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Smarandache BCH-Algebras

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Abstract: We introduce the notion of Smarandache BCH-algebra and Smarandache (fresh, clean and fantastic) ideals, some example are given and related properties are investigated. Relationship between Q-Smarandache (fresh, clean and fantastic) ideals and other types of ideals are given. Extension properties for Q-Smarandache (fresh, clean and fantastic) ideals are established.

Key words: BCK-algebra . BCH-algebra . Smarandache BCH-algebra . Smarandache (fresh, clean and fantastic) ideals

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INTRODUCTION

A Smarandache structure on a set A means a weak structure W on A such that there exists a proper subset B of A which is embedded with a strong structure S. In [10], W.B. Vasantha Kandasamy studied the concept of Smarandache groupoids, subgroupoids, ideal of groupoids, semi-normal subgroupoids, Smarandache Bol groupoids and strong Bol groupoids and obtained many interesting results about them. Smarandache semi-groups are very important for the study of congruences and it was studied by R. Padilla [9].

As it is well known, BCK/ BCI-algebras are two classes of algebras of logic. They were introduced by Imai and Iseki [3, 4, 8]. BCI-algebras are generalizations of BCK-algebra. Most of algebras related to the t-norm based logic, such as MTL-algebras, BL-algebras, hoop, MV-algebras and Boolean algebras *et al*., are extensions of BCK-algebras.

In 1983, Hu and Li [6, 7] introduced the notion of a BCH-algebra, which is a generalization of the notions of BCK and BCI-algebras and studied by many researchers [1, 2, 6].
It will be very interesting to study the Smarandache structure in these algebraic structures. In [5], Y. B. Jun discussed the Smarandache structure in BCI-algebras.

In this paper we introduce the notion of Smarandache BCH-algebra and we deal with Smarandache ideal structures in Smarandache BCH-algebras. We introduce the notion of Smarandache (fresh, clean and fantastic) ideal in a BCH-algebra and then we obtain some related results which have been mentioned in the abstract.

PRELIMINARIES

An algebra (X;*, 0) of type (2, 0) is called a BCH-algebra if it satisfies the following axioms: for every $x,y,z \in X$, [6].

(I1) $x * x = 0$,

(I2) $x * y = 0$ and $y * x = 0$ imply $x = y$,

(I3) $(x * y) * z = (x * z) * y$,

In a BCH-algebra X, the following holds for all $x, y \in X$

(J1) $x * 0 = x$,

(J2) $(x * (x * y)) * y = 0$,

(J3) $0 * (x * y) = (0 * x) * (0 * y)$,

(J4) $0 * (0 * (0 * x)) = 0 * x$,

(J5) $x \le y$ implies $0 * x = 0 * y$.

An algebra (X,*, 0) is called a BCK-algebra if it satisfies the following conditions for every $x, y, z \in X$

(a1) $((x * y) * (x * z)) * (z * y) = 0$,

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(a2) $(x*(x*y))*y=0$,

(a3) $x*0=x$,

(a4) $x*y=0$ and $y*x=0$ imply $x=y$,

(a5) $0*x=0$.

In any BCH/BCK-algebra X we can define a partial order ≤ by putting x≤y if and only if x*y = 0.

SMARANDACHE BCH-ALGEBRAS

Definition 1: A Smarandache BCH-algebra is defined to be a BCH-algebra X in which there exists a proper subset Q of X such that

(s1) $0\in Q$ and $|Q|\geq 2$,

(s2) Q is a BCK-algebra under the operation of X.

Example 2: Let X= {0, 1, 2, 3, 4}. The following table shows the BCH-algebra structure on X

*	0	1	2	3	4
0	0	0	0	0	4
1	1	0	0	1	4
2	2	2	0	0	4
3	3	3	3	0	4
4	4	4	4	4	0

Consider Q = {1, 2, 3}, thus Q is a BCK-algebra which is properly contained in X. Then (X,*,0) is a Smarandache BCH-algebra.

In what follows, let X and Q denote a Smarandache BCH-algebra and a BCK-algebra which is properly contained in X, respectively.

Definition 3: A nonempty subset I of X is called a Smarandache ideal of X related to Q (or briefly, Q-Smarandache ideal of X) if it satisfies:

(c1) $0\in I$,

(c2) $(\forall x\in Q)(\forall y\in I)(x*y\in I\Rightarrow x\in I)$.

Example 4: Let X be a Smarandache BCH-algebra of above example. It is easily checked that I = {0, 2} is a Smarandache ideal of X.

If I is a Smarandache ideal of X related to every BCK-algebra contained in X, we simply say that I is a Smarandache ideal of X.

Proposition 5: Any ideal of X is a Q-Smarandache ideal of X.

By the following example we show that the converse of above proposition is not correct in general.

Example 6: Let X={0,a,b,c,d,e,f,g,h,i,j,k,l,m,n}. The following table shows the BCH-algebra structure on X.

*	0	a	b	c	d	e	f	g	h	i	j	k	l	m	n
0	0	0	0	0	0	0	0	0	h	h	h	h	l	l	n
a	a	0	a	0	a	0	a	0	h	h	h	h	m	l	n
b	b	b	0	0	f	f	f	f	i	h	k	k	l	l	n
c	c	b	a	0	g	f	g	f	i	h	k	k	m	l	n
d	d	d	0	0	0	0	d	d	j	h	h	j	l	l	n
e	e	e	a	0	a	0	e	d	j	h	h	j	m	l	n
f	f	f	0	0	0	0	0	0	k	h	h	h	l	l	n
g	g	f	a	0	a	0	a	0	k	h	h	h	a	l	n
h	h	h	h	h	h	h	h	h	0	0	0	0	n	n	l
i	i	i	h	h	k	k	k	k	b	0	f	f	n	n	l
j	j	j	h	h	h	h	j	j	d	0	0	d	n	n	l
k	k	k	h	h	h	h	h	h	f	0	0	0	n	n	l
l	l	l	l	l	l	l	l	l	n	n	n	n	0	0	h
m	m	l	m	l	m	l	m	l	n	n	n	n	a	0	h
n	n	n	n	n	n	n	n	n	l	l	h	l	h	h	0

Consider Q= {0, a}, thus Q is a BCK-algebra. Then X is a Smarandache BCH-algebra. It is clear that I = {0, a, b} is a Q-Smarandache ideal, which is not an ideal, since d*b = 0 ∈ I and b∈ I, but d∉ I.

Proposition 7: If Q satisfies Q*X⊂Q, then every Q-Smarandache ideal of X satisfies in the following implication

$$(\forall x,y\in I)(\forall z\in Q)((z*y)*x=0\Rightarrow z\in I) \qquad (1)$$

Proof: Assume that Q*X⊂Q and let I be a Q-Smarandache ideal of X. Suppose that (z*y)*x=0, for all x,y∈ I and z∈Q, then (z*y)∈Q by assumption and (z*y)*x∈ I then z*y∈ I by (c2) since y∈I, it follows that z∈ I. This completes the proof.

Theorem 8: Let Q1 and Q2 are BCK-algebras which are properly contained in X and $Q_1\subseteq Q_2$. Then every Q_2-Smarandache ideal is a Q_1-Smarandache ideal.

The following example shows that the converse of above theorem is not true in general.

Example 9: Let X= {0, a, b, c, d, e}. The following table shows the BCH-algebra structure on X.

*	0	a	b	c	d	e
0	0	0	0	0	d	d
a	a	0	0	a	d	d
b	b	b	0	b	d	d
c	c	c	c	0	d	d
d	d	d	d	d	0	0
e	e	d	d	e	a	0

Note that the subsets Q_1 = {0, b} and Q_2 = {0, a, b, c} are BCK-algebra which are properly contained in X. Then X is a Smarandache BCH-algebra. It is easily checked that I= {0, b, c} is a Q_1-Smarandache ideal of X and it is not a Q2-Smarandache ideal of X, since $a * b = 0 \in I$ and $b \in I$, but $a \notin I$

Definition 10: A nonempty subset I of X is called a Smarandache fresh ideal of X related to Q (or briefly, Q-Smarandache fresh ideal of X) if it satisfies the condition (c1) and

$$\text{(c3)}\ \forall x,y,z \in Q((x * y) * z \in I, y * z \in I \Rightarrow x * z \in I)$$

Example 11: Let X= {0, 1, 2, 3}. The following table shows the BCH-algebra structure on X.

*	0	1	2	3
0	0	0	2	2
1	1	0	2	2
2	2	2	0	0
3	3	2	1	0

Note that Q= {0, 1} is a BCK-algebra which is properly contained in X. Then X is a Smarandache BCH-algebra. It is easily checked that I= {0, 1} is a Q-Smarandache fresh ideal of X.

Theorem 12: Let Q_1 and Q_2 are BCK-algebras which are properly contained in X and $Q_1 \subseteq Q_2$. Then every Q_2-Smarandache fresh ideal of X is a Q_1-Smarandache fresh ideal of X.

The following example shows that the converse of above theorem is not true in general.

Example 13: Let X= {0, 1, 2, 3, 4, 5}. The following table shows the BCH-algebra structure on X.

*	0	1	2	3	4	5
0	0	0	0	0	4	4
1	1	0	0	1	4	4
2	2	2	0	2	4	4
3	3	3	3	0	4	4
4	4	4	4	4	0	0
5	5	4	4	5	1	0

Note that Q_1 = {0, 1} and Q_2 = {0, 1, 2, 3} are BCK-algebra which are properly contained in X. Then X is a Smarandache BCH-algebra. It is easily checked that a subset I= {0, 2, 3} is a Q_1-Smarandache fresh ideal of X and is not a Q_2-Smarandache fresh ideal of X. Since $(1*2)*3 = 0 \in I$ and $2*3 = 2 \in I$ but $1*3 = 1 \notin I$

Proposition 14: If I is a Q-Smarandache fresh ideal of X, then

$$(\forall x,y \in Q)((x * y) * y \in I \Rightarrow x * y \in I)$$

Proof: Assume that $(x*y)*y \in I$ for all $x,y \in Q$ Since $y*y=0 \in I$. By (I1) and (c1), it follows from (c3) that $x*y \in I$. This is the desired result.

Theorem 15: Every Q-Smarandache fresh ideal which is contained in Q is a Q-Smarandache ideal.

Proof: Let I be a Q-Smarandache fresh ideal of X which is contained in Q. Let $x \in Q$ and $y \in I$ be such that $x*y \in I$. Then $(x*y)*0=x*y \in I$ and $y*0=y \in I$. Since $x \in Q$ and $y \in I \subset Q$ it follow from (c3) and (I3) that $x=x*0 \in I$ so that I is a Q-Smarandache ideal of X.

The following example shows that the converse of above theorem is not true in general.

Example 16: Let X= {0, a, b, c, d, e}. The following table shows the BCH-algebra structure on X.

*	0	a	b	c	d	e
0	0	0	0	0	0	e
a	a	0	0	0	a	e
b	b	a	0	a	b	e
c	c	c	c	0	d	d
d	d	d	d	d	0	e
e	e	e	e	e	e	0

Note that Q= {0, a, b, c, d} is a BCK-algebra which is properly contained in X. Then X is a Smarandache BCH-algebra. It is easily checked that a subset I= {0, d} is a Q-Smarandache ideal of X which is not a Q-Smarandache fresh ideal. Since $(b*a)*c=0 \in I$ and

$$a * c = 0 \in I, \text{ but } b * c = a \notin I$$

We provide conditions for a Q-Smarandache ideal to be a Q-Smarandache fresh ideal.

Theorem 17: If I is a Q-Smarandache ideal of X such that

$$(\forall x, y, z \in Q)((x * y) * z \in I \Rightarrow (x * z) * (y * z) \in I \qquad (2)$$

Then I is a Q-Smarandache fresh ideal of X.

Proof: Assume that (x*y)*z∈I and y*z∈I, for all z,y,z∈Q. Then (x*z)*(y*z) ∈I by (2) and so x*z∈I by (c2). Therefore I is a Q-Smarandache fresh ideal of X.

Proposition 18: If I is a Q-Smarandache fresh ideal of X which is contained in Q, then

$$(\forall x, y \in Q)(\forall z \in I)(((x * y) * y) * z \in I \Rightarrow x * y \in I) \qquad (3)$$

Proof: Assume that ((x*y)*)*z∈I for all x,y∈Q and z∈I. If I is a Q-Smarandache fresh ideal of X, which is contained in Q, then I is a Q-Smarandache ideal of X. Using (c2), we know that (x*y)*y∈I by Proposition 14, we get that x*y∈I.

Theorem 19: Let I and J are Q-Smarandache ideal of X and I⊂J. If I is a Q-Smarandache fresh ideal of X and I satisfies in following condition

$$(\forall x, y, z \in Q)\ ((x * y) * z) \in I \Rightarrow (x * z) * (y * z) \in I$$

Then J is a Q-Smarandache fresh ideal of X.

Proof: Let (x*y)*z∈J for all x, y, z∈Q. Using (I1) and (I3), we have ((x*((x*y)*z)) *y)*z= ((x*y)*z)*((x*y) *z) =0∈I. Since I is a Q-Smarandache fresh ideal of X and by hypothesis we get that

((x*z)*(y*z)*((x*y)*z) = ((x*((x*y)*z))*z)*(y*z)∈I⊂J

So we get that (x*y)*(y*z) ∈J. This proves that J is a Q-Smarandache fresh ideal of X.

SMARANDACHE CLEAN IDEALS

Definition 1: A nonempty subset I of X is called a Smarandache clean ideal of X related to Q (or briefly, Q-Smarandache clean ideal of X) if it satisfies the condition (c1) and

$$\text{(c4)}\ (\forall x, y \in Q)(\forall z \in I)\ ((x * (y * x)) * z \in I \Rightarrow x \in I)$$

Example 2: Let X= {0, a, b, c, d, e}. The following table shows the BCH-algebra structure on X.

*	0	a	b	c	d	e
0	0	0	0	0	0	e
a	a	0	0	0	0	e
b	b	a	0	a	0	e
c	c	c	c	0	0	e
d	d	d	d	d	0	e
e	e	e	e	e	e	0

Note that Q= {0, a, b, c, d} is a BCK-algebra which is properly contained in X. Then X is a Smarandache BCH-algebra. It is easily checked that I= {0, a, b, c} is a Q-Smarandache clean ideal of X.

Theorem 3: Every Q-Smarandache clean ideal of X is a Q-Smarandache ideal of X.

Proof: Let I be a Q-Smarandache clean ideal of X. Let x∈Q and z∈I be such that x*z∈I. Taking y=x in (c4), we have

$$(x * (x * x)) * z = (x * 0) * z = x * z \in I$$

and so x∈I. Hence I is a Q-Smarandache ideal of X.

The following example shows that converse of above theorem is not correct in general.

Example 4: Let X= {0, 1, 2, 3, 4} with the following table shows the BCH-algebra structure on X.

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	1
2	2	2	0	2	0
3	3	1	3	0	3
4	4	4	4	4	0

It is easily checked that a subset I= {0, 2} is a Q-Smarandache ideal of X, but it is not a Q-Smarandache clean ideal of X, since

$$(3 * (3 * 4)) * 2 = (3 * 3) * 2 = 0 * 2 = 0 \in I, \text{ but } 3 \notin I$$

Remark 5: If Q is an implicative BCK-algebra, that is, Q satisfies the condition:

$$(\forall x, y \in Q)(x = x * (y * x))$$

Then every Q-Smarandache ideal is a Q-Smarandache clean ideal.

The following example shows that every Q-Smarandache fresh ideal is not a Q-Smarandache clean ideal and also every Q-Smarandache clean ideal is not a Q-Smarandache fresh ideal.

Example 6: Let X= {0, 1, 2, 3}. The following table shows the BCH-algebra structure on X.

*	0	1	2	3
0	0	0	0	0
1	1	0	3	3
2	2	0	0	2
3	3	0	0	0

Note that Q= {0, 1, 2, 3} is BCK-algebra which is properly contained in X Then X is a Smarandache BCH-algebra. It is easily checked that a subset I1 = {0, 3} is a Q-Smarandache clean ideal of X, but it is not a Q-Smarandache fresh ideal of X. Since (2*3)*2=0∈I and 3*2=0∈I, but 2*3=2∉I.

Example 7: Let X= {0, 1, 2, 3, 4} with the following table shows the BCH-algebra structure on X.

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	1
2	2	2	0	2	0
3	3	1	3	0	3
4	4	4	2	4	0

it is easily checked that a subset I={0,2,4} is a Q-Smarandache fresh ideal, but is not a Q-Smarandache clean ideal of X, since (1*(3*1))*1=0, but 1∉I.

Proposition 8: Every Q-Smarandache clean ideal I of X satisfies the following implication:

$$\forall x,y\in Q,\ y*(y*x)\in I \Rightarrow x*(x*y)\in I \qquad (4)$$

Proof: Let I be a Q-Smarandache clean ideal of X. Then I is a Q-Smarandache ideal of X (Theorem 3). Assume that y*(y*x) ∈I, for all x, y ∈Q. Since ((x*(x*y))*(y*(x*(x*y))))*(y*(y*x)) =0∈I it follows from (J1) and (c2) that ((x*(x*y))*(y*(x*(x*y))))*0=(x*(x*y))*(y*(x*(x*y))) ∈I. So we get that (x*(x*y))∈I.

Theorem 9: Let I be a Q-Smarandache fresh ideal of X which is contained in Q. If I satisfy the following condition:

$$(\forall x,y\in Q)(x*(y*x))\in I \Rightarrow x\in I \qquad (5)$$

and if x≤y implies that a*z≤y*z, then I is a Q-Smarandache clean ideal of X.

Proof: Let x, y∈Q and z∈I be such that (x*(y*x))*z∈I. If I is a Q-Smarandache fresh ideal which is contained in Q, then I is a Q-Smarandache ideal. Thus x*(y*x)∈I by (c2). Since

$$((y*(y*x))*(y*x))*(x*(y*x))=0\in I$$

it follows from (c2) that (y*(y*x))*(y*x)∈I by Proposition 14, we get that y*(y*x)∈I. Hence by (5), x*(x*y) ∈I on the other hand, note that

$$((x*y)*z)*(x*(y*x))=0\in I$$

Then (x*y)*z∈I and so x*y∈I. Therefore x∈I.

Theorem 10: Let I be a Q-Smarandache ideal of X Then I is a Q-Smarandache clean ideal of X if and only if I satisfies the following condition:

$$(\forall x,y\in Q)(x*(y*x)\in I \Rightarrow x\in I) \qquad (6)$$

Proof: Suppose that (x*(y*x))*z∈I, for all x,y∈Q and z∈I. Then x*(y*x)∈I by (c2) and so by (6) x∈I.

Conversely, assume that I is a Q-Smarandache clean ideal of X and x,y∈Q be such that x*(y*x)∈I. Since 0∈I, it follows from (J1) that (x*(y*x))*0= x*(y*x)∈I, so from (c4) we get that x∈I.

SMARANDACHE FANTASTIC IDEALS

Definition 1: A nonempty subset I of X is called a Smaradache fatastic ideal of X related to Q (or brifly, Q-Smarandache fatastic ideal of X) if it satisfies the condition (c1) and

$$(c5)(\forall x,y\in Q)(\forall z\in I)(x*y)*z\in I \Rightarrow x*(y*(y*x))\in I$$

In the following example we show the relationship between the Q-Smarandache fantastic ideal of X and other types of Q-Smarandache ideals which defined before.

Example 2: Let X= {0, 1, 2, 3, 4, 5}. The following table shows the BCH-algebra structure on X.

*	0	1	2	3	4	5
0	0	0	0	0	0	5
1	1	0	1	0	1	5
2	2	2	0	2	0	5
3	3	1	3	0	3	5
4	4	4	4	4	0	5
5	5	5	5	5	5	0

Note that Q= {0, 1, 2, 3, 4} is a BCK-algebra which is properly contained in X. Then X is a Smarandache BCH-algebra. It is easily checked that a subset I_1={0,2} and I_2={0,2,4} are Q-Smarandache fantastic ideal of X, but it is not a Q-Smarandache fresh ideal of X. I_3={0,1,3} is a Q-Smarandache fresh ideal, but is not a Q-Smarandache fantastic ideal, since $(2*4)*3=0\in I_3$ and $2*(4*(4*2))=2\notin I_3$.

Also I_1= {0,2} is a Q-Smarandach fatastic ideal of X, but it is not a Q-Smarandache clean ideal, since $(1*(3*1))*0=0=(1*1)*0=0\in I_1$, but $1\notin I_1$.

Theorem 3: Let Q1 and Q2 are BCK-algebras which are properly contained in X and Q1⊆ Q2. Then every Q2-Smarandache fantastic ideal is a Q1-Smarandache fantastic ideal.

The following example shows that converse of above theorem is not correct in general.

Example 4: Let X= {0, 1, 2, 3, 4, 5}. The following table shows the BCH-algebra structure on X.

*	0	1	2	3	4	5
0	0	0	0	0	0	5
1	1	0	1	0	1	5
2	2	2	0	2	0	5
3	3	1	3	0	3	5
4	4	4	4	4	0	5
5	5	5	5	5	5	0

Note that Q_1={0,2,4} and Q_2={0,1,2,3,4} are BCK-algebra which are properly contained in X and $Q_1\subseteq Q_2$. Then I= {0, 1, 3} is a Q_1-Smarandache fantastic ideal of X, but is not a Q_2-Smarandache fantastic ideal of X.

Theorem 5: Every Q-Smarandache fantastic ideal of X is a Q-Smarandache ideal of X.

Proof: Let I be a Q-Smarandache fantastic ideal of X. Assume that $x*z\in Q$, for all $x\in Q$ and $z\in I$. Using (J1), we get that $(x*0)*z=x*z\in I$. Since $x\in Q$ and Q is a BCK-algebra, it follows from (a5), (J1) and (c5) that $x = x*(0*(0*x))\in I$, so that I is a Q-Smarandache ideal of X.

Theorem 6: Let I be a Q-Smarandache ideal of X Then I is a Q-Smarandache fantastic ideal of X if and only if I satisfies the following condition:

$$(\forall x,y\in Q)(x*y\in I \Rightarrow x*(y*(y*x))\in I$$

Proof: Suppose that I satisfies in (7), $(x*y)*z\in I$, for all $x,y\in Q$ and $z\in I$. Then $x*y\in I$ by (c2) and so $x*(y*(y*x))\in I$.

Conversely, assume that I is a Q-Smarandache fantastic ideal of X and let $x,y\in Q$ be such that $x*y\in I$. Using (J1), we have $(x*y)*0= x*y\in I$ and $0\in I$. It follows from (c5) that $x*(y*(y*x))\in I$

Theorem 7: Let I and J are Q-Smarandache ideals of X and $I\subset J\subset Q$. If I is a Q-Smarandache fantastic ideal of X and I satisfied in following condition

$$(\forall x,y,z\in Q)((x*y)*z)\in I \Rightarrow (x*z)*(y*z)\in I$$

Then J is a Q-Smarandache fantastic ideal of X.

Proof: Assume that $x*y\in J$, for all $x,y\in Q$. Since

$$(x*(x*y))*y=(x*y)*(x*y)=0\in I$$

It follows from (I3) and (7) that

$$(x*(y*(y*(x*(x*y)))))*(x*y)=$$
$$(x*(x*y))*(y*(y*(x*(x*y))))\in I\subset J$$

So from (c2), $x*(y*(y*(x*(x*y))\in J$. Since $x,y\in Q$ and Q is a BCK-algebra, we conclude that $(x*(y*(y*x)))*(x*(y*(y*(x*(x*y)))))=0\in J$ by using (a1). It follows that $x*(y*(y*x))\in J$. Hence J is a Q-Smarandache fantastic ideal of X.

Theorem 8: Let I be a Q-Smarandache fresh ideal and Q-Smarandache fantastic ideal of X. Also

$$\forall x,y,z\in Q, (x\leq y)\Rightarrow x*z\leq y*z$$

Then I is a Q-Smarandache clean ideal of X.

Proof: Suppose that I is both a Q-Smarandache fresh ideal and Q-Smarandache fantastic ideal. Let $x,y\in Q$ be such that $x*(y*x)\in I$. Since

$$((y*(y*x))*(y*x))*(x*(y*x))=0\in I$$

We get that $(y*(y*x))*(y*x)\in I$ by (c2). Since I is a Q-Smarandache fresh ideal, it follows from Proposition 14 that $y*(y*x)\in I$, then $x*y\in I$, on the other hand we have $(x*y)*(y*(y*x))=0\in I$. Since I is a Q-Smarandache fantastic ideal, we obtain $(x*(y*(y*x)\in I$ by (7) and so $x\in I$. Therefore I is a Q –Smarandache clean ideal of X.

CONCLUSION

Smarandache structure occurs as a weak structure in any structure. In the present paper, we have introduced the concept of Smarandache BCH-algebras and investigated some of their useful properties. In our opinion, these definition and main result can be similarly extended to some other algebraic systems such as BL-algebra, lattices and Lie algebras.

It is our hope that this work would other foundations for further study of the theory of BCH-algebras. Our obtained results can be perhaps applied in engineering, soft computing or even in medical diagnosis.

In our future study of Smarandache structure of BCH-algebras, may be the following topics should be considered:

- To consider the structure of quotient of Smarandache BCH-algebras;
- To get more results in Smarandache BCH-algebras and application;
- To define fuzzy structure on Smarandache BCH-algebras.

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