Ehrenfest Paradox, Sagnac Effect, and the Michelson-Morley Experiment

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Abstract

In this article the resolution of the famous Ehrenfest paradox [1] is presented. The paradox relates to a spinning disk and the Special Relativity Theory (SRT) applied to it. The paradox resolution is based on the proposition that the paradox results from an incorrect application of SRT to a system that is not in an inertial motion. The centrifugal and centripetal forces resulting from the rotation are always present and need to be accounted for. Using the author's previously derived metric for the axially symmetric space-time the effect of centrifugal and centripetal forces can be correctly included. When this is done no paradox is obtained and it is shown that the spinning disk appears to have flat space-time geometry. This finding also provides the correct interpretation of the null result of Michelson-Morley experiment, the correct explanation of the Fizeau experiments, and a simple and consistent explanation of the Sagnac effect. The theoretical descriptions of all these experiments should, therefore, always include the effect of the centrifugal force of Earth's rotation. The measured data from other experiments conducted on rotating systems are explained by the inertial mass increase as correctly described by SRT.

Key words: Ehrenfest Paradox; Mősbauer Effect; Special Relativity Theory; General Relativity Theory; Schwarzschild Metric; New Space-Time Metric; Ehrenfest Paradox; Sagnac Effect; Michelson-Morley Experiment; Transversal Fizeau effect; One-way Speed of Light.

1. Introduction

There have been many papers published on the resolution of the Ehrenfest paradox with various degrees of success and with various conclusions [1]. Most of them are typically aimed at justifying the application of only SRT to this case and the paradox resolution is often obtained by a very contorted reasoning. The paradox results from applying the Lorentz coordinate transformation to a spinning disk whose circumference should contract while the radius should not since the motion of the radius is always perpendicular to the disk rotating direction. As a result the circumference, according to SRT, is no longer equal to $L_o = 2\pi r$, which leads to a non-flat space-time geometry that is not a domain of SRT. From this consideration it is clear that only the kinematic approach to resolve this problem, as offered by SRT, is not enough. SRT deals with the systems in inertial motion and does not account for the acceleration and inertial forces. In order to resolve the paradox, it is necessary to use the metric from the General Relativity Theory (GRT) or use other space-time metrics that describe the non-flat space-time geometry, which may be adopted to include the centrifugal and centripetal forces. The well-known metric describing the space-time around a centrally gravitating body that has a mass M is the Schwarzschild metric:

$$ds^2 = (1 - R_s / r)(cdt)^2 - (1 - R_s / r)^{-1} dr^2 - r^2 \left(d\vartheta^2 + \sin \vartheta^2 d\varphi^2 \right), \tag{1}$$

where $R = 2\kappa M/c^2$ is the Schwarzschild radius, κ the gravitational constant, and c the speed of light in our local intergalactic neighborhood. However, a new metric for the axially symmetric space-time has been recently published [2], which is more suitable for studying this case:

$$ds^2 = e^{2\varphi_n / c^2} (cdt)^2 - dr^2 - r^2 e^{2\varphi_n / c^2} d\varphi^2 - dz^2 . \tag{2}$$

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The parameter φ_n is the Newtonian gravitational potential of a mass configuration with an axial symmetry. The coordinate system for this metric is cylindrical with the symmetry axis in the z direction. A brief derivation of the metric shown in Eq.2 is given in the Appendix.

2. The paradox Resolution

The new metric can now be used to resolve the paradox. An observer placed on the spinning disk circumference, and firmly holding onto it, observes, as is well known, a centrifugal force. This force is actually an inertial force, which is the reaction to the centripetal force caused by the inter-atomic cohesion forces of the disk forcing the atoms to travel in a circle with a radial acceleration. The observer is holding onto these atoms to stay on the disk and not to fly off. The centripetal force can be modeled by a gravitation-like potential whose gradient is equal to this force. The potential will be called the pseudo potential φ'_n in order to distinguish it from the standard gravitational potential φ_n. It is thus clear that from the point of view of the disk atoms it does not matter if they are forced to travel in a circle by the disk cohesion forces or by the gravitation-like force derived from the gradient of the pseudo potential acting directly on the atomic nuclei themselves. The gravitation-like pseudo potential and its gradient thus faithfully simulate the action of the centripetal force.

In the next step it will be considered that according to the Riemann principle the motion of particles travelling in a flat space-time under the influence of gravitational forces can be described by the curved space-time in which the particles travel in a free inertial-like motion along geodesic lines without being acted on by any forces. Using this concept it will be possible to correctly include the action of the centripetal force into the considerations and resolve the Ehrenfest paradox.

The pseudo potential describing the action of the centripetal force is calculated from the following considerations: for the centrifugal inertial force from the relativistic Newton's law, as observed in the laboratory coordinate system, it holds that:

$$f_{cf} = \frac{m_o v^2 / r}{\sqrt{1 - v^2 / c^2}}\,, \tag{3}$$

where m_o is the rest mass of a test body located at the circumference of the disk, and where it was considered that the inertial mass depends on velocity as follows:

$$m_i = \frac{m_o}{\sqrt{1 - v^2 / c^2}}\,. \tag{4}$$

For the compensating centripetal force of the disk material that is simulated by the gravitational-like force it will be also considered that it depends on velocity of the observer located at the circumference according to the gravitational mass velocity dependence [3]:

$$m_g = m_o \sqrt{1 - v^2 / c^2}\,. \tag{5}$$

It is necessary to emphasize here that this relation does not follow the famous Einstein's Weak Equivalence Principle (WEP) $m_i = m_g$ [4]. The pseudo potential for the simulating gravitational-like centripetal force is then found from the force equilibrium condition:

$$\frac{\partial \varphi'_n}{\partial r} m_0 \sqrt{1 - \omega^2 r^2 / c^2} = \frac{m_0 \omega^2 r}{\sqrt{1 - \omega^2 r^2 / c^2}}\,. \tag{6}$$

More details about this equation and the derivation of the formula in Eq.5 are also given in the Appendix. After integration the pseudo potential, as observed in the laboratory coordinate system, is found to be:

$$\varphi'_n = \int_0^r \frac{\omega^2 r dr}{1-\omega^2 r^2 / c^2} = -\frac{c^2}{2} \ln\left(1 - \frac{\omega^2 r^2}{c^2}\right), \tag{7}$$

where $v = \omega r$, and ω is the disk angular velocity. The Riemann curved space-time differential metric line element that describes the action of the centripetal force on the atomic nuclei of the spinning disk is then obtained by substituting the found potential in Eq.7 into Eq.2:

$$ds^2 = \frac{(cdt)^2}{(1-\omega^2 r^2 / c^2)} - dr^2 - \frac{r^2 d\varphi^2}{(1-\omega^2 r^2 / c^2)} - dz^2 . \tag{8}$$

Since the particles in this curved space-time are now moving along the geodesic lines without experiencing any acceleration it is no problem to use the Lorentz SRT length contraction formula to calculate the circumference length of the spinning disk as follows:

$$L_c = L_o \sqrt{1 - v^2 / c^2} . \tag{9}$$

However, according to the metric line element in Eq.8 it is easily seen, that after the substitution for the relation: $\omega r = v$, the circumference length as observed by the laboratory observer is:

$$L_i = L_o / \sqrt{1 - v^2 / c^2} . \tag{10}$$

The effects thus precisely cancel each other and no paradox results. The laboratory observer will see the disk periphery not contracted. The similar conclusion is obtained from the metric in Eq.8 also for time. The simulated centripetal force pseudo potential affects the time metric coefficient thus causing the time contraction while the Lorentz time dilation compensates this effect. The space-time geometry of the rotating disk as viewed by the laboratory observer is flat. It thus seems that all the SRT effects are being compensated by the curved space-time metric and the only remaining SRT effect that is not compensated for is the inertial mass increase.

Finally, the important law to verify for this metric is whether the orbiting test body satisfies the conservation of angular momentum. The first integrals of Euler Lagrange equations for the orbital motion derived from the Lagrangian corresponding to the metric in Eq.8 are as follows:

$$\frac{1}{(1-\omega^2 r^2 / c^2)} \frac{dt}{d\tau} = k , \tag{11}$$

$$\frac{r^2}{(1-\omega^2 r^2 / c^2)} \frac{d\varphi}{d\tau} = k\alpha , \tag{12}$$

where k and α are the arbitrary constants of integration. Eliminating $d\tau$ from these equations, since $d\tau$ is a computed invariant and not an observable parameter, results in the standard formula for the conservation of angular momentum:

$$r^2 \frac{d\varphi}{dt} = \alpha . \tag{13}$$

This is one of the well-recognized fundamental principles of physics that must always be satisfied and which the Schwarzschild metric of Einstein's GRT surprisingly does not satisfy.

The above derived results are supported by the experiments published elsewhere [5]. However, the most convincing argument in support of the presented Ehrenfest paradox resolution comes from the GPS data [6]. It is an experimental fact that the time rate measured anywhere on the

Earth's surface is the same. Only the small differences in the gravitational potential affect the surface located clock rate.

It is important to realize that the standard Schwarzschild metric does not offer the similar solution to the Ehrenfest paradox and does not support the conservation of angular momentum as stated in Eq.13. This is a consequence of the incorrect metric coefficient standing by the angular coordinate. The resolution of the Ehrenfest paradox using the new metric and the different dependencies of inertial and gravitational masses on velocity thus provide an important additional support for the correctness of these formulas.

The reasoning used in the above derivation can also be reversed and it could be stated as a theorem that in order to avoid the Ehrenfest paradox the metric for the axially symmetric gravitational field has to have a form given in Eq.2. It is also possible to generalize Eq.7 for any static space-time metric as follows:

$$\varphi_n = \frac{c^2}{2} \ln \sqrt{\frac{g}{g_0}}\,, \tag{14}$$

and generalize the result further by eliminating the gravitational potential:

$$g_{tt}^2 = g / g_0\,, \tag{15}$$

where g and g_o are the metric determinants of the metric line element in Eq.8 with and without rotation. However, the metric determinant g_o has a slightly different meaning in a general case as has been explained elsewhere [2]. The g_o is the determinant of the Minkowski flat physical space-time that corresponds to space-time of the curved coordinate system with the determinant g.

These derivations, however, do not agree with the solution of Einstein's field equations and the Einstein's WEP, which ultimately raises a significant doubt about the accuracy and correctness of the GRT [7].

There have been many experiments performed in the past on rotating systems to confirm various GRT phenomena, but as is clear from the above explanation only the SRT inertial mass increase, and the effects related to the inertial mass increase such as the absorption line shift in the Mossbauer Fe^{57} effect can be observed [8]. No GRT effects related to the curved space-time geometry can be measured in these experiments. It is also necessary to understand in detail the construction of the particular clock used in the experiments to make sure that it is the time what is measured and not the inertial mass increase.

3. Michelson-Morley experiment

The previous section has provided an adequate background for the correct analysis of the Michelson-Morley experiment [9] (MM) that is discussed in this section. When the typical analysis of this experiment is presented the centrifugal force of the Earth's rotation is not considered. The Earth's gravitation substantially overshadows this force; therefore, the centrifugal force is deemed not important and is neglected. In order to separate the action of the Earth's gravity from the effects of rotation it is helpful to imagine for a moment that most of the Earth's mass is not rotating and only a thin surface shell is rotating and gliding on the core. From the point of view of an observer located on the Earths' surface there would be no noticeable difference. It is thus clear that the centrally gravitating mass of Earth is just enhancing the Earth's surface cohesion and provides the force that holds the objects on the ground. The Earth's gravity thus does not have to be considered any further when the rotational effects are studied. The MM experimental apparatus is relatively small in comparison to the Earth's size and it is thus clear that these tests are always performed on the same gravitational equipotential surface. The situation is thus essentially identical to the disk case and the analysis of the MM experiments should always be conducted in the axially symmetric space-time when the Earth's rotational effects are to be included.

The experimental arrangement of the Michelson interferometer is shown in Fig.1. To simplify the drawing and the analysis only one half of the paths the light travels is shown. It is easily seen that it is necessary to calculate only the time the light travels along the path from the beam splitter, mirror M1, to the mirror M2, which is perpendicular to the motion of the interferometer, and to the mirror M3 positioned along the path parallel to the motion. The light travel time back from the mirrors to the beam splitter to create the interference is easily found using the same formulas where a simple substitution of $v \to -v$ is made.

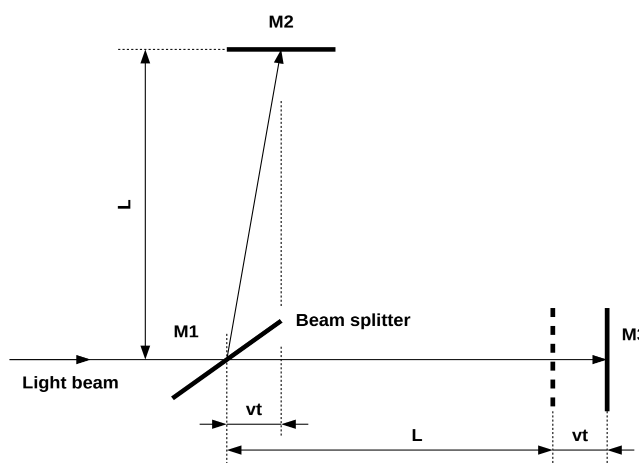

Fig.1: Schematic diagram of Michelson Interferometer. The beam splitter M1 divides the light beam into two components, one continuing horizontally in the *x* direction to the Mirror M3 and the second one reflected in the *z* direction to the mirror M2. Mirrors then reflect the light back to the beam splitter for the observation of interference.

To simplify the considerations further, the analysis of the MM experiment will be divided into two cases: In the first case it will be considered that the interferometer is located on the Earth's equator, so that the axially symmetric metric can adequately describe, in a small equator's neighborhood, the space-time without complications of the Earth's gravitational force not being perpendicular to the rotational axis; and in the second case on the Earth's pole where there is no centrifugal force and thus no centripetal reaction to it. The axial coordinate system for both cases will be the system centered on the Earth's rotational axis and will not rotate with Earth. The metric describing the space-time of the interferometer for the equatorial location thus follows from the metric derived in Eq.8.

$$ds^2 = \frac{(cdt)^2}{(1-v^2/c^2)} - \frac{dx^2}{(1-v^2/c^2)} - dz^2\,, \tag{16}$$

where v is the velocity of the Earth's surface relative to the stationary reference, $dx = Rd\varphi$ is the direction along the equator with R being the Earth's radius, and dz is the meridian direction perpendicular to the equator. However, as was explained previously, the Lorentz coordinate transformation compensates the metric coefficients effect by its length contraction and the time dilation, therefore, the t-x-z space-time of the Earth surface, as viewed from the non-rotating coordinate system, is Minkowski flat. In the z direction, however, the Lorentz length contraction is not present, therefore, the physical length of the interferometer arm is actually longer equal to:

$l' = l/\sqrt{1-v^2/c^2}$. The interferometer arm's length measurement from within the interferometer coordinate system, however, will not show this elongation, since the measuring stick is also longer in this direction. This conclusion follows from the requirement that both arms should have the same physical (invariant) length that must be used in computing the photon travel time. Following the metric in Eq.16, the physical length of the arm in the *x* direction is therefore: $l_{xph} = l/\sqrt{1-v^2/c^2} = l_{zph} = l'$. In practice, however, it may not be possible to make the arms exactly of the same length, so the apparatus is rotated by 90° during the measurement to detect any effect on the interference from the possible difference in the arm's lengths. The speed of light in the reference system in the *x* direction is $c_x = c$ and in the z direction is also $c_z = c$, as this follows directly from the flat space-time metric. The speed of light in the *x*-*z* plane, parallel to the Earth's surface, is thus isotropic. It is now easy to find the photon travel times to the respective mirrors and back to the beam splitter. For the mirror M2 the travel time is found from the relation:

$$t_2^2 = \left(l'^2 + v^2 t_2^2\right)/c^2\,, \tag{17}$$

and similarly for the travel time to the mirror M3 and back to the beam splitter from the relations:

$$t_3 = \left(l + vt_3\right)/c\,, \qquad\qquad t'_3 = \left(l - vt'_3\right)/c\,. \tag{18}$$

It is thus clear that the total photon travel times from the beam splitter to the respective mirrors and back to the beam splitter are identical and equal to:

$$t_{2tot} = t_{3tot} = 2lc/\left(c^2 - v^2\right), \tag{19}$$

where for the total travel time in the *x* direction it is: $t_{3tot} = t_3 + t'_3$. The higher order terms that result from the rotation induced slight beam direction deviation have been neglected in this analysis considering that the condition $l << R$ is always satisfied. No interference fringe shift due to the Earth's rotation relative to the hypothetical stationary medium that supports the photon propagation (ether) can thus be detected by this experiment. The null result of the MM experiment performed at the equator, therefore, does not confirm the special relativity theory, since all the SRT effects are always compensated by the centripetal force that is added to the Earth's gravity. For the verification of validity of formula in Eq.19 another derivation is given in section 5 relative to the observer located on the Earth surface.

A question could also be asked what if the interferometer were positioned vertically. For this case it is clear from the metric in Eq.8 that the coordinates *z* and *r* have the same metric coefficients and are thus interchangeable. The result for this interferometer orientation will be identical to the horizontal orientation case except perhaps for some small influence from the vertical differences in the Earth's gravitational potential.

However, for the MM experiment conducted at the Earth's pole the situation is slightly different. There is no centripetal force there to cancel the Lorentz coordinate transformation and both arms of the interferometer appear to have the same length when not moving. Of course, the experimental setup is approximately at rest relative to the coordinate system introduced previously, so no effect is expected and the photon travel times to the respective mirrors and back to the beam splitter must be identical and equal to:

$$t_{2tot} = t_{3tot} = 2l/c\,. \tag{20}$$

To consider the ether drift it is therefore necessary to select another coordinate system for the analysis. The new coordinate system can be a hypothetical stationary system referenced, for example, to the Universe background radiation relative to which Earth will now move with a velocity *u* in the *x* direction. The speed of light in this coordinate system is considered also isotropic and equal to a local c. In this case SRT and the Lorentz coordinate transformation must now be considered leading to the following expressions for the photon travel times to the respective mirrors:

$$t_2^2 = \left(l^2 + u^2 t_2^2\right)/c^2\,, \tag{21}$$

for the arrival time to the mirror M2, and for the travel time to the mirror M3 and back to the beam splitter:

$$t_3 = \left(l\sqrt{1-u^2/c^2} + ut_3\right)/c\,, \qquad t'_3 = \left(l\sqrt{1-u^2/c^2} - ut'_3\right)/c\,. \tag{22}$$

By solving these equations the total photon travel times to the respective mirrors and back to the beam splitter are:

$$t_{2tot} = t_{3tot} = t_3 + t'_3 = \frac{2l}{c\sqrt{1-u^2/c^2}}\,. \tag{23}$$

Again, both times are identical, no interference fringe shift due to the motion of Earth relative to the Cosmic background radiation coordinate system can be detected thus satisfying one of the fundamental tenets of SRT. For the confirmation of the derivation correctness it is easily seen that Eq.23 can be obtained directly from Eq.20 by simply including the Lorentz coordinate time dilation factor.

The null result of the interference fringe shift in the MM experiment is considered as a proof of SRT and the Lorentz coordinate transformation. However, as is common with all the null result experiments, the proof is weak, since there are usually other assumptions leading to the same conclusions as was clearly demonstrated above. Eq.19 is different from the classical result presented in most textbooks, since they do not consider the centrifugal force and thus derive only formula in Eq.23, which describes an effect that is actually not observable by the observer on Earth, and which is not compatible with the standard Sagnac effect formula that is discussed next.

4. Sagnac effect

The Sagnac effect [10] is considered by many opponents of SRT as a proof of its invalidity. On the other hand many supporters derived SRT equations for it. The explanation that is confirmed by the GPS data is simple to obtain. As it was shown previously the space-time in the *t*-*x*-*z* coordinates is flat due to the centrifugal force effect, so the time difference for the signals that propagate along the equator in parallel with the direction of Earth's rotation and against it or equivalently in small portable rotating systems are easily derived from the following equations:

$$t_+ = \left(2\pi R + vt_+\right)/c\,, \qquad t_- = \left(2\pi R - vt_-\right)/c\,. \tag{24}$$

The time difference needed for the evaluation of the interference fringe shift is then equal to:

$$\Delta t = t_+ - t_- = 4\pi Rv/\left(c^2 - v^2\right). \tag{25}$$

This formula is well known in the industry that builds the gyroscopes based on this effect. For Earth at the equator and at the sea level this difference is: $\Delta t_e = 413.6ns$, which the GPS has verified. It is also important to note that this result is consistent with the formula derived in Eq.19, since the underlying physics is the same. The coefficients appearing in the respective denominators must be identical, which proves the correctness of the axially symmetric metric used in the analysis. Again, no SRT effects can be detected in this experiment, since the Lorentz coordinate transformation is compensated by the curved space-time metric that results from the modeling of the centripetal force. The opponents citing this experiment as a proof that SRT is wrong are not correct, since the SRT effects are not present there. Furthermore the supporters of

SRT who modify Eq.25 to include the relativistic effects are also wrong for the same reason, since there are no SRT effects in the Sagnac experiment.

5. MM experiment relative to the observer on Earth

It might be possible to raise an objection to the above presented derivations of the interference fringe shift in the MM experiment and the Sagnac effect, since the derivations were performed relative to a non-rotating reference frame. The experiments are always observed from the surface of the rotating Earth, so a difference in time between the rotating and non-rotating coordinate systems might be expected according to the classical relativistic point of view. However, as it was clearly shown, the Earth's surface metric as viewed from the non-rotating coordinate system is flat and no time difference is observed between the clock rates on the equator and on the poles as is also confirmed by the GPS [6], the formulas for the fringe shift should therefore be identical.

Since the space-time of the experiments is flat and the Lorentz coordinate transform does not hold, the velocities, including the speed of light, must be adding classically. For the speed of light observed on Earth in the *x* direction it must therefore hold true that:

$$c_x = c \pm v\,, \tag{26}$$

and similarly for the speed of light in the z-direction it must hold true that:

$$c_z^2 = c^2 - v^2\,. \tag{27}$$

The Earth's centripetal force together with the force of gravity therefore compensate for the relativistic effects and the Lorentz coordinate transformation of objects that are supported by the Earth's surface, or move along the surface to experience the force. The only difference, as previously mentioned, is the difference in the physical length of the interferometer arm in the *z* direction caused by the metric given in Eq.16. The *z* direction interferometer arm length is:

$$l' = l / \sqrt{1 - v^2 / c^2}\,. \tag{28}$$

The time of the photon flight from the beam splitter to the mirror M3 and back as observed on Earth is then simply as follows:

$$t_3 = l/(c - v), \qquad t'_3 = l/(c + v), \tag{29}$$

and to the mirror M2 equal to:

$$t_2^2 = \frac{l^2}{(1 - v^2/c^2)} \frac{1}{(c^2 - v^2)} = \frac{l^2 c^2}{(c^2 - v^2)^2}\,. \tag{30}$$

The total photon travel times are therefore identical and equal to:

$$t_{2tot} = t_{3tot} = t_3 + t'_3 = 2lc/(c^2 - v^2). \tag{31}$$

It is important to note that this is the same formula as derived in Eq.19 with no Lorentz factor for the time transformation between the non-rotating coordinate systems and the Earth's surface coordinate system as explained previously. This also confirms the classical velocity addition in rotating systems caused by the centripetal force of rotating Earth including the "superluminal" velocity $c' = c+v$ [11].

Finally, it is also clear that for the same reason the Sagnac formula derived in the non-rotating reference coordinate system is the same as in the rotating system, so the observer rotating with the disc sees the same fringe shift as the observer in the non-rotating coordinate system.

6. Fizeau experiments

Among the remaining tests that should also be mentioned as being possibly affected by the Earth's rotation is the one that is famous for being among the first to verify SRT. This is the Fizeau experiment where the speed of light was measured in a moving water medium [12]. For the test apparatus located on the Earth's pole there will be no difference from the usual relativistic treatment using the velocity addition formula:

$$c' = \frac{c/n + v}{1 + v/cn} \approx \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right), \tag{32}$$

where n is the index of refraction of the moving medium and v is its velocity relative to the stationary observer. The expression in parenthesis in Eq.32 is the well-known Fresnel light dragging coefficient. After considering the light returning path in the direction opposite to the media flow the photon flight time difference for creating the interference will be:

$$\Delta t_p = \frac{4lv(n^2 - 1)}{c^2 - v^2 n^2} \approx \frac{4lv}{c^2}(n^2 - 1)\,. \tag{33}$$

On the equator, however, the velocities add classically outside of the medium and divided by n inside of the medium, so the resulting speed of light as observed by the laboratory observer is:

$$c' = \frac{c - v}{n} + v = \frac{c}{n} + v\left(1 - \frac{1}{n}\right). \tag{34}$$

The second term in the parenthesis can thus be considered to be the new light dragging coefficient. Similarly, for the photon flight time difference at the interference mirror the result is:

$$\Delta t_e = \frac{4lvn(n-1)}{c^2 - v^2 (n-1)^2} \approx \frac{4lv}{c^2} n(n-1)\,. \tag{35}$$

The relativistic formula thus predicts a slightly larger interference fringe shift at the Earth's pole. Using the original experimental setup parameters found in the published literature [12] and the corrected value for the water flow due to the velocity profile across the tube as published elsewhere [13], the formula in Eq.33 predicts the result: $\Delta\lambda_p/\lambda$ = 0.24099 and the formula from Eq.35 the result: $\Delta\lambda_e/\lambda$ = 0.13775, where: $\Delta\lambda = c\Delta t$. The actual measured value of the interference fringe shift was: $\Delta\lambda/\lambda$ = 0.23016. The original Fizeau experiment did not consider the water flow velocity profile across the tube diameter, thus introducing a considerable error into the measurement. It is also possible that the index of refraction may have changed depending on the water pressure. This should have been verified. Various other objections about the details of the water flow along the Earth's surface and its response to the gravitational and centrifugal forces could also be raised, since water is not a solid object. All these complications make this test not a particularly useful and convincing for the proof.

A more compelling evidence for the classical Galilean velocity addition on rotating platforms, however, comes from the recent measurements of the transversal Fizeau effect [14], where for the lateral image displacement Δ due to the source rotation as observed through a stationary disk of a thickness L it was experimentally determined that: $\Delta = L(n-1)v/c$. This formula is easily derived for the case of the rotating disk and a stationary source using the new light dragging coefficient from Eq.34. Considering that for the first order approximation for a relatively slow rotational velocity in comparison to c the photon travel time across the disk thickness is still: $t_L = nL/c$, the lateral image displacement is equal to:

$$\Delta = t_L v\left(1-\frac{1}{n}\right) = L\frac{v}{c}(n-1). \tag{36}$$

An interesting aspect of this calculation is that this result agrees with the case when the light source is moving and the glass disk is stationary thus having no centrifugal force in it. An advantage of such an experimental configuration is that the disk is free of any rotation caused distortions and possible stress induced index of refraction changes. The image displacement due to the light travel time in the glass for this case is then:

$$\Delta = L\frac{v}{c}n - L\frac{v}{c} = L\frac{v}{c}(n-1). \tag{37}$$

The term Lv/c represents the image shift when no glass medium is present, which needs to be subtracted. The effect is thus perfectly symmetrical satisfying the source-observer relativity as expected. It thus seems that the image of the centrifugal force effect on the addition of velocities in rotating platforms is absolutely necessary in order to satisfy the basic tenet of relativity. The experimental data confirming this result were also presented at a conference in Rochester New York in June 13, 2007 [15]. Finally, this result suggests that when Maxwell's equations are used to describe the EM fields and the wave propagation in solid rotating media, it is also necessary to account for the centrifugal forces and the resulting curved space-time metric.

7. Conclusions

The new space-time metric was used in resolving the Ehrenfest paradox and the related experimental verifications. The metric allowed for a correct inclusion of the centripetal force and its pseudo-potential into the considerations, which compensates for the time dilation and the length contraction effects of SRT. The inertial mass increase, however, is not compensated for, which explains the published experimental results [7]. The space-time geometry resulting from the new metric also clearly explained the Sagnac effect and the failure to detect the ether wind in the Michelson-Morley interferometer experiments. The resolution of the Ehrenfest paradox, the explanation of the Sagnac effect, and the explanation of the null result of Michelson-Morley experiments thus validate the new metric correctness and the different dependencies of inertial and gravitational masses on velocity. Finally, it was clearly shown, supported by the GPS data [6], and also supported by the recent experiments on transparent cylinders and rotating light sources [14, 15], that the velocities in rotating systems, due to the centripetal force, add classically including the speed of light. This is an interesting fact and a fundamentally very important finding that is contrary to the standard SRT point of view. The curved space-time effects caused by centripetal force thus must be considered when evaluating the EM fields and the wave propagation using Maxwell's equations in rotating solid body platforms.

Appendix: Metric for the space-time with the axial symmetry

The detail metric derivation for this space-time is available elsewhere [2]. The derivation presented here uses a slightly different approach.

The general form of the metric line element for a static axially symmetric space-time with the gravitating axis positioned along the z direction is as follows:

$$ds^2 = g_{tt}(cdt)^2 - g_{rr}dr^2 - g_{\varphi\varphi}d\varphi^2 - g_{zz}dz^2, \tag{A1}$$

where the metric coefficients can depend only on r.

In the next steps the metric coefficients g_{tt}, g_{rr}, and g_{zz} will be found by analyzing motion of a small test body in the r-z plane. The Lagrangian describing such a motion is equal to:

$$L = g_{tt}\left(\frac{cdt}{d\tau}\right)^2 - g_{rr}\left(\frac{dr}{d\tau}\right)^2 - g_{zz}\left(\frac{dz}{d\tau}\right)^2 \ . \tag{A2}$$

Since the Lagrangian itself is also the first integral ($L = c^2$), and since for the first integrals of Euler- Lagrange (E-L) equations corresponding to the time and z coordinates holds that:

$$d\tau = g_{tt}dt \ , \tag{A3}$$

$$g_{zz}\frac{dz}{d\tau} = k \ , \tag{A4}$$

where k is an arbitrary constant of integration, it is possible to write the following equation that the test body motion must satisfy:

$$\left(\frac{dr}{d\tau}\right)^2 + \frac{g_{zz}}{g_{rr}}\left(\frac{dz}{d\tau}\right)^2 = \frac{c^2}{g_{tt}g_{rr}} - \frac{c^2}{g_{rr}} \ . \tag{A5}$$

By differentiating Eq.A5 with respect to τ the following result is obtained:

$$\frac{d^2r}{d\tau^2} + \left[\frac{d}{d\tau}\left(\frac{g_{zz}}{g_{rr}}\right)\frac{1}{2}\left(\frac{dz}{d\tau}\right)^2 + \left(\frac{g_{zz}}{g_{rr}}\right)\frac{dz}{d\tau}\frac{d^2z}{d\tau^2}\right]\left(\frac{dr}{d\tau}\right)^{-1} = -\frac{c^2}{2}\frac{\partial}{\partial\varphi_n}\left(\frac{1}{g_{rr}} - \frac{1}{g_{rr}g_{tt}}\right)\frac{\partial\varphi_n}{\partial r} \ . \tag{A6}$$

In this equation it is assumed that the metric coefficients are functions of the Newton gravitational potential. It is also clear that the acceleration in the r direction cannot depend on velocity, particularly on the velocity in the z direction, and since there is no gravitational field in that direction the parameterized acceleration (z differentiated twice with respect to τ) must also be zero. Both terms in the square parenthesis thus must be equal to zero. Considering also from Eq.A4, that: $dz/d\tau = k/g_{zz}$, the following conditions must be satisfied:

$$g_{rr} = g_{zz} \ , \qquad\qquad g_{zz} = 1. \tag{A7}$$

The found metric coefficients simplify Eq.A6 to read:

$$g_{tt}\frac{d^2r}{d\tau^2} = -\left(\frac{c^2}{2}\frac{1}{g_{tt}}\frac{\partial g_{tt}}{\partial\varphi_n}\right)\frac{\partial\varphi_n}{\partial r} \ . \tag{A8}$$

This equation has the expected simple form, since the parameterized acceleration in a static axially symmetric space-time can depend only on the gradient of $\varphi_n(r)$. The term in the parenthesis, however, must be equal to unity in order to satisfy the well-known and many times experimentally verified Einstein's equivalence principle (the Einstein elevator) with the acceleration equal to force of gravity:

$$g_{tt}\frac{d^2r}{d\tau^2} = -\frac{\partial\varphi_n}{\partial r} \ . \tag{A9}$$

This equation also satisfies the well-known covariance principle of tensor calculus with the total of covariant quantities equal on both sides. This leads to the following condition:

$$\frac{c^2}{2}\frac{1}{g_{tt}}\frac{\partial g_{tt}}{\partial\varphi_n} = 1. \tag{A10}$$

By integrating this result using the boundary condition at infinity where the potential is set to zero, the g_{tt} metric coefficient is found equal to:

$$g_{tt} = e^{2\varphi_n / c^2} . \tag{A11}$$

The metric coefficient standing by the angular coordinate is found by considering again a small test body orbital motion in the space-time defined by the following metric line element:

$$ds^2 = g_{tt}(cdt)^2 - dr^2 - g_{\varphi\varphi} d\varphi^2 - dz^2 . \tag{A12}$$

The Lagrangian describing this motion is then:

$$L = g_{tt}\left(\frac{cdt}{d\tau}\right)^2 - \left(\frac{dr}{d\tau}\right)^2 - g_{\varphi\varphi}\left(\frac{d\varphi}{d\tau}\right)^2 - \left(\frac{dz}{d\tau}\right)^2 . \tag{A13}$$

The first integral of E-L equation corresponding to the angular coordinate is:

$$g_{\varphi\varphi}\frac{d\varphi}{d\tau} = \alpha , \tag{A14}$$

where the suitable integration constants was used. The first integral for the time coordinate is the same as in Eq.A3. It is well known and experimentally confirmed that the orbital motion must satisfy the conservation of angular momentum. From Eq.A3 and Eq.A14 then follows that:

$$\frac{g_{\varphi\varphi}}{g_{tt}}\left(\frac{d\varphi}{dt}\right) = \alpha , \tag{A15}$$

and from this result then also follows that for the angular metric coefficient it is:

$$g_{\varphi\varphi} = r^2 g_{tt} , \tag{A16}$$

since the metric coefficient standing by the radial coordinate is unity. The radial coordinate distance is equal to the radial physical distance in this case. Substituting these results into Eq.A12 the metric line element used in Eq.2 is obtained:

$$ds^2 = e^{2\varphi_n / c^2}(cdt)^2 - dr^2 - r^2 e^{2\varphi_n / c^2} d\varphi^2 - dz^2 . \tag{A17}$$

Finally a short comment is necessary related to Eq.6: The right hand side of Eq.A3 can be factored out into the two identical components:

$$d\tau = dt\sqrt{g_{tt}}\sqrt{g_{tt}} , \tag{A18}$$

which makes it possible to explicitly show the compatibility with the Lorentz coordinate transformation. This is accomplished by substituting for one of the terms the expression for g_{tt} obtained from Eq.A5, for $dz/d\tau = 0$, which can be rearranged and rewritten using Eq.A3 as:

$$v^2 / c_r^2 = 1 - g_{tt} , \tag{A19}$$

where c_r denotes the local radial speed of light $c_r = c\sqrt{g_{tt}}$, and where $v = dr/dt$. Eq.A18 can then be generalized for any direction of motion including the disc circular motion as follows:

$$d\tau = dt\sqrt{g_{tt}}\sqrt{1 - v^2 / c^2} . \tag{A20}$$

It is now obvious that this formula is compatible with the Lorentz time coordinate transformation for the flat space-time when $g_{tt} = 1$ and also with the time coordinate transformation when the test body is stationary in a gravitational field with $v = 0$. Multiplying both sides of Eq.A9 by the test body rest mass m_o and substituting for the invariant $d\tau$ from Eq.A20, the result is:

$$g_{tt}\frac{d}{dt}\left(\frac{v\ m_o}{\sqrt{g_{tt}}\sqrt{1-v^2/c^2}}\right)=-\frac{\partial\varphi_n}{\partial r}m_o\sqrt{g_{tt}}\sqrt{1-v^2/c^2}\ . \qquad \text{(A21)}$$

From this result then follow the dependencies of the inertial and gravitational masses on velocity:

$$m_i=\frac{m_o}{\sqrt{g_{tt}}\sqrt{1-v^2/c^2}}\ , \qquad \text{(A22)}$$

$$m_g=m_o\sqrt{g_{tt}}\sqrt{1-v^2/c^2}\ . \qquad \text{(A23)}$$

Since for the previously introduced simulating gravitational-like potential or the pseudo potential φ' it is not important whether the forces are generated by the mass located along the z axis or by a rotation around the z axis, the simulated gravitational-like force equilibrium with the inertial centrifugal force should, therefore, follow Eq.A21 and be more generally written as:

$$\frac{m_o\omega^2 r}{\sqrt{g_{tt}}\sqrt{1-\omega^2r^2/c^2}}=\frac{1}{g_{tt}}\frac{\partial\varphi'_n}{\partial r}m_o\sqrt{g_{tt}}\sqrt{1-\omega^2r^2/c^2}\ , \qquad \text{(A24)}$$

where $dv/dt = v^2/r$ and $v = \omega r$ for the circular orbits. However, all the terms containing g_{tt} cancel out making Eq.6 correct also. The derivation of these general formulas for the inertial mass and the gravitational mass dependence on velocity can be also found elsewhere [16, 17, 18].

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