The Seesaw Mechanism and the Structure of Spacetime above the Electroweak Scale

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Abstract

According to the seesaw mechanism, neutrino masses arise from the existence of heavy Majorana neutrinos postulated to emerge near the grand unification scale (GUT) of about 10^{16} GeV. Despite its theoretical appeal, this scenario involves either physics at inaccessible scales or tuning the Yukawa couplings to un-naturally low values. Our work sidesteps the seesaw mechanism and shows that neutrino masses follow from placing the Standard Model on a spacetime support equipped with arbitrarily small deviations from four dimensions.

Key words: Seesaw mechanism, Neutrino masses, Majorana neutrino, Standard Model, Minimal fractal manifold.

1. Basics of the seesaw scenario

The seesaw mechanism assumes that left handed (LH) neutrinos (ν_L) are associated with heavy fermion partners (ν_R), which are undetectable but not strictly forbidden by the Standard Model of particle physics (SM) [see e.g. 1-6]. The heavy partners are right handed (RH) Majorana neutrinos conjectured to exist as a result of symmetry breaking near the GUT scale. Although heavy particles are not directly observable at the SM scale, they are allowed to occur as off-shell virtual states.

Based on these premises, neutrinos contribute with a massive Dirac term given by

$$L_m^D = m_D \overline{\nu_R} \nu_L + \text{h.c.} = \frac{1}{2} m_D (\overline{\nu_R} \nu_L + \overline{\nu_L^c} \nu_R^c) + \text{h.c.} \tag{1}$$

where “h.c.” stands for “hermitian conjugate”. In addition to (1), Majorana mass terms with both LH and RH components are also allowed and expressed as

$$L_m^L = \frac{1}{2} m_L \overline{\nu_L^c} \nu_L + \text{h.c.} \tag{2a}$$

$$L_m^R = \frac{1}{2} m_R \overline{\nu_R^c} \nu_R + \text{h.c.} \tag{2b}$$

One can subsequently construct a LH neutrino vector from the light and heavy fields,

$$n_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}, \quad \overline{n_c^L} = \begin{pmatrix} \overline{\nu_L^c} & \overline{\nu_R} \end{pmatrix} \tag{3}$$

In the most general case, the mass matrix for neutrinos M_ν can be formulated as

$$M_\nu = \begin{pmatrix} m_L & m_D \\ m_D^T & m_R \end{pmatrix} \tag{4}$$

The overall neutrino mass Lagrangian reads

$$L_m = L_m^D + L_m^L + L_m^R = \frac{1}{2} \overline{n_L^c} M_\nu n_L \tag{5}$$

with positive eigen-states for the light and heavy neutrinos given by

$$m_{1,2} = \left| \frac{1}{2} (m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4 m_D^2}) \right| \tag{6}$$

The Dirac mass is hypothesized to lie near the electroweak scale $m_D = O(M_{EW})$, where $M_{EW} = 246.2$ GeV is the vacuum expectation value of the Higgs boson. The RH neutrino field is heavy $m_R >> M_{EW}$ and, since the light neutrino ν_L carries non-zero isospin and hypercharge, the LH Majorana is excluded by the symmetries of the SM, therefore $m_L = 0$. The only possible solutions of (6) that comply with the SM symmetries are supplied by

$$m_1 = \frac{m_D^2}{m_R} \tag{7a}$$

$$m_2 = m_R \tag{7b}$$

Notwithstanding its appeal, the seesaw scenario involves either physics at inaccessible scales or tuning the Yukawa couplings to un-naturally low values. In response, *extended seesaw models* advocate lowering the heavy scale by invoking the contribution of higher-dimensional effective operators or the physics beyond SM.

The object of next section is to describe how the seesaw paradigm can be circumvented altogether by placing the SM on a spacetime support having arbitrarily small deviations from four dimensions.

2. The low fractal structure of spacetime above the electroweak scale

A rather counterintuitive outcome of field theory is that the exact continuum limit of a local quantum field theory (QFT) formulated on flat spacetime has, strictly speaking, *no correlate to physical reality* [7]. The Minkowski metric of Special Relativity underlies the most basic aspect of QFT, namely the space-like commutativity of local observables, yet is considered only an "emergent" phenomenon and an approximate description of an underlying fundamental theory.

Considerations based on the Renormalization Group program (RG) suggest that the smooth four-dimensional spacetime turns into a manifold with arbitrarily small deviations from four dimensions near M_{EW} [8-13]. Topological structures of this kind are called *minimal fractal manifolds* (MFM) and are defined as continuous spacetimes of dimension $D = 4 \pm \varepsilon$, where $\varepsilon << 1$. The cross-over regime between $\varepsilon \neq 0$ and $\varepsilon = 0$ is the *only sensible setting* where the dynamics of interacting fields asymptotically meets all consistency requirements imposed by QFT and the SM. Hence, a key feature of the MFM is that the assumption ε << 1, postulated near or above the electroweak scale, is the only possible way of asymptotically matching the SM in the limit of vanishing fractality ε = 0. Large deviations from four dimensions ($\varepsilon \sim O(1)$) are likely to stand in direct conflict with both QFT and SM. Particular attention needs to be paid, for example, to the potential violation of Lorentz invariance in Quantum Gravity theories advocating the emergence of spacetime of lower dimensionality at high energy scales. Similarly, large departures from four-dimensionality imply non-differentiability of spacetime trajectories in the conventional sense. This in turn, spoils the very concept of “speed of light” and it becomes manifestly incompatible with Lorentz symmetry [10].

Analysis of the RG equations on the MFM provides a plausible account for the mass and flavor hierarchies of the SM [8-9, 13]. Near the electroweak scale, the normalized masses of SM fermions (m_f), vector bosons (M) and electroweak gauge charges (g_0) can be shown to scale as

$$m_f \sim \varepsilon \tag{8}$$

$$g_0^2 \sim \varepsilon \tag{9}$$

$$g_0^2 M^2 = const \to M^2 \sim \varepsilon^{-1} \tag{10}$$

The system of nonlinear RG equations leads in general to transition to chaos via period-doubling bifurcations as $\varepsilon \to 0$ [9, 13]. The sequence of critical values ε_n, $n = 1, 2, ...$ driving this transition to chaos satisfies the geometric progression

$$\varepsilon_n - \varepsilon_\infty = \varepsilon_n - 0 \sim k_n \overline{\delta}^{-n} \tag{11}$$

Here, $n >> 1$ is the index counting the number of cycles created through the period-doubling cascade, $\overline{\delta}$ is the rate of convergence and k_n is a coefficient that becomes asymptotically independent of n as $n \to \infty$. Period-doubling cycles are characterized by $n = 2^i$, for i >> 1. Substituting (8) to (10) in (11) yields the following ladder-like progression of critical couplings

$$m_{f,i} \sim g_{0,i}^2 \sim \overline{\delta}^{-2^i} \tag{12}$$

Scaling (12) recovers the full mass and flavor content of the SM, including neutrinos, together with the coupling strengths of gauge interactions. Specifically,

- The trivial FP of the RG equations consists of the massless photon (γ) and the massless UV gluon (g).
- The non-trivial FP of the RG equations is degenerate and consists of massive quarks (q), massive charged leptons and their neutrinos (l, ν) and massive weak bosons (W, Z).
- Gauge interactions develop near the non-trivial FP and include electrodynamics, the weak interaction and the strong interaction.

Hence, it is seen that (12) implies the existence of massive neutrinos without invoking the seesaw mechanism. The same conclusion can be reached starting from [14] if we demand that field theory asymptotically approaches scale invariance at both ends of the overall energy spectrum. Then, the far infrared (IR) scale of field theory set by the cosmological constant value ($\Lambda_{cc}^{1/4}$), the electroweak scale (M_{EW}) and the far ultraviolet (UV) scale fixed either by the Planck mass $(\Lambda_{UV} = M_{Pl})$ or by the GUT scale $(\Lambda_{UV} = M_{GUT})$ satisfy the constraint

$$\frac{\Lambda_{cc}^{1/4}}{M_{EW}} \sim \frac{M_{EW}}{\Lambda_{UV}} = O(\varepsilon) \tag{13}$$

which implies

$$\Lambda_{cc}^{1/4} \sim \frac{M_{EW}^2}{\Lambda_{UV}} \tag{14}$$

Comparing (7a) to (14) leads to the straightforward conclusion *that the neutrino mass scale must be comparable to the cosmological constant scale*, $m_1 \sim \Lambda_{cc}^{1/4}$. This result is consistent with the observation that,

- Neutrinos have the lowest known mass in the fermion spectrum,
- The cosmological constant is rooted in the long-distance behavior of neutrino oscillations, as discussed in [15].

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