Similarity measure between possibility neutrosophic soft sets and its applications

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Abstract. In this paper, a similarity measure between possibility neutrosophic soft sets (PNS-set) is defined, and its properties are studied. A decision making method is established based on proposed similarity measure. Finally, an application of this similarity measure involving the real life problem is given.

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1. INTRODUCTION

The concept of soft sets was proposed by Molodtsov [14] as a mathematical tool for dealing with uncertainty in 1999. Maji et al. [15, 16] applied soft set theory to decision making problem in 2003 and they introduced some new operations between soft sets. After Maji’s work, studies on soft set theory and its applications have been progressing rapidly. see [1, 7, 8, 11, 19]. Neutrosophic set was defined by Samarandache [18], as a new mathematical tool for dealing with problems involving incomplete, indeterminant, inconsistent knowledge. In 2013, Maji [17] introduced concept of neutrosophic soft set and some operations of neutrosophic soft sets. Karaaslan [12] redefined concept and operations of neutrosophic soft sets different from Maji’s neutrosophic soft set definition and operations. Recently, the properties and applications on the neutrosophic soft sets have been studied increasingly [3, 4, 5, 9, 10, 20].

Possibility fuzzy soft sets and operations defined on these sets were firstly introduced by Alkhazaleh et al [2]. In 2012, concept of possibility intuitionistic fuzzy soft set and its operations were defined by Bashir et al. [6]. Also, Bashir et al. [6] discussed similarity measure of two possibility intuitionistic fuzzy soft sets. They
also gave an application of this similarity measure. In 2014, concept of possibility neutrosophic soft set and its operations were defined by Karaaslan [13].

In this study, after giving some definition related to the possibility neutrosophic soft sets (PNS-set), we define a similarity measure between two PNS-sets. We finally an application of this similarity measure is given to fill an empty position with an appropriate person in a firm.

2. Preliminary

In this section, we recall some required definitions related to the PNS-sets [13].

Throughout paper $U$ is an initial universe, $E$ is a set of parameters and $\Lambda$ is an index set.

Definition 2.1. [12] A neutrosophic soft set (or namely ns-set) $f$ over $U$ is a neutrosophic set valued function from $E$ to $\mathcal{N}(U)$. It can be written as

$$f = \left\{ \left( e, \left\{ (u, t_{f(e)}(u), i_{f(e)}(u), f_{f(e)}(u)) : u \in U \right\} \right) : e \in E \right\}$$

where, $\mathcal{N}(U)$ denotes set of all neutrosophic sets over $U$. Note that if $f(e) = \left\{ (u, 0, 1, 1) : u \in U \right\}$, the element $(e, f(e))$ is not appeared in the neutrosophic soft set $f$. Set of all ns-sets over $U$ is denoted by $\mathcal{NS}$.

Definition 2.2. [13] Let $U$ be an initial universe, $E$ be a parameter set, $\mathcal{N}(U)$ be the collection of all neutrosophic sets of $U$ and $I^U$ is collection of all fuzzy subset of $U$. A possibility neutrosophic soft set (PNS-set) $f_p$ over $U$ is defined by the set of ordered pairs

$$f_p = \left\{ \left( e_i, \left\{ \left( \frac{u_j}{f_p(e_i)(u_j)}, \mu(e_i)(u_j) \right) : u_j \in U \right\} : e_i \in E \right\}$$

where, $i, j \in \Lambda$, $f$ is a mapping given by $f : E \rightarrow \mathcal{N}(U)$ and $\mu(e_i)$ is a fuzzy set such that $\mu : E \rightarrow I^V$. Here, $f_p$ is a mapping defined by $f_p : E \rightarrow \mathcal{N}(U) \times I^U$.

For each parameter $e_i \in E$, $f(e_i) = \left\{ (u_j, t_{f(e_i)}(u_j), i_{f(e_i)}(u_j), f_{f(e_i)}(u_j)) : u_j \in U \right\}$ indicates neutrosophic value set of parameter $e_i$ and where $t, i, f : U \rightarrow [0, 1]$ are the membership functions of truth, indeterminacy and falsity respectively of the element $u_j \in U$. For each $u_j \in U$ and $e_i \in E$, $0 \leq t_{f(e_i)}(u_j) + i_{f(e_i)}(u_j) + f_{f(e_i)}(u_j) \leq 3$. Also $\mu(e_i)$, degrees of possibility of belongingness of elements of $U$ in $f(e_i)$. So we can write

$$f_p(e_i) = \left\{ \left( \frac{u_1}{f_p(e_i)(u_1)}, \mu(e_i)(u_1) \right), \left( \frac{u_2}{f_p(e_i)(u_2)}, \mu(e_i)(u_2) \right), ..., \left( \frac{u_n}{f_p(e_i)(u_n)}, \mu(e_i)(u_n) \right) \right\}$$

From now on, we will show set of all possibility neutrosophic soft sets over $U$ with $\mathcal{PS}(U,E)$ such that $E$ is parameter set.

Example 2.3. Let $U = \{u_1, u_2, u_3\}$ be a set of three cars. Let $E = \{e_1, e_2, e_3\}$ be a set of qualities where $e_1 =$cheap, $e_2 =$equipment, $e_3 =$fuel consumption and let $\mu : E \rightarrow I^U$. We define a function $f_p : E \rightarrow \mathcal{N}(U) \times I^U$ as follows:

$$f_p = \begin{cases} f_p(e_1) = \left\{ \left( \frac{u_1}{(0,0,0,0,0)}, 0.8 \right), \left( \frac{u_2}{(0,0,0,0,0)}, 0.4 \right), \left( \frac{u_3}{(0,0,0,0,0)}, 0.7 \right) \right\} \\
 f_p(e_2) = \left\{ \left( \frac{u_1}{(0,0,0,0,0)}, 0.6 \right), \left( \frac{u_2}{(0,0,0,0,0)}, 0.7 \right), \left( \frac{u_3}{(0,0,0,0,0)}, 0.6 \right) \right\} \\
 f_p(e_3) = \left\{ \left( \frac{u_1}{(0,0,0,0,0)}, 0.2 \right), \left( \frac{u_2}{(0,0,0,0,0)}, 0.6 \right), \left( \frac{u_3}{(0,0,0,0,0)}, 0.5 \right) \right\} \end{cases}$$
also we can define a function \( g_v : E \rightarrow \mathcal{N}(U) \times I^U \) as follows:

\[
g_v = \left\{ \begin{array}{ll}
g_v(e_1) = \left\{ \frac{0.6}{0.6,0,0.6}, \frac{0.8}{0.6,0,0.6} \right\},
g_v(e_2) = \left\{ \frac{0.5}{0.5,0.5,0.5}, \frac{0.7}{0.5,0.5,0.5} \right\},
g_v(e_3) = \left\{ \frac{0.1}{0.1,0.1,0.1}, \frac{0.8}{0.1,0.1,0.1} \right\}
\end{array} \right.
\]

For the purpose of storing a possibility neutrosophic soft set in a computer, we can use matrix notation of possibility neutrosophic soft set \( f_\mu \). For example, matrix notation of possibility neutrosophic soft set \( f_\mu \) can be written as follows: for \( m, n \in \Lambda, \)

\[
f_\mu = \begin{pmatrix}
((0.5,0.2,0.6),0.8) & ((0.7,0.3,0.5),0.4) & ((0.4,0.5,0.8),0.7) \\
((0.8,0.4,0.5),0.6) & ((0.5,0.7,0.2),0.8) & ((0.7,0.3,0.9),0.4) \\
((0.6,0.7,0.5),0.2) & ((0.5,0.3,0.7),0.6) & ((0.6,0.5,0.4),0.5)
\end{pmatrix}
\]

where the \( m \)--th row vector shows \( f(e_m) \) and \( n \)--th column vector shows \( u_n \).

**Definition 2.4.** [13] Let \( f_\mu \in P\mathcal{N}(U) \), where \( f_\mu(e_i) = \{(f(e_i)(u_j), \mu(e_i)(u_j)) : e_i \in E, u_j \in U \} \) and \( f(e_i) = \{(u, t_{f(e_i)}(u_j), i_{f(e_i)}(u_j), f_{f(e_i)}(u_j))\} \) for all \( e_i \in E, u \in U \). Then for \( e_i \in E \) and \( u_j \in U, \)

1. \( f^t_\mu \) is said to be truth-membership part of \( f_\mu \),
   \[ f^t_\mu = \{(f^t_{ij}(e_i), \mu_{ij}(e_i))\} \] and \( f^t_{ij}(e_i) = \{(u_j, t_{f(e_i)}(u_j))\} \)

2. \( f^i_\mu \) is said to be indeterminacy-membership part of \( f_\mu \),
   \[ f^i_\mu = \{(f^i_{ij}(e_i), \mu_{ij}(e_i))\} \] and \( f^i_{ij}(e_i) = \{(u_j, i_{f(e_i)}(u_j))\} \)

3. \( f^l_\mu \) is said to be truth-membership part of \( f_\mu \),
   \[ f^l_\mu = \{(f^l_{ij}(e_i), \mu_{ij}(e_i))\} \] and \( f^l_{ij}(e_i) = \{(u_j, f_{f(e_i)}(u_j))\} \)

We can write a possibility neutrosophic soft set in form \( f_\mu = (f^t_\mu, f^i_\mu, f^l_\mu) \).

3. Similarity measure of possibility neutrosophic soft sets

In this section, we introduce a measure of similarity between two \( PNS \)-sets.

**Definition 3.1.** Similarity between two \( PNS \)-sets \( f_\mu \) and \( g_v \), denoted by \( S(f_\mu, g_v) \), is defined as follows:

\[
S(f_\mu, g_v) = M(f(e), g(e))M(\mu(e), \nu(e))
\]

such that

\[
M(f(e), g(e)) = \frac{1}{n} \sum_{i=1}^{n} M_i(f(e), g(e)),
\]

\[
M(\mu(e), \nu(e)) = \frac{1}{n} \sum_{i=1}^{n} M(\mu(e_i), \nu(e_i)),
\]

where

\[
M_i(f(e), g(e)) = 1 - \frac{1}{\sqrt{n}} \sqrt{\sum_{i=1}^{n} (\phi_{f_\mu(e_i)}(u_j) - \phi_{g_v(e_i)}(u_j))^p}, 1 \leq p \leq \infty
\]

\[3\]
such that and

\[ \phi_{f_{\mu}(e_i)}(u_j) = \frac{f_{ij}^\mu(e_i) + f_{ij}^v(e_i) + f_{ij}^\nu(e_i)}{3}, \quad \phi_{g_{\mu}(e_i)}(u_j) = \frac{g_{ij}^\mu(e_i) + g_{ij}^v(e_i) + g_{ij}^\nu(e_i)}{3}, \]

\[ M(\mu(e_i), \nu(e_i)) = 1 - \frac{\sum_{j=1}^{n} |\mu_{ij}(e_i) - \nu_{ij}(e_i)|}{\sum_{j=1}^{n} |\mu_{ij}(e_i) + \nu_{ij}(e_i)|} \]

**Definition 3.2.** Let \( f_{\mu} \) and \( g_{\nu} \) be two PNS-sets over \( U \). We say that \( f_{\mu} \) and \( g_{\nu} \) are significantly similar if \( S(f_{\mu}, g_{\nu}) \geq \frac{1}{2} \)

**Proposition 3.3.** Let \( f_{\mu}, g_{\nu} \in PN(U, E) \). Then,

1. \( S(f_{\mu}, g_{\nu}) = S(g_{\nu}, f_{\mu}) \)
2. \( 0 \leq S(f_{\mu}, g_{\nu}) \leq 1 \)
3. \( f_{\mu} = g_{\nu} \Rightarrow S(f_{\mu}, g_{\nu}) = 1 \)

**Proof.** The proof is straightforward and follows from Definition 3.1. \( \square \)

**Example 3.4.** Let us consider PNS-sets \( f_{\mu} \) and \( g_{\nu} \) in Example 2.3 given as follows:

\[
\begin{align*}
\{ f_{\mu}(e_1) & = \left\{ \left[ \begin{array}{c} \mu_1 \\
\mu_1 \\
\mu_1 \end{array} \right] \right\} \} \\
\{ f_{\mu}(e_2) & = \left\{ \left[ \begin{array}{c} \mu_2 \\
\mu_2 \\
\mu_2 \end{array} \right] \right\} \} \\
\{ f_{\mu}(e_3) & = \left\{ \left[ \begin{array}{c} \mu_3 \\
\mu_3 \\
\mu_3 \end{array} \right] \right\} \}
\end{align*}
\]

and

\[
\begin{align*}
\{ g_{\nu}(e_1) & = \left\{ \left[ \begin{array}{c} \nu_1 \\
\nu_1 \\
\nu_1 \end{array} \right] \right\} \} \\
\{ g_{\nu}(e_2) & = \left\{ \left[ \begin{array}{c} \nu_2 \\
\nu_2 \\
\nu_2 \end{array} \right] \right\} \} \\
\{ g_{\nu}(e_3) & = \left\{ \left[ \begin{array}{c} \nu_3 \\
\nu_3 \\
\nu_3 \end{array} \right] \right\} \}
\end{align*}
\]

then,

\[
M(\mu(e_1), \nu(e_1)) = 1 - \frac{\sum_{j=1}^{3} |\mu_{ij}(e_1) - \nu_{ij}(e_1)|}{\sum_{j=1}^{3} |\mu_{ij}(e_1) + \nu_{ij}(e_1)|}
= 1 - \frac{|0.8 - 0.4| + |0.4 - 0.7| + |0.7 - 0.8|}{|0.8 + 0.4| + |0.4 + 0.7| + |0.7 + 0.8|} = 0.79
\]

Similarly we get \( M(\mu(e_2), \nu(e_2)) = 0.74 \) and \( M(\mu(e_3), \nu(e_3)) = 0.75 \), then

\[
M(\mu, \nu) = \frac{1}{3} (0.79 + 0.75 + 0.74) = 0.76
\]
\[
M_1(f(e), g(e)) = 1 - \frac{1}{\sqrt{n}} \sqrt{\sum_{i=1}^{n} (\phi_{f,\mu}(e_i) - \phi_{g,\nu}(e_i))^2}
\]
\[
= 1 - \frac{1}{\sqrt{3}} \sqrt{(0.43 - 0.57)^2 + (0.50 - 0.53)^2 + (0.57 - 0.40)^2} = 0.73
\]
\[
M_2(f(e), g(e)) = 0.86
\]
\[
M_3(f(e), g(e)) = 0.94
\]
\[
M(f(e), g(e)) = \frac{1}{3}(0.73 + 0.86 + 0.94) = 0.84
\]
and
\[
S(f_\mu, g_\nu) = 0.84 \times 0.76 = 0.64
\]

4. Decision-making method based on the similarity measure

In this section, we give a decision making problem involving possibility neutrosophic soft sets by means of the similarity measure between the possibility neutrosophic soft sets.

Let our universal set contain only two elements "yes" and "no", that is \( U = y, n \). Assume that \( P = \{p_1, p_2, p_3, p_4, p_5\} \) are five candidates who fill in a form in order to apply formally for the position. There is a decision maker committee. They want to interview the candidates by model possibility neutrosophic soft set determined by committee. So they want to test similarity of each of candidate to model possibility neutrosophic soft set.

Let \( E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\} \) be the set of parameters, where \( e_1=\text{experience} \), \( e_2=\text{computer knowledge} \), \( e_3=\text{training} \), \( e_4=\text{young age} \), \( e_5=\text{higher education} \), \( e_6=\text{marriage status} \) and \( e_7=\text{good health} \).

Our model possibility neutrosophic soft set determined by committee for suitable candidates properties \( f_\mu \) is given in Table 1.

<table>
<thead>
<tr>
<th>( f_\mu )</th>
<th>( e_1, \mu )</th>
<th>( e_2, \mu )</th>
<th>( e_3, \mu )</th>
<th>( e_4, \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>((1, 0, 0), 1)</td>
<td>((1, 0, 0), 1)</td>
<td>((0, 1, 1), 1)</td>
<td>((0, 1, 1), 1)</td>
</tr>
<tr>
<td>( n )</td>
<td>((0, 1, 1), 1)</td>
<td>((1, 0, 0), 1)</td>
<td>((0, 1, 1), 1)</td>
<td>((1, 0, 0), 1)</td>
</tr>
</tbody>
</table>

Table 1: The tabular representation of model possibility neutrosophic soft set

<table>
<thead>
<tr>
<th>( g_\nu )</th>
<th>( e_1, \nu )</th>
<th>( e_2, \nu )</th>
<th>( e_3, \nu )</th>
<th>( e_4, \nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>((0.7, 0.2, 0.5), 0.4)</td>
<td>((0.5, 0.4, 0.6), 0.2)</td>
<td>((0.2, 0.3, 0.4), 0.5)</td>
<td>((0.8, 0.4, 0.6), 0.3)</td>
</tr>
<tr>
<td>( n )</td>
<td>((0.3, 0.7, 0.1), 0.3)</td>
<td>((0.7, 0.3, 0.5), 0.4)</td>
<td>((0.6, 0.5, 0.3), 0.2)</td>
<td>((0.2, 0.1, 0.5), 0.4)</td>
</tr>
</tbody>
</table>

Table 2: The tabular representation of possibility neutrosophic soft set for \( p_1 \)
In future, these seem to have natural applications as image encryption.

Table 3: The tabular representation of possibility neutrosophic soft set for $p_2$

<table>
<thead>
<tr>
<th>$h_{\delta}$</th>
<th>$e_1, \delta$</th>
<th>$e_2, \delta$</th>
<th>$e_3, \delta$</th>
<th>$e_4, \delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>(0.8, 0.2, 0.1, 0.3)</td>
<td>(0.4, 0.2, 0.6, 0.1)</td>
<td>(0.7, 0.2, 0.4, 0.2)</td>
<td>(0.3, 0.2, 0.7, 0.6)</td>
</tr>
<tr>
<td>$n$</td>
<td>(0.2, 0.4, 0.3, 0.5)</td>
<td>(0.6, 0.3, 0.2, 0.3)</td>
<td>(0.4, 0.3, 0.2, 0.1)</td>
<td>(0.8, 0.1, 0.3, 0.3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$h_{\delta}$</th>
<th>$e_5, \delta$</th>
<th>$e_6, \delta$</th>
<th>$e_7, \delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>(0.5, 0.2, 0.4, 0.5)</td>
<td>(0.1, 1, 0.5)</td>
<td>(0.3, 0.2, 0.5, 0.4)</td>
</tr>
<tr>
<td>$n$</td>
<td>(0.4, 0.5, 0.6, 0.2)</td>
<td>(1, 0.0, 0.2)</td>
<td>(0.7, 0.3, 0.4, 0.2)</td>
</tr>
</tbody>
</table>

Now we find the similarity between the model possibility neutrosophic soft set and possibility neutrosophic soft set of each person as follow

$S(f_\mu, g_\nu) \cong 0.49 < \frac{1}{2}$, $S(f_\mu, h_\delta) \cong 0.47 < \frac{1}{2}$, $S(f_\mu, r_0) \cong 0.51 > \frac{1}{2}$, $S(f_\mu, s_\alpha) \cong 0.54 > \frac{1}{2}$, $S(f_\mu, m_\gamma) \cong 0.57 > \frac{1}{2}$.

Consequently, $p_5$ is should be selected by the committee.

5. Conclusion

In this paper we have introduced a similarity measure between the PNS-sets. An applications of proposed similarity measure have been given to solve a decision making problem. In future, these seem to have natural applications as image encryption and correlation of between PNS-sets.
REFERENCES


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