

On Religious Prosociality, Fairness, and Beauty

Ramzi Suleiman^{a,b}

^a Department of Psychology, University of Haifa

^b Department of Philosophy, Al-Quds University

First, unedited draft 15.10.2014

Please address all correspondence to Dr. Ramzi Suleiman, University of Haifa, Haifa 31509, Israel. Email: suleiman@psy.haifa.ac.il, Mobiles: 972-(0)50-5474- 215, Fax: 972-(0)4-8240-966.

On Religious Prosociality, Fairness, and Beauty

Abstract

Previous theoretical and empirical studies of the effects of religion on adherents' prosociality have focused, almost entirely on the intragroup and intergroup aspects of religion, as facilitators of prosocial attitudes and behaviors. The moral values preached by various religions were treated inseparably, as part of a whole of religious practices, which contribute to the evolution of cohesive, more adapted, large groups. The present research takes another direction by addresses the question pertaining to the *intrinsic* effects of religious moral principle on its adherents' economic behavior. For symmetric and asymmetric economic interactions between two rational individuals who nonetheless follow the moral maxim "treat others as you treat yourself", we solved for the divisions of profit which guarantee equal levels of satisfaction of the interacting parties

As expected, for the symmetric case, modeled as a symmetric two-person resource dilemma (CPR) game, we find that adherence to the discussed moral principle prescribes an equal division of the shared resource. For the asymmetric game, modelled as a dictator game, the solution depends on the assumption made about how outcome satisfaction is measured. Under the conventional definition of level of satisfaction as the difference between actual and aspired outcomes, the model prescribes a division of $(\frac{2}{3}, \frac{1}{3})$, for the allocator and recipient, respectively. Strikingly, under a new definition which compares the actual and aspired outcomes on a *ratio scale*, rather than a *difference scale*, the model prescribes a division $(\Phi, 1 - \Phi)$, or $\approx (0.62, 0.38)$, for the allocator and recipient, respectively, where Φ is the famous Golden Ratio. We discuss the obtained solutions, suggest experimental tests of their predictions and comment about possible connection between human's perception of fairness and their aesthetic tastes.

Keywords: Prosociality, Religion, Morals, Fairness, Pay satisfaction, Aesthetics, Golden Ratio.

1. Introduction

Recent research on prosocial behavior has witnessed a growing interest in the experimental and theoretical investigation on the effect of religion on prosociality. Aside from the fact that religious texts of all major religions explicitly encourage prosociality in their adherents [1-3], empirical evidence suggests that religion can promote intragroup cooperation. For examples, individuals who frequently pray and attend religious services reliably report more prosocial behavior, such as charitable donations and volunteerism [1, 4]. Sosis and Ruffle [5] tested the relationship between religious ritual and cooperation. They conducted an experiment in which participants, sampled from Israeli religious and secular kibbutz members, were requested to simultaneously divide a given sum of money between themselves and an anonymous participant from their own kibbutz. Their findings provided support for the hypothesis that collective ritual can promote cooperation. Controlling for a host of significant predictors, religious males on average were significantly more cooperative than religious females, secular females, and secular males. Worth noting that several studies have suggested that the positive association between religion and prosociality is most evident when the situation calls for maintaining a favorable social reputation within the group [2].

Recent evolutionary theories of religion [6-11] agree that selective pressures over the course of human evolution can explain the wide cross-cultural reoccurrence, historical persistence, and predictable cognitive structure of religious beliefs and behaviors. In a paper published in *Science*, Norenzayan and Shariff [12] argued that religion's effect on prosocial tendencies depends on reputational sensitivity. They further argue that "cognitive awareness of gods is likely to heighten prosocial reputational concerns among believers, just as the cognitive awareness of human watchers does among believers and non-believers alike. However, supernatural monitoring, to the degree that it is genuinely believed and cognitively salient, offers the powerful advantage that cooperative interactions can be observed even in the absence of social monitoring" (p. 58).

Despite the importance of empirical studies on the relationship between religion and prosociality, and its evolutionary origins, the research focus of survey and experimental studies is confined to comparison between prosocial attitudes and behaviors of religious vs. non-religious individuals. Most theoretical research, on the other hand, has been directed to investigating the evolution of religions as a social groups, and the instrumentality of religious morals, ceremonies and other worship practices, in enhancing large groups' cohesiveness, resulting in increased prosocial behaviors between ingroup members, which

in turn is bound to increase the groups' fitness and competitive edge with other groups [12-13]. Taken together, the above mentioned lines of empirical and theoretical research have almost conclusively looked at religions as a distinct type of *social groups*, and on their characteristic social and psychological intragroup and intergroup processes. Religious morals were considered only in the extent that they are instrumental in promoting group cohesiveness, which in turn is expected to increase in-group prosociality in service of better addictiveness. Sosis and Alcorta [13] express a similar evaluation by stating that "Whether positing that the function of religion is to foster social cohesion, preserve the social order, or maintain population-resource homeostasis, all of these researchers have interpreted religion from a group perspective" (p.266).

In the present paper we take a different perspective by directing attention to the *intrinsic* effects of religious laws and commandments on believers' pro-social attitudes and behaviors. For this purpose we contemplate on adherence to the moral rule "treat others as you treat yourself", preached by most religions ⁽¹⁾, on the behavior and outcomes of rational adherents to the rule. To keep the analysis simple, we focus on a simple economic interaction involving the division of a given resource between two individuals. We assume the two parties in the interaction are by their nature rational, self-interested individuals whose rationality is constrained by adherence to the above stated moral. We discuss two situations: A *symmetric* situation, in which the two interacting individuals have equal decision control on dividing the goods, and an *asymmetric* situation in which one individual has complete decision control, and the other has no decision control. In game theory, the first situation could be modeled by a two-person simultaneous common pool resource dilemmas game (CPR) [14], while the second situation could be modeled by a dictator game [15]. In real life cases, division of wealth between two individuals who have equal ownership or division of irrigation water between farmers with equal consumption rights are CPR situation, while the division of a profit between an employer and his or her temporary employee, in the absence of regulations, and when no agreement or pre-commitment has been made, is a dictator game.

(1) The Christian Bible teaches that "the whole law is fulfilled in one word: 'You shall love your neighbor as yourself.' (Galatians 5:14). In the Islamic "*Hadith*", which contains the oral teachings of Prophet Mohamed, the same rule appears as: "*None of you truly believes until he loves for his brother what he loves for himself.*" And in the Bible of Judaism, the same rule appears as "thou shalt love thy neighbor as thyself" (Leviticus 19:18).

For the two described situations, assuming that the individuals' rationality is constrained by the above moral rule, we theorize about the allocation decision and their consequent outcomes for the involved individuals and for the collective as a whole. Expectedly, we find that constraining of rational, self-favoring motives by the above moral, leads to fair, although unequal shares, expressed in simple, and aesthetically pleasing proportions between prominent algebraic numbers.

The remainder of the paper is organized as follows. In section 2 and 3 we formalize the above discussed situations, and propose possible solutions for the allocation decisions, depending on the assumption made on how outcome satisfaction is defined. In section 4 we conclude with a brief discussion and suggestions for experimental research.

2. A Symmetric resource allocation case (CPR game)

Consider a situation in which two individuals have equal property rights on a sum of S monetary units (MUs), which they must divide between themselves. Suppose that they bargain on the shared amount by having each one propose a division, until an agreement is reached, otherwise no deal is done. Assume that the two individuals are self-interested "players", but whose economic decisions are constrained by the maxim "treat others as you treat yourself". In this case symmetry considerations leads to the intuitive solution of the equal split. This prediction agrees with the equality principle as a prominence of equality as a decision heuristic [16]. A formal solution based on the assumption that a best settlement is one which guarantees equal levels of outcome satisfaction yields the same result. We prove that by solving for the players' decisions under using two general definitions of levels of satisfaction: the conventional definition, used in psychological [17-19] and evolutionary [20] research, which posits the level of satisfaction of an individual (LS) is proportional to the difference between actual and aspired outcome, or $x - A$, where A is the player's aspiration level, and a novel definition [21], in which the individual LS is proportional to the ratio between the actual and aspired outcomes (x/A). As self-interested players, each of the two interacting individuals would aspire for receiving the entire sum S , or S minus an infinitesimal amount ϵ to be transferred to the other player. However, abiding to the above mentioned maxim dictates that if a player i ($i = 1, 2$) asks for x_i (MUs), he or she should transfer an amount of $y_i = x_i$ ($i = 1, 2$) to his or her counterpart. Since agreement entails equal levels of outcome satisfaction, using the conventional difference measurement scale for levels of aspiration, under the assumption of linearity, we can write:

$$x_1 - A_1 = x_2 - A_2 \quad \dots (1)$$

For two rational players, assuming $S=1$ for simplicity, we have:

$$A_1 = A_2 = 1 - \varepsilon \ (\varepsilon \rightarrow 0), \quad \dots (2)$$

$$\text{Thus: } x_1 = x_2, \ y_1 = x_1 = 1 - x_1, \ \text{And } y_2 = x_2 = 1 - x_2, \quad \dots (3)$$

Yielding:

$$x_1 = x_2 = \frac{1}{2} \ (\text{and } x_1 = x_2 = \frac{1}{2}) \quad \dots (4)$$

It is easily seen that using a ratio measurement scale yields the same result, of each player receiving half of the entire amount.

3. Asymmetric resource allocation case (Dictator game)

Now consider an asymmetric case in which player 1 is a dictator who have total decision control on dividing the amount, while player 2 is a passive recipient. In this case we cannot assume that both players have the same aspiration levels. As a rational player the allocator would aspire for receiving the entire amount minus ε ($\varepsilon \rightarrow 0$), or $A_1=1 - \varepsilon$. The recipient can only aspire that the allocator, as adherent of the moral rule, would allocate to him the same amount that she would keep to herself. In formal terms, the recipient's aspiration level is only $A_2= x$, where x the amount that the allocator would keep to herself. In terms of levels of satisfaction, the allocation x that would guarantee an equal treatment should satisfy: $LS_1 = LS_2$, or:

$$x - A_1 = (1-x) - A_2 \quad \dots (5)$$

Substituting $A_1= 1 - \varepsilon$ and $A_2 =x$ and solving for x we have:

$$x = \frac{2}{3} (1 - \varepsilon) \quad \dots (6)$$

and

$$1-x = \frac{1}{3}(1 + 2 \varepsilon) \quad \dots (7)$$

For $\varepsilon \rightarrow 0$ we have the division of $(\frac{2}{3}, \frac{1}{3})$, for the allocator and recipient, respectively.

If we defining the levels of satisfaction using a *ratio* scale, the players' levels of satisfaction become:

$$LS_1 = \text{allocator actual payoff / aspired payoff} = \frac{x}{A_1} = \frac{x}{1-\varepsilon}$$

$$LS_2 = \text{recipient actual payoff / aspired payoff} = \frac{1-x}{A_2} = \frac{1-x}{x}$$

Equal levels of satisfaction are achieved when:

$$\frac{x}{1-\varepsilon} = \frac{1-x}{x} \quad \dots (8)$$

Or:

$$x^2 + (1-\varepsilon)x - (1-\varepsilon) = 0 \quad \dots (9)$$

Which solves for:

$$x = \frac{\sqrt{(1-\varepsilon)^2 + 4(1-\varepsilon)} - (1-\varepsilon)}{2}, \quad \dots (10)$$

and for $\varepsilon \rightarrow 0$ we get:

$$x = \frac{\sqrt[3]{5}-1}{2} = \Phi, \text{ and } 1-x = 1-\Phi, \quad \dots (11)$$

Where Φ is the famous Golden Ratio [22-23], equaling $\approx 0.618^{(2)}$, with corresponding division of ≈ 0.618 and 0.382 for the allocator and the recipient, respectively.

(2) The Golden ratio is commonly known as $\varphi = \frac{\sqrt[3]{5}+1}{2} \approx 1.618$. The two Golden Ratios Φ and φ are related by the relationships $\Phi = 1 - \varphi = \frac{1}{\varphi}$.

4. Summary and concluding remarks

Previous theoretical and empirical studies of the effects of religion on adherents' prosociality have focused, almost entirely on the intragroup and intergroup aspects of religion, as facilitators of prosocial attitudes and behaviors [12-13]. The moral values preached by religions were treated inseparably, as part of a whole lump of religious practices, which contribute to the evolution of cohesive, more adapted, large groups. The present research is the first, known to us, which addresses the question pertaining to the *intrinsic* effects of religious moral principle on its adherents economic behavior. For a hypothetical society comprised of self-interested individuals who nonetheless follow the moral maxim "treat others as you treat yourself", under simplifying assumptions of linearity we solved for the division of profit between two individuals in simple symmetric and asymmetric interactions. As expected, for a symmetric case, modeled as a two-person resource dilemma (CPR) with equal property rights, we found that adherence to the moral principle enforces an division of the shared resource. For an asymmetric case, modelled as a dictator game, the solution, under the moral principle constraint depends on the assumption made about how outcome satisfaction is defined and measured. Under the conventional definition of level of satisfaction as the difference between actual and aspired outcomes, yielded a division of $(\frac{2}{3}, \frac{1}{3})$, for the allocator and recipient, respectively. Strikingly, under our novel definition which compares the actual and aspired outcomes on a ratio scale, rather than a difference scale, the discussed moral constraint yielded the division $(\Phi, 1 - \Phi)$, or $\approx (0.62, 0.38)$, for the allocator and recipient, respectively, where Φ is the famous Golden Ratio.

We contend that the ratio scale is preferable than the commonly used difference scale for defining outcome satisfaction. The ratio scale is most common in physics, biology, evolution and other exact sciences. It is also the standard practice in psychophysics, starting from Fechner's law [24-25] and Steven's power law [26-27], to more recent theories of audio and visual perception [28-29] and signal detection [30]. Moreover, the ratio scale has some unique advantages: (1) it is dimensionless and does not depend on the measurement units of the divided goods. (2) All types of statistical measures are applicable to ratio scales, and only with these scales may we properly indulge in logarithmic transformations such as are involved in astronomical measures of redshift, in music, sound analysis (decibels), and more. In fact the primary ratio scale is the scale of integer number itself [26].

Regardless of the definition adopted for level of aspiration, the important point to stress here, is that in the two measurement scales, the obtained solutions game is a reasonably

equitable, ranging between an equal split in the symmetric case, to $\frac{2}{3}$ or ≈ 0.618 for the player who has complete control in the asymmetric case, depending on how levels of aspiration are measured. No less important, assuming that the individuals adhere to the discussed moral principle, the relatively fair allocations are predicted without assuming reputation effects or social rewards and punishments.

In the above analysis we assumed that a rational allocator, would aspire for the entire amount. However, it is more realistic to assume that rational allocators, who cannot assume that the recipient are rational, might expect that positive but low offers are likely to be rejected [31-32]. Relaxing the model, by assuming that allocators might aspire for any amount between the entire amount, $S = 1$, and $S = 1 - \alpha$, where $0 \leq \alpha \leq 0.5$, is a "security factor" [33], reveals that under plausible assumptions about the allocators' aspiration levels, the resulting solutions are only 2%-3% higher than the solutions derived under the complete rationality assumption [21]. Moreover, extension of the solution to account for nonlinearity in the aspiration levels function is straightforward [34].

Fair divisions and aesthetics

Interestingly, all the derived solutions are very aesthetically pleasing numbers, and as such are found in various fields of the arts. Most prominent is the Golden Ratio, known for its numerous appearances in arts and aesthetics [22-23, 35-36], design [37], and music [38], as well as in the physical and biological sciences [39-41], market behavior [42], ethical judgment [43], brain functioning [44-45], and more. The most famous works of art with Golden Ratio symmetries is Leonardo da Vinci's "Mona Lisa" and "The Last Supper" [46], and in Salvador Dali's "The Sacrament of the Last Supper" (see Image 1). While some researchers doubt that da Vinci was conscious to the Golden Ratio in his paintings [22], there is very little doubt that Salvador Dali did deliberately include the Golden Ratio in his art. The other ratios which emerged as possible solutions are also aesthetically pleasing. The ratio $\frac{1}{2}$ is a focal symmetrical point in all measurement scales. In geometry it appears in the formulas of areas of triangles, trapezoids. Similarly, the ratio $\frac{1}{3}$ appears in the formulas of the volumes of pyramids and cones. In music, the two ratios are important measures of tonality (i.e., the half-tone and third-tone).

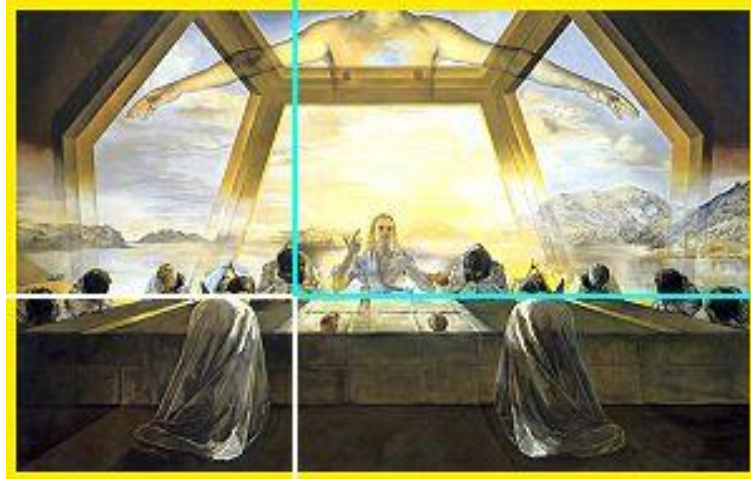


Image 1: The Sacrament of the Last Supper. Salvador Dali framed his painting in a golden rectangle.



Image2: Sunflower (photo by Yves Couder). The numbers and arrangements of petals, leaves, sections and seeds in most plants are Fibonacci numbers. The sunflower in the picture has 55 petals spiral

The Golden Ratio is also intimately connected with the Fibonacci series found in the physical and life sciences (see Fig. 2), as well as in human work of art and design [22-23]. In the Fibonacci series: 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, ..., each term f_n is the sum of the two preceding terms.

Interestingly, the three ratios which emerges as solutions ($\frac{1}{2}$ and $\frac{2}{3}$ and the Golden Ratio) could be expressed as ratios between successive elements ($\frac{f_n}{f_{n+1}}$) in the Fibonacci series, where f_n denotes the nth. element in the series. For $n = 1$ we have $\frac{f_1}{f_2} = \frac{1}{2}$, for $n=2$ we have $\frac{f_2}{f_3} = \frac{2}{3}$, and for $n \rightarrow \infty$ we have $\lim_{n \rightarrow \infty} \frac{f_n}{f_{n+1}} = \Phi \approx 0.618$. In fact, convergence to the Golden Ratio is quite fast, such that for $n \geq 7$, $\frac{f_n}{f_{n+1}} \approx 0.618$.

The proposed solution yields interesting testable hypothesis. As example, based on our main conclusion, we hypothesize that believers in the maxim "treat others as you treat yourself", will behave more fairly than non-believers, and that priming the above mentioned maxim, either consciously, by a pre-experimental discussion, or subliminally by standard implicit priming methods, would increase the amount transferred by believers and non-believers in resource allocation games. The suggested relationship between fairness and aesthetics could also be subjected to direct empirical test. For example, we hypothesize that a fair division will arouse similar physiological and neurological responses, to the responses aroused when perceiving aesthetically pleasing visual or auditory stimuli. Chapman et al. [47] had recently confirmed a comparable hypothesis. In a study published in *Science*, the authors demonstrated that photographs of disgusting contaminants, and receiving unfair offers, evoked similar activation of the muscle region of the face characteristic of an oral-nasal rejection response.

It is important to note that proposition that abiding to the discussed religious moral increases fairness and cooperation is not meant as a statement on the role played by religion as a whole. What we suggest is that abiding to the maxim "treat others as you treat yourself" is in itself sufficient for producing fair allocation, regardless of whether the interacting parties are religious or non-religious. In fact, one might argue, that religion, as a social institution, promotes inter-group conflicts, no less that it promoter in-group solidarity. In fact, human history is saturated with intergroup conflicts in which religion was a major ingredient, and wars perpetuated by religious extremity have recently erupted in many countries in the middle-east and elsewhere around the globe. We also agree with view expressed in [12] that although religions continue to be powerful facilitators of prosociality in large groups,

reliable secular institutions, such as secular educational civil society institutions, have significant roles in promoting prosocial attitudes and behaviors.

References

- [1] Monsma, S. V. Religion and philanthropic giving and volunteerism: Building blocks for civic responsibility. *Interdisciplinary Journal of Religious Research*, 3, 2-28, 2007.
- [2] Batson, C. D., Schoenrade, P. & Ventis, W. L. *Religion and the individual*, Oxford University Press, New York, 1993.
- [3] Durkheim, E. *The elementary forms of religious life*, Free Press, New York, 1995.
- [4] A. Brooks, Who Really Cares? *The surprising truth about compassionate conservatism*, Basic Books, New York, 2006.
- [5] Sosis, R., and Ruffle, B. J. Religious Ritual and Cooperation: Testing for a Relationship on Israeli Religious and Secular Kibbutzim. *Current Anthropology*, 44 (5), 713-722, 2003.
- [6] S. Atran, A. Norenzayan, Behav. Religion's evolutionary landscape: Counterintuition, commitment, compassion, communion. *Behavioral and Brain Sciences*, 27, 713- 770, 2004.
- [7] Boyer, P. *Religion Explained*, Basic Books, New York, 2001.
- [8] Boyd, R, and Richerson, P. Group Beneficial Norms Can Spread Rapidly in a Structured Population. *Journal of Theoretical Biology*, 215 (3), 287 - 296, 2002.
- [10] Henrich, N., and Henrich, J. *Why Humans Cooperate: A Cultural and Evolutionary Explanation*. Oxford University Press, 2007.
- [11] Wilson, D. S. *Darwin's Cathedral*, University of Chicago Press, 2002.
- [12] Norenzayan, A., and Shariff, A. F. The origin and evolution of religious prosociality. *Science*, 322 (5898), 58- 62, 2008.
- [13] Sosis, R., and Alcorta, C. Signaling, Solidarity, and the Sacred: The Evolution of Religious Behavior. *Evolutionary Anthropology*, 12, 264–274, 2003.
- [14] Ostrom, E., Gardner, R., and Walker, J. *Rules, games, and common-pool resources*. University of Michigan Press. 1994.
- [15] Bardsley, N. Dictator game giving: altruism or artefact? *Experimental Economics*, 11, 122 -133, 2008.
- [16] Messick DM. 1993. Equality as a decision heuristic. In B. Mellers & J. Baron (Eds.). *Psychological Perspectives on Justice: Theory and Applications* (pp. 11–31). Cambridge University Press, 1993.
- [17] Simon, H. A. Theories of decision-making in economics and behavioral science. *The American Economic Review*, 49 (3), 253-283, 1959.

- [18] Lopes, L. L., & Oden, G. C. The role of aspiration level in risky choice: A comparison of cumulative prospect theory and SP/A theory. *Journal of Mathematical Psychology*, 43, 86-313, 1999.
- [19] Rieger, M. O. (2010). SP/A and CPT: A reconciliation of two behavioral decision theories. *Economics Letters*, 108 (3), 327–329, 2010.
- [20] Chen, X. and Wang, L. Promotion of cooperation induced by appropriate payoff aspirations in a networked small-world game. *Physical Review E*, 77, 017103, 2008
- [21] Suleiman, R. What is a fair salary? Paper under consideration, 2014. <http://vixra.org/pdf/1409.0075v5.pdf>.
- [22] Livio, M. *The Golden Ratio: The Story of Phi, the World's Most Astonishing Number*. New-York: Broadway Books, 2002.
- [23] Posamentier, A. S., & Lehmann, I. *The fabulous Fibonacci Numbers*. Prometheus Books, 2007.
- [24] Fechner, G. T. *Elemente der Psychophysik* (Elements of Psychophysics), 1860.
- [25] Gescheider, G. *Psychophysics: the fundamentals* (3rd ed.). Lawrence Erlbaum Associates, 1997.
- [26] Stevens, S. S. On the theory of scales of measurement, *Science*, 103 (2684), 677-680, 1946.
- [27] Stevens, S.S. The psychophysics of sensory function. *American Scientist*, 48 (2), 226-253, 1960.
- [28] Luce, R. D., Steingrimsson, R., and Narens, L. Are psychophysical scales of intensities the same or different when stimuli vary on other dimensions? Theory with experiments varying loudness and pitch. *Psychological Review*, 117 (4), 1247-1258, 2010.
- [29] Palmer, S. E. *Vision Science - Photons to Phenomenology*, MIT Press, Cambridge, MA, 1999.
- [30] Posch, M. Win–Stay, Lose–Shift Strategies for Repeated Games—Memory Length, Aspiration Levels and Noise, *Journal of Theoretical Biology*, 198, 183–195, 1999.
- [31] Harrison, G.W. and K.A. McCabe, Expectations and fairness in a simple bargaining experiment. *International Journal of Game Theory*, 25, 303-327, 1996.
- [32] Hoffman, E., McCabe, K., Shachat, K., & Smith, V.. Preferences, property rights and anonymity in bargaining games. *Games and Economic Behavior*, 7, 346-380, 1994.
- [33] Lopes, L. L. Between hope and fear: The psychology of risk. *Advances in Experimental Social Psychology*, 20, 255-295, 1987.
- [34] Suleiman, R. An Aspirations-Homeostasis Theory of Interactive Decisions. Paper under

consideration, 2014. <http://vixra.org/pdf/1403.0029v1.pdf>.

[35] Olsen, S. *The Golden Section*. New-York: Walker & Co., 2006.

[36] Green, C. D. All that glitters: a review of psychological research on the aesthetics of the golden section. *Perception*, 24 (8), 937 - 968, 1995.

[37] Pittard, N., Ewing, M., & Jevons, C. Aesthetic theory and logo design: examining consumer response to proportion across cultures. *International Marketing Review*, 24(4), 457- 473, 2007.

[38] Hammel, G. T., & Vaughan, K. C. *Math and Music: Harmonious Connections*. Dale Seymour Publications, 1995.

[39] Klar, A. J. S. Fibonacci's flowers. *Nature*, 417, 595, 2002.

[40] Coldea, R., Tennant, D. A., Wheeler, E. M., Wawrzynska, E., Prabhakaran, D., Telling, M., Habicht, K., Smeibid, P., & Kiefer, K. Quantum criticality in an Ising chain: Experimental evidence for emergent E8 symmetry. *Science*, 327 (5962), 177-180, 2010.

[41] Suleiman, R. The Dark Side Revealed: A Complete Relativity Theory Predicts the Content of the Universe. *Progress in Physics*, 4, 34-40, 2013.

[42] Nikolic, S.T., Cosic, L., Pecujlija, M., & Miletic, A. The effect of the Golden Ratio on consumer behavior. *African Journal of Business Management*, 5, 8347-8360, 2011.

[43] Lefebvre, V. A. The golden section and an algebraic model of ethical cognition. *Journal of Mathematical Psychology*, 29 (3), 289 -310, 1985.

[44] Weiss, H., & Weiss, V. The golden mean as clock cycle of brain waves. *Chaos, Solitons, & Fractals*, 18(4), 643- 652, 2003.

[45] Roopun, A. K., Kramer, M. A., Carracedo, L. M., Kaiser, M., Davies, C. H., Traub, R. D., Kopell, N. J., & Whittington, M. A. Temporal interactions between cortical rhythms. *Frontiers in Neuroscience*, 2 (2), 145-154, 2008.

[46] Atalay, B., *Math and the Mona Lisa: The art and science of Leonardo da Vinci*, Smithsonian Books, 2004.

[47] Chapman, H. A., Kim, D. A. Kim, Susskind, J. M., & Anderso, A. K. In Bad taste: evidence for the oral origins of moral disgust. *Science*, 323, 1222-1226, 2009.