A formula for generating a certain kind of semiprimes based on the two known Wieferich primes

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Abstract. In one of my previous papers, “A possible infinite subset of Poulet numbers generated by a formula based on Wieferich primes” I pointed an interesting relation between Poulet numbers and the two known Wieferich primes (not the known fact that the squares of these two primes are Poulet numbers themselves but a way to relate an entire set of Poulet numbers by a Wieferich prime). Exploring further that formula I found a way to generate primes, respectively semiprimes of the form q1*q2, where q2 – q1 is equal to a multiple of 30.

Note:

In the paper I was talking about in Abstract I conjectured that there exist, for every Wieferich prime W, an infinity of Poulet numbers which are equal to n*W – n + 1, where n is integer, n > 1. Examples of such Poulet numbers are 3277 = 1093*3 – 2, 4369 = 1093*4 – 3, 5461 = 1093*5 – 4, respectively 49141 = 1093*14 – 13. In other words, I conjectured that there exist an infinity of pairs of Poulet numbers (P1, P2) such that P2 – P1 + 1 = 1093, respectively an infinity of pairs of Poulet numbers (P1, P2) such that P2 – P1 + 1 = 3511. Examples of such pairs of Poulet numbers are (1729, 2821), (3277, 4369), (4369, 5461). Playing with this formula I noted that in many cases the number P + W – 1, where P is a Poulet number and W a Wieferich prime, is equal to a semiprime q1*q2, where q2 – q1 = 30 (examples of such semiprimes are 37*67 = 1387 + 1093 – 1 and 43*73 = 2047 + 1093 – 1). But, more than that, I noticed that often the numbers of the type q1*q2 – W + 1 (and implicitely, as we will see further, of the type q1*q2 + W – 1), where q1 and q2 are primes such that q2 – q1 = 30*k, where k positive integer, are often equal to q3*q4, where q3 and q4 are primes such that q4 – q3 = 30*h, where h positive integer.

Conjecture 1:

For every prime p, p > 5, there exist an infinity of primes q, q = p + 30*n, where n positive integer, such that the number p*q + 1092 is equal to a semiprime pi*qi, where qi – pi = 30*m, where m positive integer.

Conjecture 2:

For every prime p, p > 5, there exist an infinity of primes q, q = p + 30*n, where n positive integer, such that the number p*q + 1092 is equal to a prime.

The first three such semiprimes corresponding to p = 17:

: 17*47 + 1092 = 31*61;
: 17*107 + 1092 = 41*71;
: 17*137 + 1092 = 11*311.

The first three such primes corresponding to p = 17:

: 17*167 + 1092 = 3931, prime;
: 17*197 + 1092 = 4441, prime;
: 17*137 + 1092 = 4951, prime.

The first three such semiprimes corresponding to p = 23:

: 23*173 + 1092 = 11*461;
: 23*353 + 1092 = 61*151;
: 23*443 + 1092 = 29*389.

The first three such primes corresponding to p = 23:

: 23*53 + 1092 = 2311, prime;
: 23*83 + 1092 = 3001, prime;
: 23*113 + 1092 = 3691, prime.

Conjecture 3:

For every prime p, p > 5, there exist an infinity of primes q, q = p + 30*n, where n positive integer, such that the number p*q + 3510 is equal to a semiprime pi*qi, where qi – pi = 30*m, where m positive integer.

Conjecture 4:

For every prime p, p > 5, there exist an infinity of primes q, q = p + 30*n, where n positive integer, such that the number p*q + 3510 is equal to a prime.

The first three such semiprimes corresponding to p = 17:

: 17*107 + 3510 = 73*73;
: 17*167 + 3510 = 7*907;
: 17*347 + 3510 = 97*97.

The first three such primes corresponding to p = 17:

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:    17*137 + 3510 = 5839, prime;
:    17*227 + 3510 = 7369, prime;
:    17*257 + 3510 = 7879, prime.
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The first three such semiprimes corresponding to p = 23:

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:    23*293 + 3510 = 37*277;
:    23*383 + 3510 = 97*127;
:    23*503 + 3510 = 17*887.
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The first three such primes corresponding to p = 23:

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:    23*53 + 3510 = 4729, prime;
:    23*83 + 3510 = 5419, prime;
:    23*173 + 3510 = 7489, prime.
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