A brief essay on numerology of the mass ratio of proton to electron

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Abstract

There are few mathematical expressions for calculation proton to electron mass ratio presented. Some of them are new and some are not. They have been analyzed in terms of their simplicity, numerical significance and precision. Expressions are listed in the structured manner with comments. The close attention should be paid to a comparison of the formula similarity via their precision. A brief review of the different attempts in similar search is given.
Keywords: proton to electron mass ratio, fundamental dimensionless constant, numerology in physics.

1. Introduction

The founding of the analytical expression for fundamental dimensionless constant was a dream of a physical science for many years. There are many papers in literature trying to derive or explain fine structure constant from pure numerical theories. Such hypothetical theories can be divided into two types. The first ones propose that the dimensionless constants of the Nature are not actually constant and suggest using some close numbers which deviate from the original ones. These types of the theories require further experimental research because deviations of the dimensionless constants are still unknown with good precision. For example G. Gamov following Eddington's belief explained fine structure constant suggesting that it is equal to exactly 137 but it differs from exact number because of some quantum effects or fluctuations [7]. The second types of the theories are less common, they suggest exact relation for the dimensionless constants which is close to current experimental value. Usually such hypotheses derive formulas which are huge, unnatural and lack of elegancy and explain-ability. Moreover physical justification for such expressions doesn't have enough arguments or the physical model is absent.

The part of the physics which involves dimensionless constant is very prone to invasion of numerology. However such cooperation has not been shown to be efficient yet. Though it is worth to notice that numerology itself stays very close to algebra and number theory of mathematics. Numerology itself can be considered as ancient prototype of the modern algebra (as well as alchemy was a base for a modern chemistry) and as it was said by I. J. Good: "At one time numerology meant divination by numbers, but during the last few decades it has been used in a sense that has nothing to do with the occult and is more fully called *physical numerology*"[8]. At this perspective, physical numerology seems to be a way through backdoor which researches also try to enter and finding a key by trying to pickup right numbers.

2. Background

The search for mathematical expression for this dimensionless number motivated many serious scientists. A sufficient theory on particle masses and their ratios is not yet ready. The mass ratio of proton to electron – two stable particles which belong to two different types of elementary particles (leptons and hadrons) - remains the mystery among other dimensionless numbers.

In 1929 Reinhold Furth hypothesized that μ can derived from the quadratic equation which involves the fine structure constant [5]. Later on in 1935, A. Eddington who accepted some of Furth's ideas presented the equation for proton to electron mass ratio calculation ($10\mu^2$-136μ+1=0) which appears in his book «New Pathways in Science» [6]. However both approaches can not be used nowadays as they give very high deviation from the currently known experimental value of μ, so they are not reviewed in present work. Later on in 1951, it was Lenz [12] (but not Richard P. Feynman!) who noted that μ can be approximated by $6\pi^5$. In 1990-th I. J. Good, British mathematician has assembled eight conjectures of numerology for the ratio of the rest masses of the proton and the electron. Nowadays proton to electron mass ratio is known with much greater precision: μ= m_p/m_e =1836.152672 45(75), with uncertainty 4.1×10^{-10} (CODATA 2010, [4]). Recently the professional approach to mathematically decode m_p/m_e mass ratio was done by Simon Plouffe [17]. He used a large database of mathematical constants and specialized program to directly find an expression. Alone with his main remarkable result for the expression for μ via Fibonacci and Lucas numbers and golden ratio he also noted that expression for μ using π can be improved as $6\pi^5 + 328/\pi^8$, but he noted that "it hardly can be explained in terms of primes and composites".

2. Variability

The possible variability of the μ can not prevent further search for the numerical expression. It even motivates stronger because the variation means one have to find mean value of its oscillation or the beginning value from where it has started to change. And such variation would give a wider space for the further numerical sophistication because such value can not be verified immediately as we currently lack experimental verification of the amount of such change

Reinhold et al. [18] using the analysis of the molecular hydrogen absorption spectra of quasars Q0405-443 and Q0347-373 concluded that μ could have decreased in the past 12 Gyr and $\Delta\ \mu/\mu = (2.4\pm0.6)\times10^{-5}$. This corresponds to entry value of μ= 1836.19674. King et al. [9] reanalyzed the spectral data of Reinhold et al. and collected new data on another quasar, Q0528-250. They estimated that $\Delta\mu/\mu=(2.6\pm3.0)\times10^{-6}$, different from the estimates of Reinhold et al. (2006). So the corresponding value for maximal deviated μ to be something around 1836.1574.

The later results from Murphy et al. [15] and Bagdonaite et al. [2] gave a stringent limit $\Delta\mu/\mu < 1.8\times10^{-6}$ and $\Delta\mu/\mu = (0.0\pm1.0)\times10^{-7}$ respectively. However these deviations could be valid only for the half of the Universe's current age or to the past of 7 Gyr which may not be enough for full understanding of the evolution of such variation. The results obtained by Planck gave $\Delta\alpha/\alpha=(3.6\pm3.7)\times10^{-3}$ and $\Delta m_e/m_e=(4\pm11)\times10^{-3}$ at the 68% confidence level [1] which provided not so strong limit comparing to found in [18] and [9].

The subject of variability of μ is still under heavy debate and further confirmation and the experimental data are required. If the fundamental constants are floating and the Nature is fine-tuned by slight the ratio changes from time to time, even so, there should be middle value as the best balance for such fluctuations. In this sense numerologists are free to use more relaxed conditions for their search, and the precision for μ with uncertainty of 2×10^{-6} (as discussed above) may suffice for their numerical experiments. The formulas listed after number 7 in the table below do fall into this range.

3. The table

Expression	Value	Comment below
$\mu = \left(\frac{7}{2}\right)^6$	**183**8.2656 (1×10^{-3})	1.
$\mu = \sin\left(\frac{\pi}{5}\right) * 5^5$	**1836**.8289 (4×10^{-4})	2.
$\mu = \frac{17}{4}432$	**1836**.0000 (8×10^{-5})	3.
$\mu = 150^{\frac{3}{2}} - 1$	**1836.1**173 (2×10^{-5})	4.
$\mu = 6\pi^6$	**1836.1**181 (2×10^{-5})	5.
$\mu = \frac{2^{300}}{7^{103}}$	**1836.1**179 (2×10^{-5})	6.
$\mu = \frac{22}{5^2 * 3^2}\alpha^{-2}$	**1836.15**56 (2×10^{-6})	7.
$\mu = \frac{5*7^3}{6*67}137\pi$	**1836.15**14 (6×10^{-7})	8.
$\mu = \frac{2^4 3^5}{5\alpha^{-1}}103\pi$	**1836.152**20 (3×10^{-7})	9.
$\mu = \frac{e^8 - 10}{\phi}$	**1836.15**301 (2×10^{-7})	10.
$\mu = \frac{40}{3}\alpha^{-1} + \frac{800}{9\pi^2}$	**1836.152**98 (2×10^{-7})	11.
$\mu = \frac{86^4}{31^3}$	**1836.152**39 (2×10^{-7})	12.
$\mu = \frac{2267^2}{5*7*11*\alpha^{-1}}6\pi$	**1836.152**5639 (6×10^{-8})	13.
$\mu = \frac{11^2 * 5^{\frac{4}{5}} * 7^{\frac{2}{5}} e^3}{6 * 2^{\frac{4}{5}}}$	**1836.15267**03 (1×10^{-9})	14.
$\mu = \frac{55 * 5^{\frac{3}{2}} * 11^{\frac{15}{32}}}{\phi^{\frac{1}{16}}}$	**1836.15267**48 (1×10^{-9})	15.
$\mu = \frac{3^{\frac{15}{4}} 5^{\frac{9}{4}} 14^{\frac{3}{2}}}{\pi^3 e^{\frac{3}{4}}}$	**1836.15267**19 (1×10^{-10})	16.

Comments

1. This expression is not very precise and given for its simplicity only. The number 7/2 definitely has certain numerological significance. It is not trivial task to improve the formula accuracy, but why not for example: $\mu = \left(\frac{7}{2}\right)^8 \frac{9*13}{10\pi * \alpha^{-1}}$ (relative error: 10^{-6}).

2. It is well known [17] that m_p/m_n ratio can be well approximated as $cos\left(\frac{\pi}{60}\right)$ with relative uncertainty of $6*10^{-6}$. So this is an attempt to build the formula for m_p/m_e ratio with similar form

3. It was Werner Heisenberg in 1935 [10] who suggested to use number $2^4 3^3$ (which is equal to 432) to calculate alpha as $\alpha^{-1}=432/\pi$, so m_p/m_e ratio can be also obtained approximately via 432. The expression can be also rewritten as 1836=17*108 (the number 108 is considered to be sacred by several Eastern religions). There are other possible representations for the number 1836 which were noticed in the past, for example: 1836=(136*135)/10 (see review in [8] and [19]).

4. This expression has some certain theoretical base related to original R.Furth ideas [5], but it won't be discussed here. The precision has the same order as famous $6\pi^5$.

5. Reviewed above.

6. The simplest way to approximate m_p/m_e ratio using powers of 2 and 7. The value of the expression better fits to the value of the m_n/m_e ratio (relative uncertainty is $2*10^{-4}$).

7. The elegant expression which uses kabalistic numbers 22, 5, 3 and fine structure constant.

8. Parker-Rhodes in 1981, see [16] and review in [8]. McGoveran D.O. [14] claimed that this formula does not have anything in common with numerology as it was derived entirely from their discrete theory.

9. This elegant expression uses only alpha, power of 2, 3, 5 and the number 103. As J.I. Good said: "the favoured integers seem all to be of the form $2^a 3^b$." [8]

10. By unknown source. No comment.

11. The expression can be also rewritten in the form $\mu = 2*\left(\frac{20}{3}\alpha^{-1} + \left(\frac{20}{3\pi}\right)^2\right)$. It can be noted that the number 20/3 appears in the author previous work [11] in the expression for gravitational constant G.

12. One of the found expressions by author's specialized program. The search was performed for the expression of the view: $\mu = p_1^{n1} p_2^{n2} p_3^{n3} p_4^{n4}$, where p_i – some prime numbers, n_i – some natural numbers.

13. Number 2267 has many interesting properties; it is a prime of the form 30n-13 and 13n+5, it is congruent to 7 mod 20. It is father primes of order 4 and 10 e t c. In the divisor of this formula there are sequential primes 5,7,11. There are other possible expressions of the similar form with similar precision (10^{-8}), for example $\mu = \frac{9*5*49*53^2}{8*29*\alpha^{-1}} 5\pi$. It is also hard to justify why α^{-1} in expressions 9 and 13 stays opposite to π as by definition they supposed to be on the same side: $\alpha^{-1}=\hbar c/ke^2$ or ($2\pi\ \alpha^{-1}$) $=hc/ke^2$. But the author did not succeed in finding similar expressions with alpha and π on the same side with the same uncertainty. There are some few other nice looking formulas which use big prime numbers, for example: $\mu = \sqrt{4^3 * 52679}$ (9×10^{-8}).

14. Another possible expression was found using web based program Wolframalpha[20]. The precision is the same as in 14.
15. Simon Plouffe's approximation using Fibonacci and Lucas numbers [17] - slightly adjusted from its original look. Another beautiful form of this formula is following:

$$\mu^{32} = \frac{11^{47} 5^{80}}{\phi^2}$$

16. This formula has the best precision alone the listed. Though, powers of π and e seem to despoil its possible physical meaning.

Conclusions

At the present moment big attention is paid to experimental verification of possible proton-electron mass ratio variation. If experimental data will provide evidence for the ratio constancy then only few expressions (14-16 from the listed) may pretend to express proton-electron mass ratio as they fall closely into current experimental uncertainty range (4.1×10^{-10} as per CODATA 2010). Of course Simon Plouffe's formula (14) seems as a pure winner among them in terms of the balance between it simplicity and precision. However, some future hope for the other formulas remains if the variability of the proton to electron mass ratio is confirmed. Important to note that there could be unlimited numbers of numerical approximations for dimensionless constant. Some of them may look more simple and "natural" than others. It is easy to see that expression simplicity and explain-ability in opposite determines its precision. As all formulas with uncertainty 10^{-8} and less become obviously more complex. And at the end "What is the chance that seemingly impressive formulae arise purely by chance?" [3].

Remembering mentoring words said by Seth Lloyd [13] "not to follow in Dirac's footsteps and take such numerology too seriously" the author encourages the reader to continue such mathematical experiments and in order to extend the table of the formulas and submit your expressions to the author. Special attention will be brought to simple expressions with relations to: power of two (2^n), prime numbers and properties of Archimedean solids. Besides that it may be interesting mathematical exercise it may also reveal some hidden properties of the numbers. But how complexity of the mathematical expression can be connected to the complexity of the numbers? What is the origin of the Universe complexity? How much we can encode by one mathematical expression?

The mass ratio of proton to electron – two stable particles - defines approximately 95% of the visible Universe mass and those can be related to the total value Computational capacity of the Universe (see [13]). So as a pure numbers they supposedly have to be connected to prime numbers, entropy, binary and complexity. So, possibly, their property should be investigated further by looking through the prism of the algorithmic information theory.

Let's hope that presented material can be a ground for someone in his future investigation of this area.

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