Few notable observations on the prime factors of the Fibonacci numbers involving deconcatenation and congruence modulo

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Abstract. In one of my previous papers, namely "A conjecture about a large subset of Carmichael numbers related to concatenation", I obtained interesting results combining the method of deconcatenation with the method of congruence modulo. Applying the same methods to the prime factors of the Fibonacci numbers I found also notable patterns.

Observation 1:

The first fifty distinct prime factors (of the Fibonacci numbers) that have the last digit 1 share the following property: the numbers obtained removing the last digit 1 are all equal to 0(mod 6), 1(mod 6), 3(mod 6) or 4(mod 6).

Verifying the observation:

: 1 = 1(mod 6);
: 6 = 0(mod 6);
: 4 = 4(mod 6);
: 42 = 0(mod 6);
: 300 = 0(mod 6);
: 28 = 4(mod 6);
: 1980 = 0(mod 6);
: 357 = 0(mod 6);
: 14196 = 0(mod 6);
: 222 = 0(mod 6);
: 13572 = 0(mod 6);
: 21 = 0(mod 6);
: 1094 = 0(mod 6);
: 46 = 4(mod 6);
: 5594574 = 0(mod 6);
: 66 = 0(mod 6);
: 47454 = 0(mod 6);
: 352368 = 0(mod 6);
: 448 = 4(mod 6);
: 1473620616= 0(mod 6);
: 990 = 0(mod 6);
: 7 = 1(mod 6);

: 91 = 1(mod 6);
: 5401852 = 1(mod 6);
: 23068650 = 0(mod 6);
: 2913460 = 4(mod 6);
: 98868 = 0(mod 6);
: 483252 = 0(mod 6);
: 160 = 1(mod 6);
: 304 = 1(mod 6);
: 1944 = 0(mod 6);
: 37024845 = 0(mod 6);
: 952 = 1(mod 6);
: 341591404 = 0(mod 6);
: 14448 = 0(mod 6);
: 88 = 1(mod 6);
: 166508832180048 = 0(mod 6);
: 18 = 0(mod 6);
: 54 = 0(mod 6);
: 453110055090 = 0(mod 6);
: 6773500 = 1(mod 6);
: 57060 = 0(mod 6);
: 51912 = 0(mod 6);
: 828882348 = 0(mod 6);
: 11921885137 = 1(mod 6);
: 824206505006176 = 4(mod 6);
: 33 = 3(mod 6);
: 3916 = 4(mod 6);
: 145900030551372 = 0(mod 6);
: 1074508848= 0(mod 6).

Observation 2:

The first fifty distinct prime factors (of the Fibonacci numbers) that have the last digit 3 share the following property: the numbers obtained removing the last digit 3 are all equal to 1(mod 6), 2(mod 6), 4(mod 6) or 5(mod 6).

Verifying the observation:

: 1 = 1(mod 6);
: 23 = 5(mod 6);
: 11 = 5(mod 6);
: 2 = 2(mod 6);
: 5 = 5(mod 6);
: 7 = 1(mod 6);
: 4 = 4(mod 6);
: 297121507 = 1(mod 6);
: 110 = 2(mod 6);
: 95 = 5(mod 6);
: 1450 = 4(mod 6);
: 5483 = 5(mod 6);

: 35 = 5(mod 6);
: 142991 = 5(mod 6);
: 667 = 1(mod 6);
: 4616537107= 1(mod 6);
: 9218047149475= 1(mod 6);
: 8 = 2(mod 6);
: 26 = 2(mod 6);
: 15960799 = 1(mod 6);
: 19 = 1(mod 6);
: 1854680513= 5(mod 6);
: 564419 = 5(mod 6);
: 10 = 4(mod 6);
: 124783 = 1(mod 6);
: 127008388 = 4(mod 6);
: 556805304822773221007 = 5(mod 6);
: 851 = 5(mod 6);
: 4229 = 5(mod 6);
: 8552672293768909 = 1(mod 6);
: 29 = 5(mod 6);
: 34750205267= 5(mod 6);
: 8114347796 = 2(mod 6);
: 717532311495056459 = 5(mod 6);
: 31 = 1(mod 6);
: 3336551939= 5(mod 6);
: 3256622320813= 1(mod 6);
: 80048 = 2(mod 6);
: 1810470079= 1(mod 6);
: 6574058 = 2(mod 6);
: 389678749007629271532 7 = 1(mod 6);
: 42245 = 5(mod 6);
: 8175789237238547574551461 09 = 1(mod 6);
: 142973479719757578008 = 2(mod 6);
: 950637219386 = 2(mod 6);
: 37 = 1(mod 6);
: 56 = 2(mod 6);
: 487072367131 = 1(mod 6);
: 7578108062569891284399757 9 = 1(mod 6);
: 1186257524870= 2(mod 6).

Observation 3:

The first twenty distinct prime factors (of the Fibonacci numbers) that have the last digit 7 share the following property: the numbers obtained removing the last digit 7 are all equal to 0(mod 6), 1(mod 6), 3(mod 6) or 4(mod 6).

Verifying the observation:

: 1 = 1(mod 6);
: 4 = 4(mod 6);

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:     159         =    3(mod 6);
:     3           =    3(mod 6);
:     2865        =    3(mod 6);
:     55          =    1(mod 6);
:     241         =    1(mod 6);
:     220         =    4(mod 6);
:     10          =    4(mod 6);
:     43349443    =    1(mod 6);
:     30          =    0(mod 6);
:     55500349    =    1(mod 6);
:     6           =    0(mod 6);
:     13          =    1(mod 6);
:     1807        =    1(mod 6);
:     8602071     =    3(mod 6);
:     15          =    3(mod 6);
:     9919485309475549 = 1(mod 6);
:     382126393 =    1(mod 6);
:     19          =    1(mod 6).
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Observation 4:

The first twenty distinct prime factors (of the Fibonacci numbers) that have the last digit 9 share the following property: the numbers obtained removing the last digit 9 are all equal to 1(mod 6), 2(mod 6), 4(mod 6) or 5(mod 6).

Verifying the observation:

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:     2           =    2(mod 6);
:     1           =    1(mod 6);
:     19          =    1(mod 6);
:     10          =    4(mod 6);
:     51422       =    2(mod 6);
:     14          =    2(mod 6);
:     278         =    2(mod 6);
:     593         =    5(mod 6);
:     13          =    1(mod 6);
:     616870      =    4(mod 6);
:     577         =    1(mod 6);
:     5           =    5(mod 6);
:     19489       =    1(mod 6);
:     301034      =    2(mod 6);
:     26          =    2(mod 6);
:     11684       =    2(mod 6);
:     82          =    4(mod 6);
:     937582      =    4(mod 6);
:     7           =    1(mod 6);
:     85          =    1(mod 6).
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