust.

C. Accuracy and Precision

Self confidence is strongly related to both precision and accuracy. A source can be self confident in both the precision of its generated data (consistency or variability in its reports - such as reported variance) as well as the accuracy of its reports (the reported bias or the reported distance of the mean value of the generated data form true value). In other words, to make sense of the term the self-confidence of a source, the data encapsulate a combination of precision and accuracy. A distinction is made between precision and accuracy reported by the machine (such as the estimated mean and variance at the output of the Kalman filter) and the actual precision and accuracy of the data emanating from the source. The URREF ontology categorizes accuracy, precision, and self-confidence as types of criteria to evaluate data [1].

Statistical methods of uncertainty analysis from measurement systems include accuracy and precision, shown in Figure 4. The use of distance metrics (accuracy) and precision metrics (standard deviations) help to analyze whether the measurement is calibrated and repeatable. We would desire the same analysis for human semantic analysis with precise meanings, consistent understanding, and accurate terminology.

Figure 3 – Uncertainty as a function of accuracy and precision.

Human Confidence-Accuracy: Traditionally known as the confidence-accuracy (CA) relationship, the assumption is that as one’s confidence increases so does their level of accuracy which is affected by memory, consistency, and ability [41]. Issues include absolute versus relative assessment, feedback, and performance.

The confidence-accuracy relationship was shown to be a by-product of the consistency-correctness relationship: It is positive because the answers that are consistently chosen are generally correct, but negative when the wrong answers tend to be favored. The overconfidence bias stems from the reliability-validity discrepancy: Confidence monitors reliability (or self-consistency), but its accuracy is evaluated in calibration studies against correctness. Also, the response
speed is a frugal cue for self-consistency and depends on the validity of self-consistency in predicting performance [42].

Koriat [42] explains that Sensing tasks are dominated by Thurstonian uncertainty (local rank ordering with stochastic noise) within an individual and exhibit an under-confidence bias. However, general knowledge tasks are dominated by Brunswikian Uncertainty (global probabilistic model from limited sample sets to infer general knowledge [43]) that supports inter-person ecological relations.

Consistency is then the repeatability of the information, which should imply no conflicts in decision-making. We can use the proportional conflict redistribution (PCR6) to get a measure of repeated consistency such that favored wrong answers are corrected in confidence analysis [44]. PCR6 is more general and efficient than PCR5 when combining more than two sources altogether. Moreover, PCR6 has been proved compatible with frequency probabilities when working with binary BBA’s, whereas PCR5 and DS are not compatible with frequency probabilities [44].

Self-confidence could be measured with a Receiver Operating Characteristic (ROC) curve as once a decision can be made, we can then assess its impact on confidence. A low self-confidence could be measured with a Receiver Operating Characteristic (ROC) curve as once a decision can be made, and efficient than PCR5 when combining more than two sources altogether. Moreover, PCR6 has been proved compatible with frequency probabilities when working with binary BBA’s, whereas PCR5 and DS are not compatible with frequency probabilities [44].

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It is noted that a choice in time is not just the product of the current analysis, but the accumulated evidence. For example, in Figure 5, we see that the signal is moving between semantic boundaries from doubtful to probable evidence. Given the history, then the decision maker could be self-confident in the current measurement given their perception of the entire processing of machine decision making measures for each time.

A Piercian hypothesis [48] implies confidence is a multiplicative function of the quantity of the information needed to make a decision ($\theta$, or the distance traveled by the diffusion process) and the quality of the information ($\delta$, or the rate of evidence accumulation in the diffusion process) accumulated in Dynamic Signal Detection [48]. Without bias, the authors of [48] show that:

$$\text{conf}(\text{self}) = \beta \cdot \frac{1}{2} \ln \left[ \frac{P(R_{H,j} | S_{H,j})}{P(R_{H} | S_{H,j})} \right] = \frac{\delta \theta}{\sigma^2}$$  \hspace{1cm} (4)

where $\beta$ is a scaling parameter. A decision, $\theta$, is related to a response ($R$) of detection to a stimuli ($S$). Given the ability to model self-confidence as a measure of precision, we extend the methodology using subjective-logic and DSmT [44] for robust decision making.

### III. SELF-CONFIDENCE

Signal detection theory provides a measure of confidence in decision making that by assuming a limited hypothesis set is actually a measure of self-confidence. One classic example is Wald’s Sequential Probability Ratio Test (SPRT) [45]. Assuming evidence is sampled at discrete time intervals, then the human or cognitive agent compares the conditional probabilities $x(t + \Delta t)$ for two hypothesis $H_j$ ($j = 1, 2$). Using then SPRT, then

$$y(t) = h[x(t)] = \ln \left[ \frac{f_1[x(t)]}{f_2[x(t)]} \right]$$  \hspace{1cm} (1)

If $y(t) > 0$, then evidence supports $H_1$, and if $y(t) < 0$, then $H_2$ is more likely. As time accumulates for decision making, there is an aggregation of the log likelihood ratios:

$$L(t + \Delta t) = L(t) + \ln \left[ \frac{f_1[x(t + \Delta t)]}{f_2[x(t + \Delta t)]} \right]$$  \hspace{1cm} (2)

where, for a stochastic system $L(t) \sim \mathcal{N}(y(t), \sigma^2(t))$. Eq (2) can be written in Bayesian log odds:

$$\ln \left[ \frac{p(H_1 | D)}{p(H_2 | D)} \right] = \ln \left[ \frac{f_1[x(t)]}{f_2[x(t)]} \right] + \ln \left[ \frac{p(H_1)}{p(H_2)} \right]$$  \hspace{1cm} (3)

One then collects information to make a decision such that $-\theta_1 < L(t) < \theta_1$. The chosen threshold is then a measure of a decision, which can be conservative or aggressive for the case of a human agent [46]. Figure 5 shows the case in which evidence is accumulated and a decision is made with associated standard boundaries for semantic decision making. Also in Figure 5 we related decision boundaries for semantic confidence classification [47].

### IV. SUBJECTIVE OPINIONS

Subjective opinions [49] are special cases of belief functions as they correspond to bba defined on 2D frames of type $\Theta = \{A, \neg A\}$ assuming Shafer’s model or DSmT. Subject opinions lend themselves to simple mathematical expressions of fusion models. We therefore use the opinion representation for describing the various fusion models, but the expressions can easily be mapped to traditional belief functions.

A subjective opinion expresses belief about statements in a frame. Let $X$ be a frame of cardinality $\kappa$. An opinion distributes belief mass over the reduced powerset $R(X)$ of cardinality $\kappa$. The reduced powerset $R(X)$ is defined as:

$$R(X) = P(X) \setminus \{X, \emptyset\}.$$  \hspace{1cm} (5)

where $P(X) = 2^X$ denotes the powerset of $X$. All proper subsets of $X$ are elements of $R(X)$, but the frame $\{X\}$ and empty set $\{\emptyset\}$ are not elements of $R(X)$. 

![Figure 4 – Evidence Accumulation for Decision Confidence.](image-url)
Let $\overrightarrow{b}_X$ be a belief vector over the elements of $R(X)$, $u_X$ be the complementary uncertainty mass, and $\overrightarrow{a}$ be a base rate vector over $X$. Whenever relevant, a superscript such as $A$ denotes the opinion owner. Then a subjective opinion $\omega^A_X$ is the composite function expressed as:

$$\omega^A_X = (\overrightarrow{b}_X, u_X, \overrightarrow{a}_X).$$  \hfill (6)

The attribute $A$ is thus the belief source, and $X$ is the target frame. The belief, uncertainty and base rate parameters satisfy the following additivity constraints.

- **Belief additivity:**
  $$u_X + \sum_{x_i \in R(X)} \overrightarrow{b}_X (x_i) = 1, \quad \text{where} \ x \in R(X)$$  \hfill (7)

- **Base rate additivity:**
  $$\sum_{i=1}^{k} \overrightarrow{a}_X (x_i) = 1, \quad \text{where} \ x \in X$$  \hfill (8)

The belief vector $\overrightarrow{b}_X$ has $k = (2^k - 2)$ parameters, whereas the base rate vector $\overrightarrow{a}_X$ only has $k$ parameters. The uncertainty parameter $u_X$ is a simple scalar. A general opinion thus contains $2k$ parameters. However, given that Eq.(7) and Eq.(8) remove one degree of freedom each, opinions over a frame of cardinality $k$ only have $(2^k + k - 3)$ degrees of freedom. The probability projection of hyper opinions is the vector denoted as $\overrightarrow{E}_X$:

$$\overrightarrow{E}_X = \sum_{x_i \in R(X)} \overrightarrow{a}_X (x_i \mid x_j) \overrightarrow{b}_X (x_i) + \overrightarrow{a}_X (x_i) u_X, \ \forall x_i \in R(X)$$  \hfill (9)

where $\overrightarrow{a}_X (x_i \mid x_j) = \frac{\overrightarrow{a}_X (x_i \cap x_j)}{\overrightarrow{a}_X (x_i)}$, $\forall x_i, x_j \subset X$.  \hfill (10)

denotes relative base rate, i.e. the base rate of subset $x_i$ relative to the base rate of (partially) overlapping subset $x_j$.

General opinions are also called hyper opinions. A multinomial opinion is when belief mass only applies to singleton statements in the frame. A binomial opinion is when the frame is binary. A dogmatic opinion is an opinion without uncertainty, i.e. where $u = 0$. A vacuous opinion is an opinion that only contains uncertainty, i.e. where $u = 1$. Likewise, we can make the case that confidence in the opinion is biased by a subjective opinion of the source self-confidence. Thus, self-confidence is $SC_U = 1$ owing to rank-order decision-making on a subset of the world, and the lack of self-confidence is $SC_U = 0$; where:

$$SC_U(\omega^A_X) = \overrightarrow{a}_X [1 - u_X]$$  \hfill (11)

and

$$\omega^A_X \leftarrow SC_U(\omega^A_X) \cdot \overrightarrow{b}_X$$  \hfill (12)

Equivalent probabilistic representations of opinions, e.g. as a Beta pdf (probability density function) in case of binomial opinions, as a Dirichlet pdf in case of multinomial opinions, or as a hyper Dirichlet pdf in case of hyper opinions offer an alternative interpretation of subjective opinions in terms of traditional statistics [6].

**Cumulative Fusion:**

The cumulative fusion rule is equivalent to a posteriori updating of Dirichlet distributions. Its derivation is based on the bijective mapping between the belief and evidence notations described in [6].

Let $\omega^A$ and $\omega^B$ be opinions respectively held by agents $A$ and $B$ over the same frame $X$ of cardinality $k$ with reduced powerset $R(X)$ of cardinality $\kappa$. Let $\omega^{A \oplus B}$ be the opinion where:

**CASE I:** For $u^A \neq 0$ and $u^B \neq 0$ (with Confidence)

$$\left\{
\begin{array}{l}
\overrightarrow{b}^{A \oplus B}(x_i) = \frac{\overrightarrow{b}^A(x_i) u^B + \overrightarrow{b}^B(x_i) u^A}{u^A + u^B - u^A u^B} \\
u^{A \oplus B} = \frac{u^A u^B}{u^A + u^B - u^A u^B}
\end{array}
\right.$$  \hfill (13)

**CASE II:** For $u^A = 0$ or $u^B = 0$ (without Confidence)

$$\left\{
\begin{array}{l}
\overrightarrow{b}^{A \oplus B}(x_i) = \gamma^A \overrightarrow{b}^A(x_i) + \gamma^B \overrightarrow{b}^B(x_i) \\
u^{A \oplus B} = 0
\end{array}
\right.$$  \hfill (14)

where:

$$\gamma^A = \lim_{u^A \to 0; u^B \to 0} \frac{u^B}{u^A + u^B}$$

$$\gamma^B = \lim_{u^A \to 0; u^B \to 0} \frac{u^A}{u^A + u^B}$$

Note: the case without confidence averages the results from self-confidence reports which weights effectively both the same. Confidence allows the user to weight the self-confidence of the reports based on the Brunswikian uncertainty about the world knowledge.

Then $\omega^{A \oplus B}$ is the cumulatively fused opinion of $\omega^A$ and $\omega^B$, representing the combination of independent opinions of $A$ and $B$. By using the symbol $\oplus$ to designate this belief operator, cumulative fusion is expressed as:

**Cumulative Belief Fusion:** $\omega^{A \oplus B}_X = \omega^A_X \oplus \omega^B_X$  \hfill (15)

The cumulative fusion operator is commutative, associative and non-idempotent. In Eq.(15), the associativity depends on the preservation of relative weights of intermediate results through the weight variable $\gamma$, in which case the cumulative rule is equivalent to the weighted average of probabilities.

**V. Results**

Assume we have two agent opinion makers $\omega^A$ and $\omega^B$, who each make a decision for network security [50]. Let $\omega^A$ be a machine Algorithm and let $\omega^B$ come from a human Being. After reporting their opinion, $\omega^A$ is asked for their self-confidence. The result modifies their belief $\overrightarrow{b}_X$, such that the cumulative belief fusion product is a weighted function of
For many situations, a machine can process large amounts of data, while a human agent can only comprehend a subset of the data. Thus, a machine processes the data as outputs to a user. The interaction with the user is continually updated and a decision from the user is required. For situations in which the user has more time (forensics), then his/her self-confidence in the data would be high. For quick decisions, an observe-orient-decide-act (OODA) decision might be required [51] which reduces self-confidence. We seek methods of the latter as uncertainty is higher in rapid decision making which is a subset of problems in the Dynamic Data-Driven Application Systems (DDDAS) paradigm [52, 53].

For the analysis, we have two opinion makers (machine and man). Using signal detection theory, their individual measures of analysis provide a likelihood function. We then fuse the results with confusion matrix fusion [54] as a method of combination using Bayesian, Dempster-Shafer, or DSmT results [55]. We utilize two cases in which there is a high and low-confident observer (Case 1) and then the situation in which both have comparable analysis (Case 2). With two highly self-confident observers (Case 2), the results are similar to one of the observers which could be used for opinion validity. However, the user could be looking at the results and further analyzing context to provide a more appropriate analysis of their decision (e.g., based on culture, data completeness, etc). Using subjective logic the human being could modify their opinion, \( \omega_B^{SC} \), which results in a larger value (e.g., know something) or lower value (e.g., recognize limitation of analysis).

We assume that if the user provides no assessment of self-confidence, we provide equal weight to the results (average fusion). On the other hand, if a machine provides a measure of confidence, it could be derived from the dynamic-data, which we don’t simulate here.

Example (High self-confidence with low self-confidence) Assume that we have a highly self-confident opinion maker, \( \omega_A \), that includes many sources and reliable analysis. On the other hand, we have a low-confident opinion maker, \( \omega_B \), who is making a decision. When making their decision, \( \omega_B \) is guessing or almost chance, assuming that context provides pragmatic understanding of the world events.

In Figure 7, there are two opinion makers, the red curve of a human agent suggesting that the result is “improbable,” while the more self-confident is in blue reporting “probable”. The fused result, shown in green, using self-confidence better reflects the true state; versus the average fusion of the opinion makers shown in magenta. The key issue is that self-confidence can help weight evidence.

VI. CONCLUSIONS

In this paper, we assessed self-confidence as a criterion in the URREF. Self-confidence is typically associated with a source and relates a subjective quality on the rendering of their beliefs over data. For stochastic observations, we use the SPRT in a self-confidence analysis. However, to get the case of partial information, we use subjective logic for decision-makers. We demonstrated that the PCR6 is superior to DS for decision for a scenario in which a high self-confident observer opinion is fused with a low self-confident observer. Ultimately it is the user trust in the data they have available and opinions towards self-confidence; whereas a machine only reports confidence.

Further directions include using the analysis with real operators doing intelligence analysis over data and associating semantic boundaries to their subjective decision-making.

In the scenario, we used sequential fusion of two sources and because of this, PCR5=PCR6, i.e. when combining 2 sources only PCR5 coincides with PCR6.