Superluminal Signalling by Path Entanglement

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Abstract

Entanglement studies dwell on multi-particle systems by definition – one particle, via a global symmetry/conservation law is correlated to another. It has often been wondered via EPR/Bell/Aspect/Dopfer-Zeilinger/Zbinden whether: first, a communication scheme is possible by entangled quantum state collapse and secondly, whether such a scheme would work over spacelike separations. This study follows on from the author's earlier scheme of sending classical data over a Bell Channel, to now, using an unentangled source. The rationale for this is that single particles are entangled with the vacuum state in path entanglement by the principle of conservation of probability: measurement of a photon in one path causes a collapse of the wavefunction in all the others. The new communication scheme represents an improvement over using expensive and complicated entangled sources of poor purity, for common-or-garden coherent sources.

1. Whence entanglement in single particle systems?

The phenomenon of Quantum Entanglement is the fascinating and logical interplay of global conservation laws and indeterminacy in measurement. For instance Bell's analysis[1, 2] and Aspect's experiment[3] focused on spin, which corresponds to angular momentum and its conservation. Franson[4] utilised entanglement resulting from a two level system and this is a manifestation of the conservation of energy.

In non-Relativistic Quantum Theory there is "Conservation of Probability". Recounting the author's earlier paper[5]:

"The probability density of a normalised wavefunction in QM is given by the square of the wavefunction:

$$\rho(\mathbf{r},t) = |\psi(\mathbf{r},t)|^2 \quad and \quad \int \rho(\mathbf{r},t)d^3r = 1 \qquad \text{eqn. 1}$$

If there is any sense in the concept, probability is conserved and would obey the continuity equation:

$$\frac{\partial \rho(\mathbf{r},t)}{\partial t} + \nabla \cdot \mathbf{j}(\mathbf{r},t) = 0 \qquad \text{eqn. 2}$$

Where the probability current density $\mathbf{j}$ is derived on application of the Schrödinger equation to the above relations as:

$$\frac{\hbar}{2mi}\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right) \qquad \text{eqn. 3}$$

Take a spherical source of particles (figure 1) emitted slowly enough to be counted one at a time. Arranged on a sphere one light-year in diameter (say) is a surface of detectors. Only one particle will be counted per detection event as the light-year diameter wavefunction collapses (becomes localised) randomly so that probability is conserved. The wavefunction, mistakenly in current thought, is not perceived as something that is 'real' but is then discarded and a classical path is ascribed from the source to the detector that registered the event to say the particle, *retrospectively* went along that path."

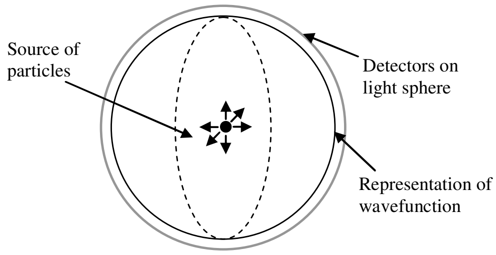


Figure 1 – Conservation of probability

The appendix analyses a beamsplitter from the Stokes relations[6, 7] to arrive at the quantum mechanical treatment. Numbering the input ports 1 and 2 and then the output ports 3 and 4, a photon through port 1 (created from the vacuum eqn. 25, appendix) of a 50:50 beamsplitter evolves thus:

$$|1\rangle_1 |0\rangle_2 \rightarrow \psi_{out} = \frac{1}{\sqrt{2}}|1\rangle_3 |0\rangle_4 + \frac{i}{\sqrt{2}}|0\rangle_3 |1\rangle_4 \qquad \text{eqn. 4}$$

This shows path entanglement and a coherent superposition of the Fock states on both ports. The expectation measurement of the photon count at port 3 or 4 is:

$$\left\langle \hat{N}_3 \right\rangle = \psi_{out}^\dagger \hat{a}_3^\dagger \hat{a}_3 \psi_{out} \qquad \left\langle \hat{N}_4 \right\rangle = \psi_{out}^\dagger \hat{a}_4^\dagger \hat{a}_4 \psi_{out} \qquad \text{eqn. 5}$$

This computes to ½. The joint expectation measurement $\psi^{\dagger}_{out}\hat{a}_3^{\dagger}\hat{a}_3\hat{a}_4^{\dagger}\hat{a}_4\psi_{out}$ is zero and shows that the photon cannot be at both ports.

2. The apparatus to transmit classical data over a quantum channel

The state at the output ports in eqn. 4 is coherent and the outputs can be made to interfere constructively or destructively by path length adjustment. However, if a measurement, that is, a non-unitary operation is performed on one (or both) of the output ports[8], the state will collapse into the mixed state:

$$\rho_{mixed} = \frac{1}{2}|1\rangle_3|0\rangle_4\,{}_4\langle 0|\,{}_3\langle 1| + \frac{1}{2}|0\rangle_3|1\rangle_4\,{}_4\langle 1|\,{}_3\langle 0| \quad \text{eqn. 6}$$

This is well known from simple "which path" experiments with interferometers, where in figure 2, an obstruction is suggested at the output of one of the beamsplitter ports:

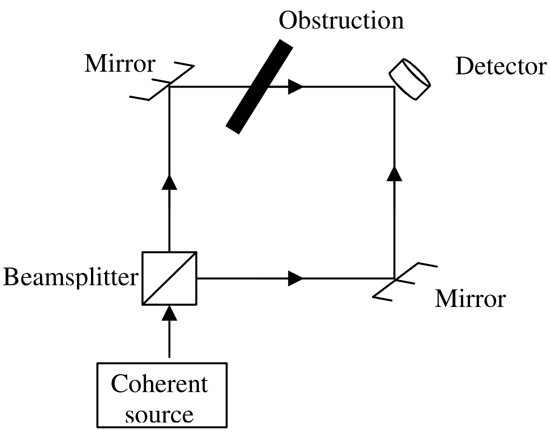


Figure 2

A re-arrangement of the interferometer can then effect the communication of classical digital data over a quantum channel[5] by interfering one output port of the interferometer with another coherent source.

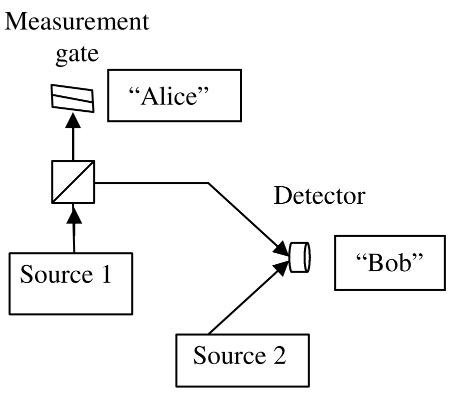


Figure 3

Obviously the two coherent sources would need good relative coherence, we shall discuss a way to use just one source later.

If we can imagine that the output ports of the beamsplitter and source are made to separate in opposite directions, the spatial arrangement of entangled and mixed states after measurement is some what akin to a ticker tape, as the wavefunction propagates through space:

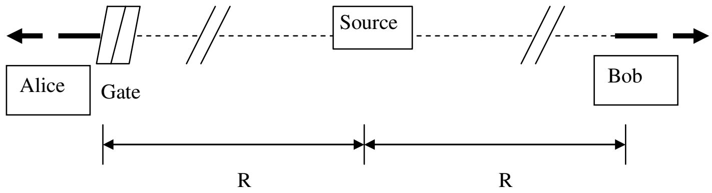


Figure 4

That is, before measurement, the wavefunction exists in space in a pure entangled state. "Bob's" interferometer will only correctly perceive the mixed state of "Alice's" measuring gate and modulations when he is equidistant or greater from the source. We note too that the coherence length/time of the sources must be greater than the "bit" time of the classical digital protocol over the quantum channel:

Alice	Bob
0: No measurement	No signal, destructive interference from pure state
1: Measurement	Signal from mixed state

Table 1 – Classical digital data over a quantum channel. Bob uses destructive interference

3. Apparatus using just one coherent source

Figure 3 required two coherent sources and it is unlikely that two sources would keep good relative coherence for long. We shall show several means to use just one coherent source.

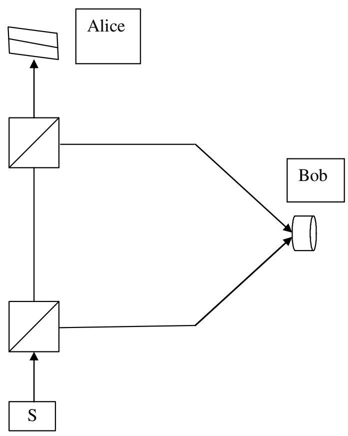


Figure 5

Figure 5 shows an arrangement with two beamsplitters that can have their reflectivities and transmissivities tailored to ensure good destructive interference at Bob's detector.

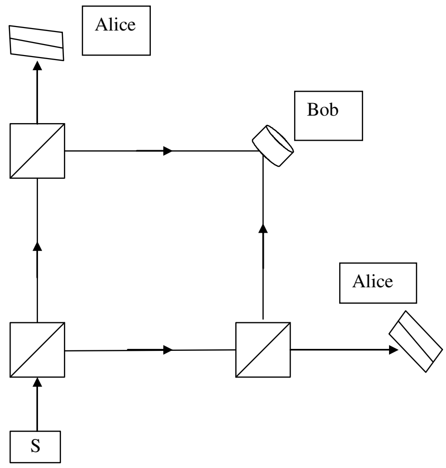


Figure 6

Figure 6 shows an arrangement with three beamsplitters and two measuring gates (though only one gate need be used). It is not to difficult to imagine ways that the paths of Alice and Bob can be brought out from the closed arrangement and sent in opposite directions by mirrors.

Figure 7 shows another method where the beam is expanded such that the widened beam is partially incident on the beamsplitter. The transmitted beamsplitter output and the part of the widened beam that skirted past the splitter are brought into interference at Bob's detector by path length adjustment (there are several means, such as a dielectric medium). The convergence of the beams can be achieved by lens or skewing the paths, as shown.

Figure 8a

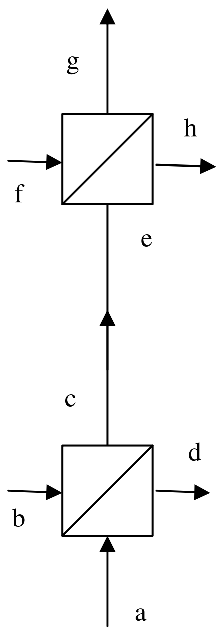


Figure 7

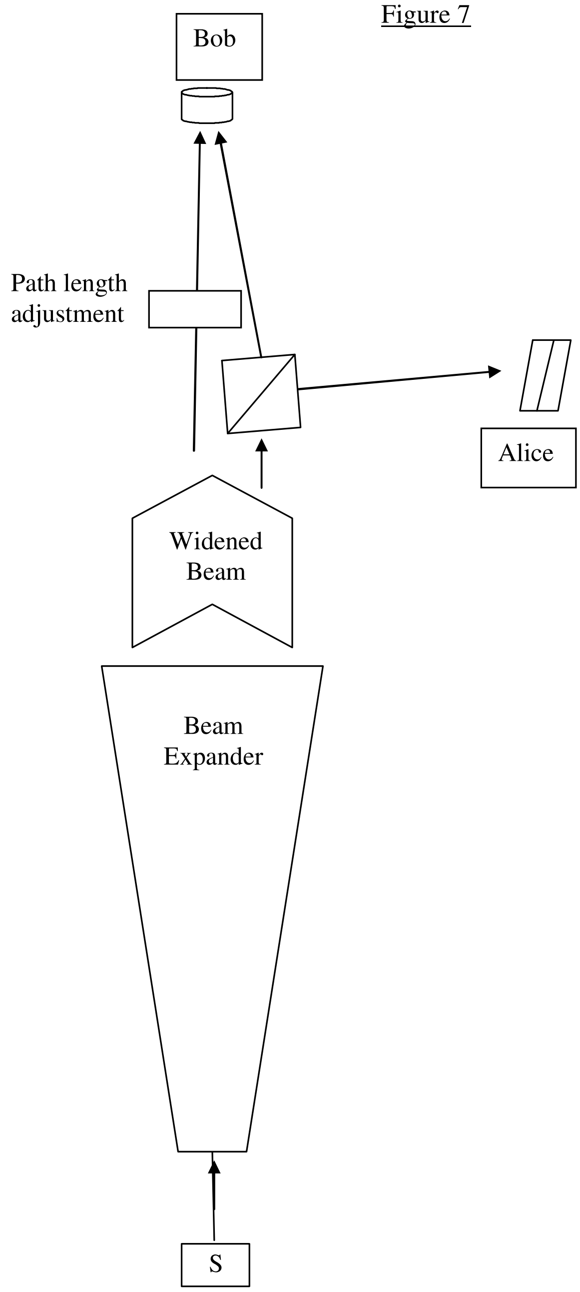
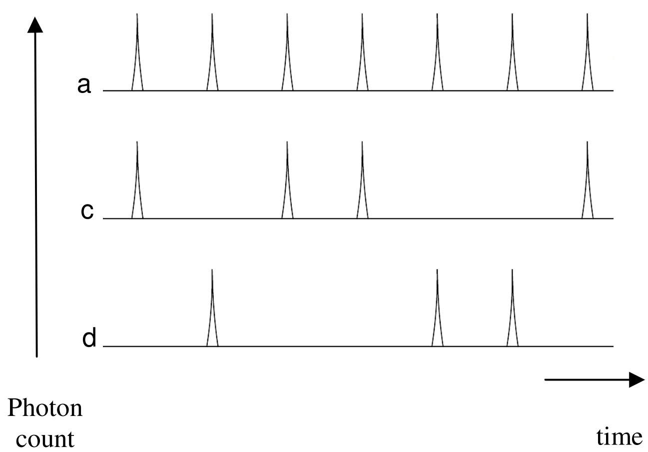


Figure 8b



4. Analysis

Let us analyse these arrangements. Figure 8a shows the enumeration scheme for the input and output ports for two sequential beamsplitters. We know that the transmittivity matrix will transform creation operators[9] at input port "a" to creation operators at output ports "c" and "d" thus (for a 50-50 beamsplitter):

$$\hat{a}_a^\dagger |0\rangle_a |0\rangle_b \rightarrow \frac{1}{\sqrt{2}}\left(\hat{a}_c^\dagger + i\hat{a}_d^\dagger\right)|0\rangle_c |0\rangle_d \qquad \text{eqn. 7}$$
$$= \frac{1}{\sqrt{2}}\left(|1\rangle_c |0\rangle_d + i|0\rangle_c |1\rangle_d\right)$$

The output is entangled (non-factorisable) and so the states are in coherent superposition. Referencing figure 8b, we can see that a sub-Poissonian/non-classical light source producing single photons (a "photon gun") at port "a" will upon detection of the outputs "c" and "d" have the photons randomly distributed across both output ports. In the parlance of the state vector approach, we might write the combined output state vector after measurement (each output port incident on separate detectors) as:

$$\psi_{cd} = \frac{1}{\sqrt{2}}\left(|1\rangle_c |0\rangle_d + e^{i\theta(t)}|0\rangle_c |1\rangle_d\right) \qquad \text{eqn. 8}$$

Where θ(t) is a random phase and random in time. Clearly the output of the detectors cannot coherently interfere. The density matrix treatment gives essentially the same reading:

$$|\psi_{cd}\rangle\langle\psi_{cd}| = \begin{pmatrix} \frac{1}{2} & \frac{1}{2}e^{-i\theta(t)} \\ \frac{1}{2}e^{i\theta(t)} & \frac{1}{2} \end{pmatrix} \qquad \text{eqn. 9}$$

The off-diagonal elements show the level of coherence and on the time-scale of the measurement, with the θ(t) term varying very fast, there is no interference.

Moving onto concatenated beamsplitters, the output state vector is spanned by the tensor product of all the output ports, such that a photon present at port "a" is transformed thus:

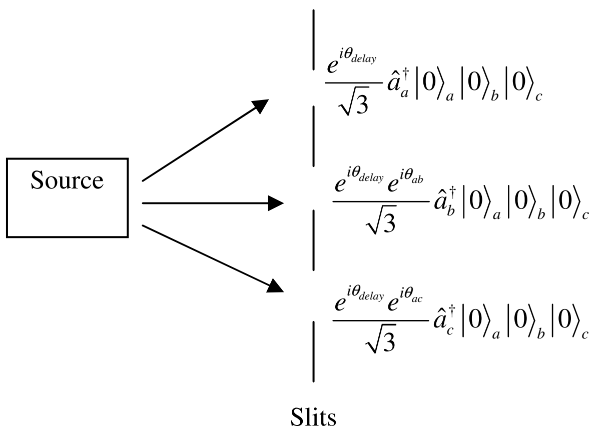
$$\hat{a}_a^\dagger |0\rangle_c |0\rangle_d \rightarrow \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\left(\hat{a}_g^\dagger + i\hat{a}_h^\dagger\right) + i\hat{a}_d^\dagger\right)|0\rangle_c |0\rangle_d |0\rangle_g |0\rangle_h$$
$$= \frac{1}{\sqrt{4}}|0\rangle_c |0\rangle_d |1\rangle_g |0\rangle_h + \frac{i}{\sqrt{4}}|0\rangle_c |0\rangle_d |0\rangle_g |1\rangle_h + \frac{i}{\sqrt{2}}|0\rangle_c |1\rangle_d |0\rangle_g |0\rangle_h$$
$$\text{eqn. 10}$$

If three detectors are placed at output ports "d", "g" and "h" the probability of detection, by Born's rule for distinguishable paths, would be:

$$P(photon) = \left|\frac{i}{\sqrt{2}}\right|^2 + \left|\frac{1}{\sqrt{4}}\right|^2 + \left|\frac{i}{\sqrt{4}}\right|^2$$
$$= 0.5 + 0.25 + 0.25 \qquad \text{eqn. 11}$$
$$= 1$$

Which is to be expected. However, in figure 5, the quantum rules for the probability at "Bob's" detector depend on "Alice's" measurement.

Figure 8c – 3-way splitter



A CRUCIAL DIFFERENCE

Attention is now drawn to the subtle difference between concatenated beamsplitters and 3-way splitters or n-way splitters in general. Consider first a 2-way splitter as described by eqn. 7: one can see how the creation operator is mapped to the output ports; we can elaborate that to show that there is a phase factor (a delay) $e^{i\theta_{delay}}$ between the input and output operators to allow for wave propagation and time of response for the materials to re-emit photons:-

$$\hat{a}_a^\dagger |0\rangle_a |0\rangle_b \rightarrow \frac{e^{i\theta_{delay}}}{\sqrt{2}}\left(\hat{a}_c^\dagger + i\hat{a}_d^\dagger\right)|0\rangle_c |0\rangle_d \qquad \text{eqn. 12}$$
$$= \frac{e^{i\theta_{delay}}}{\sqrt{2}}\left(|1\rangle_c |0\rangle_d + i|0\rangle_c |1\rangle_d\right)$$

As already mentioned, after measurement random phase factors will be introduced ("vacuum noise") into the non-entangled output states, which are either:

$$\frac{e^{i\theta_c(t)}}{\sqrt{2}}|1\rangle_c |0\rangle_d \quad \text{or} \quad \frac{e^{i\theta_d(t)}}{\sqrt{2}}|0\rangle_c |1\rangle_d \qquad \text{eqn. 13}$$

But the difference in the phases will always be equal to the input-output delay:

$$\theta_{delay} = \left|\theta_c(t) - \theta_d(t)\right| \qquad \text{eqn. 14}$$

Consider now a 3-way splitter (with only one input port), as indicated in figure 8c, made from a source and 3 slits. The mapping function is:

$$\hat{a}_0^\dagger|0\rangle_0 \to \frac{e^{i\theta_{delay}}}{\sqrt{3}}\left(\hat{a}_a^\dagger + e^{i\theta_{ab}}\hat{a}_b^\dagger + e^{i\theta_{ac}}\hat{a}_c^\dagger\right)|0\rangle_a|0\rangle_b|0\rangle_c \quad \text{eqn. 15}$$
$$= \frac{e^{i\theta_{delay}}}{\sqrt{3}}\left(|1\rangle_a|0\rangle_b|0\rangle_c + |0\rangle_a|1\rangle_b|0\rangle_c + |0\rangle_a|0\rangle_b|1\rangle_c\right)$$

(Clearly we show just the outputs normalised to each other and not to the source). The overall delay between input and output is once again $e^{i\theta_{delay}}$ and relative delays between the outputs (relative to port a) are $e^{i\theta_{ab}}$ and $e^{i\theta_{ac}}$.

Now upon measurement of one port, unlike the concatenated beamsplitters of figure 8a, the other two ports <u>will still be entangled</u>:

$$\frac{e^{i\theta_a(t)}}{\sqrt{3}}|1\rangle_a|0\rangle_b|0\rangle_c \quad \text{or} \quad \frac{e^{i\theta_{abc}(t)}}{\sqrt{3}}\left(|0\rangle_a|1\rangle_b|0\rangle_c + e^{i\theta_{bc}}|0\rangle_a|0\rangle_b|1\rangle_c\right)$$
eqn. 16

Where random phases $e^{i\theta_a(t)}$ and $e^{i\theta_{abc}(t)}$ have been introduced to reflect the process of randomness from wavefunction collapse and vacuum noise, subject, once again to:

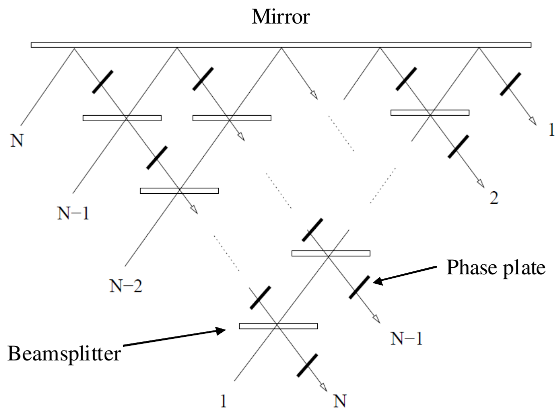
$$\theta_{delay} = \left|e^{i\theta_a(t)} - e^{i\theta_{abc}(t)}\right| \qquad \text{eqn. 17}$$

With a fixed relative phase between ports "b" and "c", $e^{i\theta_{bc}}$. This is consistent with the diagrams of figure 8b and the introduction of vacuum noise into the beamsplitter output upon measurement and how it become a particle splitter, though in this case with a 3-way splitter, a particle and entangled particle splitter.

So this begs the question: how is the concatenated, essentially 3-way splitter, of figure 8a (output ports "d", "g" and "h") different than the 3-way splitter of figure 8c or the general n-way splitter made from beamsplitters and phase plates of figure 8d? Why does measurement on one port of figure 8a render the other ports particle in nature ("distinguishable" paths) and not entangled, as figure 8c and 8d?

The answer to this is that with the concatenated beamsplitter of figure 8a vacuum noise (figure 8b) is admitted at port "f" when a measurement is performed. This renders, upon measurement at port "g", the path to port "h" distinguishable. Therefore ports "d" and "h" won't be in entanglement superposition when "g" is measured. One can clearly see this doesn't occur in figure 5c when measurement on any one port is conducted, so the remaining paths are indistinguishable nor does it occur in figure 8d.

<u>Figure 8d – N-way splitter</u>



<u>DISTINGUISHABLE AND INDISTINGUISHABLE PATHS WITH CONCATENATED BEAMSPLITTERS AND THE COMMUNICATION SCHEME</u>

Returning to figure 5, measurement implies that the paths are distinguishable and this implies adding probabilities (for 50:50 beamsplitters, other ratios can lead to tuneable probabilities),

$$\begin{aligned} \mathrm{P}(\text{Measurement, bit 1}) &= \left|\frac{i}{\sqrt{2}}\right|^2 + \left|\frac{i}{\sqrt{4}}\right|^2 \\ &= 0.5 + 0.25 \\ &= 0.75 \end{aligned} \qquad \text{eqn. 18}$$

No-measurement allows interference to occur by altering the path length, which is shown by the phase factor $e^{i\theta}$,

$$\begin{aligned} \mathrm{P}(\text{No-measurement, bit 0}) &= \left|\frac{i}{\sqrt{2}}\right|^2 + \left|\frac{e^{i\theta}}{\sqrt{4}}\right|^2 + 2\left|\frac{i}{\sqrt{2}}\right|\left|\frac{e^{i\theta}}{\sqrt{4}}\right|\cos(\arg\theta) \\ &= 0.5 + 0.25 + \frac{1}{\sqrt{2}}\cos(\arg\theta) \\ &= 0.75 \pm 0.707\cos(\arg\theta) \\ &= 0.043 \text{ minimum} \end{aligned}$$
eqn. 19

Figure 6 with its "balanced" set-up is analysed for the no-measurement and measurement conditions thus:

$$P(\text{Measurement, bit 1}) = \left|\frac{i}{\sqrt{4}}\right|^2 + \left|\frac{i}{\sqrt{4}}\right|^2$$
$$= 0.25 + 0.25$$
$$= 0.5 \qquad \text{eqn. 20}$$

And

$$P(\text{No-measurement, bit 0}) = \left|\frac{i}{\sqrt{4}}\right|^2 + \left|\frac{e^{i\theta}}{\sqrt{4}}\right|^2 + 2\left|\frac{i}{\sqrt{4}}\right|\left|\frac{e^{i\theta}}{\sqrt{4}}\right|\cos(\arg\theta)$$
$$= 0.25 + 0.25 + \frac{1}{\sqrt{4}}\cos(\arg\theta) \qquad \text{eqn. 21}$$
$$= 0.5 \pm 0.5\cos(\arg\theta)$$
$$= 0 \text{ minimum}$$

Conclusion

The focus of Cornwall's researches has been on utilising (or trying to obtain) expensive entangled sources. Apart from their obvious technical limitations, it has been proven in this paper that two particle entangled sources are not needed to affect Cornwall's[5] protocol (table 1). In due course, results of the experimentation will be presented.

Appendix: Analysis of the beamsplitter

A beamsplitter[10] can be considered a four port device. We consider the source only entering one port but the beam could enter via the other. Let us call the input ports 1 and 2 and the output ports 3 and 4. To a good approximation, a beamsplitter is lossless and hence unitary, we write the evolution of electric fields (or magnetic as) :

$$\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \to \begin{pmatrix} T & R \\ R & T \end{pmatrix}\begin{pmatrix} E_3 \\ E_4 \end{pmatrix} \qquad \text{eqn. 22}$$

Where the beamsplitter 2x2 matrix coefficients are complex numbers subject to the constraints:

$$|T|^2 + |R|^2 = 1$$
$$R^*T + RT^* = 0 \qquad \text{eqn. 23}$$

The reflected and transmitted intensities are given by $|R|^2$ and $|T|^2$ respectively. If we let T = 1 and R = $|R|e^{i\theta}$, then a 50:50 beamsplitter can be represented by the matrix:

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \qquad \text{eqn. 24}$$

Thus a reflected photon suffers a phase shift. Photon input at a port is represented by the creation operator associated with the port acting on the vacuum state. Thus we can write, respectively, for a photon at port 1 and then port 2 as:

$$\hat{a}_1^\dagger|0\rangle = |1\rangle_1|0\rangle_2$$
$$\hat{a}_2^\dagger|0\rangle = |0\rangle_1|1\rangle_2 \qquad \text{eqn. 25}$$

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