Two exciting classes of odd composites defined by a relation between their prime factors

Marius Coman
Bucuresti, Romania
email: mariuscoman13@gmail.com

Abstract. In this paper I will define two interesting classes of odd composites often met (by the author of this paper) in the study of Fermat pseudoprimes, which might also have applications in the study of big semiprimes or in other fields. This two classes of composites n = p(1)*...*p(k), where p(1), ..., p(k) are the prime factors of n are defined in the following way: p(j) – p(i) + 1 is a prime or a power of a prime, respectively p(i) + p(j) – 1 is a prime or a power of prime for any p(i), p(j) prime factors of n such that p(1) ≤ p(i) < p(j) ≤ p(k).

Definition 1:

We name the odd composites n = p(1)*...*p(k), where p(1), ..., p(k) are the prime factors of n, with the property that p(j) – p(i) + 1 is a prime or a power of a prime for any p(i), p(j) prime factors of n such that p(1) ≤ p(i) < p(j) ≤ p(k), Coman composites of the first kind. If n = p*q is a squarefree semiprime, p < q, with the property that q – p + 1 is a prime or a power of a prime, then n it will be a Coman semiprime of the first kind.

Examples:

: 2047 = 23*89 is a Coman semiprime of the first kind because 89 – 23 + 1 = 67, a prime;
: 4681 = 31*151 is a Coman semiprime of the first kind because 151 – 31 + 1 = 121, a power of a prime;
: 1729 = 7*13*19 is a Coman composite of the first kind because 19 – 7 + 1 = 13, a prime, 19 – 13 + 1 = 7, a prime, and 13 – 7 + 1 = 7, a prime.
: 2821 = 7*13*31 is a Coman composite of the first kind because 13 – 7 + 1 = 7, a prime, 31 – 13 + 1 = 19, a prime, and 31 – 7 + 1 = 25, a power of a prime.

Note that not incidentally I chose Fermat pseudoprimes to base two with two prime factors (2-Poulet numbers) and absolute Fermat pseudoprimes as examples: they are often Coman composites.

Definition 2:

We name the odd semiprimes $n_1 = p_1*q_1$, $p_1 < q_1$, with the property that $n_2 = q_1 - p_1 + 1 = p_2*q_2$, $p_2 < q_2$, is a Coman semiprime of the first kind, a Coman semiprime of the first kind of the second degree, also the odd semiprimes n_2 with the property that $n_3 = q_2 - p_2 + 1$ is a Coman semiprime of the first kind of the second degree, a Coman semiprime of the first kind of the third degree and so on.

Examples:

: 679 = 7*97 is a Coman semiprime of the first kind of the second degree because 97 – 7 + 1 = 91, a Coman semiprime of the first kind because 91 = 7*13 and 13 – 7 + 1 = 7, a prime;

: 8983 = 13*691 is a Coman semiprime of the first kind of the third degree because 691 – 13 + 1 = 679, which is a Coman semiprime of the first kind of the second degree.

Definition 3:

We name the odd composites n = p(1)*...*p(k), where p(1), ..., p(k) are the prime factors of n, with the property that p(j) + p(i) - 1 is a prime or a power of a prime for any p(i), p(j) prime factors of n such that p(1) ≤ p(i) < p(j) ≤ p(k), Coman composites of the second kind. If n = p*q is a squarefree semiprime, p < q, with the property that q + p - 1 is a prime or a power of a prime, then n it will be a Coman semiprime of the second kind.

Examples:

: 341 = 11*31 is a Coman semiprime of the second kind because 11 + 31 - 1 = 41, a prime;

: 1729 = 7*13*19 is a Coman composite of the second kind because 7 + 13 – 1 = 19, a prime, 13 + 19 – 1 = 31, a prime, and 7 + 19 – 1 = 25, a power of a prime.

Definition 4:

We name the odd semiprimes $n_1 = p_1*q_1$, $p_1 < q_1$, with the property that $n_2 = q_1 + p_1 - 1 = p_2*q_2$, $p_1 < q_1$, is a Coman semiprime of the second kind, a Coman semiprime of the second kind of the second degree, also the odd semiprimes n_2 with the property that $n_3 = q_2 + p_2 - 1$ is a Coman semiprime of the second kind of second degree, a Coman semiprime of the second kind of the third degree and so on.

Notes:

: The odd semiprimes of the type n = p*q, p < q, where abs{p – q + 1} or q^2 – p + 1 or abs{q – p^2 + 1} or q^2 – p^2 + 1 or abs{p^2 – q^2 + 1} or, respectively, p^2 + q – 1 or p + q^2 – 1 or p^2 + q^2 – 1 is also prime, seems also to be interesting to be studied;

: The numbers of the type n = p^2 + q^2 – 1 respectively n = q^2 – p^2 + 1, where p, q primes, p < q, are often, if not primes, Coman composites;

: The seventh Fermat number, 18446744073709551617 = 274177*67280421310721 is a Coman semiprime of the second kind because the number 67280421584897 = 67280421310721 + 274177 – 1 is a prime;

: Many Mersenne numbers are Coman composites: 2047 = 23*89 is a Coman semiprime of the first kind because 89 – 23 + 1 = 67, a prime; 32767 = 7*31*151 is a Coman composite of the second kind because 7 + 31 – 1 = 37, a prime, 7 + 151 – 1 = 157, a prime, and 31 + 151 – 1 = 181, a prime; 33554431 = 31*601*1801 because 31 + 601 – 1 = 631, a prime, 31 + 1801 – 1 = 1831, a prime, and 601 + 1801 – 1 = 2401 = 7^4, a power of a prime;

: In the papers from the references given below there are few conjectures about Coman composites.

References:

: A formula which conducts to primes or to a type of composites that could form a class themselves, Marius Coman;
: An elementary formula which seems to conduct often to primes, Marius Coman;
: Seven conjectures on a certain way to write primes including two generalizations of the twin primes conjecture, Marius Coman;
: Ten conjectures about certain types of pairs of primes arising in the study of 2-Poulet numbers, Marius Coman;
: The notion of chameleonic numbers, a set of composites that “hide” in their inner structure an easy way to obtain primes, Marius Coman;
: Twenty-four conjectures about “the eight essential subsets of primes”, Marius Coman;
: Two types of pairs of primes that could be associated to Poulet numbers, Marius Coman.