Interpretation of Mössbauer experiment in a rotating system: a new proof for general relativity

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Abstract

A historical experiment by Kündig on the transverse Doppler shift in a rotating system measured with the Mössbauer effect (Mössbauer rotor experiment) has been recently first re-analyzed and then replied by an experimental research group. The results of re-analyzing the experiment have shown that a correct re-processing of Kündig's experimental data gives an interesting deviation of a relative redshift between emission and absorption resonant lines from the standard prediction based on the relativistic dilatation of time. That prediction gives a redshift $\frac{\nabla E}{E} \simeq -\frac{1}{2}\frac{v^2}{c^2}$ where v is the tangential velocity of the absorber of resonant radiation, c is the velocity of light in vacuum and the result is given to the accuracy of first-order in $\frac{v^2}{c^2}$. Data re-processing gave $\frac{\nabla E}{E} \simeq -k\frac{v^2}{c^2}$ with $k = 0.596 \pm 0.006$. Subsequent new experimental results by the reply of Kündig experiment have shown a redshift with $k = 0.68 \pm 0.03$ instead.

By using Einstein Equivalence Principle, which states the equivalence between the gravitational "force" and the *pseudo-force* experienced by an observer in a non-inertial frame of reference (included a rotating frame of reference) here we re-analyze the theoretical framework of Mössbauer rotor experiments directly in the rotating frame of reference by using a general relativistic treatment. It will be shown that previous analyses missed an important effect of clock synchronization and that the correct general relativistic prevision in the rotating frame gives $k \simeq \frac{2}{3}$ in perfect agreement

with the new experimental results. Such an effect of clock synchronization has been missed in various papers in the literature with some subsequent claim of invalidity of relativity theory and/or some attempts to explain the experimental results through "exotic" effects. Our general relativistic interpretation shows, instead, that the new experimental results of the Mössbauer rotor experiment are a new, strong and independent, proof of Einstein general relativity.

In the final Section of the paper we discuss an analogy with the use of General Relativity in Global Positioning Systems.

1 Introduction

The Mössbauer effect (discovered by R. Mössbauer in 1958 [14]) consists in resonant and recoil-free emission and absorption of gamma rays, without loss of energy, by atomic nuclei bound in a solid. It resulted and currently results very important for basic research in physics and chemistry. In this work we will focus on the so called Mössbauer rotor experiment. In this particular experiment, the Mössbauer effect works through an absorber orbited around a source of resonant radiation (or vice versa). The aim is to verify the relativistic time dilation time for a moving resonant absorber (the source) inducing a relative energy shift between emission and absorption lines.

In a couple of recent papers [1, 2], the authors first re-anayzed in [1] the data of a known experiment of Kündig on the transverse Doppler shift in a rotating system measured with the Mössbauer effect [3], and second, they carried out their own experiment on the time dilation effect in a rotating system [2]. In [1] they found that the original experiment by Kündig [3] contained errors in the data processing. A puzzling fact is that, after correction of the errors of Kündig, the experimental data gave the value [1]

$$\frac{\nabla E}{E} \simeq -k\frac{v^2}{c^2}, \tag{1}$$

where $k = 0.596 \pm 0.006$, instead of the standard relativistic prediction $k = 0.5$ due to time dilatation. The authors of [1] stressed that the deviation of the coefficient k in equation (1) from 0.5 exceeds by almost 20 times the measuring error and that the revealed deviation cannot be attributed to the influence of rotor vibrations and other disturbing factors. All these potential disturbing factors have been indeed excluded by a perfect methodological trick applied by Kündig [3], i.e. a first-order Doppler modulation of the energy of $\gamma-$quanta on a rotor at each fixed rotation frequency. In that way, Kündig's experiment can be considered as the most precise among other experiments of the same kind [4–8], where the experimenters measured only the count rate of detected $\gamma-$quanta as a function of rotation frequency. The authors of [1] have also shown that the experiment [8], which contains much more data than the ones in [4–7], also confirms the supposition $k > 0.5$. Motived by their results in [1] the authors carried out their own experiment [2]. They decided to repeat neither the scheme of the Kündig experiment [3] nor the schemes of other known

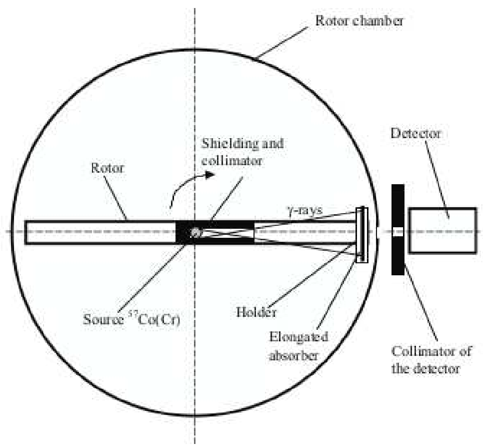

Figure 1: Scheme of the new Mössbauer rotor experiment, adapted from ref. [2]

experiments on the subject previously mentioned above [4–8]. In that way, they got independent information on the value of k in equation (1). In particular, they refrained from the first-order Doppler modulation of the energy of $\gamma-$quanta, in order to exclude the uncertainties in the realization of this method [2]. They followed the standard scheme [4–8], where the count rate of detected $\gamma-$quanta N as a function of the rotation frequency ν is measured. On the other hand, differently from the experiments [4–8], they evaluated the influence of chaotic vibrations on the measured value of k [2]. Their developed method involved a joint processing of the data collected for two selected resonant absorbers with the specified difference of resonant line positions in the Mössbauer spectra [2]. The result obtained in [2] is $k = 0.68 \pm 0.03$, confirming that the coefficient k in equation (1) substantially exceeds 0.5. The scheme of the new Mössbauer rotor experiment is in Figure 1, while technical details on it can be found in [2].

In this paper, Einstein Equivalence Principle, which states the equivalence between the gravitational "force" and the *pseudo-force* experienced by an observer in a non-inertial frame of reference (included a rotating frame of reference) will be used to re-analyze the theoretical framework of Mössbauer rotor experiments directly in the rotating frame of reference by using a general relativistic treatment. The results will show that previous analyses missed an important effect of clock synchronization and that the correct general relativistic prevision gives $k \simeq \frac{2}{3}$ in perfect agreement with the new experimental results of [2]. In that way, the general relativistic interpretation of this paper shows that the new experimental results of the Mössbauer rotor experiment are a new, strong and independent proof of Einstein general relativity. We stress that various papers in the literature missed the effect of clock synchronization [1]-[8], [11]-[13] with some subsequent claim of invalidity of relativity theory and/or some attempts to explain the experimental results through "exotic" effects [1, 2, 11, 12, 13].

2 General relativistic interpretation of time dilatation

Following [9] let us consider a transformation from an inertial frame, in which the space-time is Minkowskian, to a rotating frame of reference. Using cylindrical coordinates, the line element in the starting inertial frame is [9]

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - dz^2. \tag{2}$$

The transformation to a frame of reference $\{t', r', \phi' z'\}$ rotating at the uniform angular rate ω with respect to the starting inertial frame is given by [9]

$$t = t' \quad r = r' \quad \phi = \phi' + \omega t' \qquad z = z' \ . \tag{3}$$

Thus, eq. (2) becomes the following well-known line element (Langevin metric) in the rotating frame [9]

$$ds^2 = \left(1 - \frac{r'^2\omega^2}{c^2}\right) c^2 dt'^2 - 2\omega r'^2 d\phi' dt' - dr'^2 - r'^2 d\phi'^2 - dz'^2. \tag{4}$$

The transformation (3) is both simple to grasp and highly illustrative of the general covariance of GR as it shows that one can work first in a "simpler" frame and then transforming to a more "complex" one [15]. As we consider light propagating in the radial direction ($d\phi' = dz' = 0$), the line element (4) reduces to

$$ds^2 = \left(1 - \frac{r'^2\omega^2}{c^2}\right) c^2 dt'^2 - dr'^2. \tag{5}$$

Einstein Equivalence Principle permits to interpret the line element (5) in terms of a curved spacetime in presence of a static gravitational field. Setting the origin of the rotating frame in the source of the emitting radiation, we have a first contribution which arises from the "gravitational redshift" that can be directly computed using eq. (25.26) in [10] , which in the twentieth printing 1997 of [10] is written as

$$z \equiv \frac{\Delta\lambda}{\lambda} = \frac{\lambda_{received} - \lambda_{emitted}}{\lambda_{emitted}} = |g_{00}(r'_1)|^{-\frac{1}{2}} - 1 \tag{6}$$

and represents the redshift of a photon emitted by an atom a rest an a gravitational field and received by an observer at rest at infinity. Here we use a slightly different equation with respect to eq. (25.26) in [10] because here we are considering a gravitational field which increases with increasing radial coordinate r' while eq. (25.26) in [10] concerns a gravitational field which decreases with increasing radial coordinate. Also, we set the zero potential in $r' = 0$ instead of at infinity and we use the proper time instead of the wavelength λ. Thus, combining eq. (5), we get

$$z_1 \equiv \frac{\nabla\tau_{10}-\nabla\tau_{11}}{\tau} = 1 - |g_{00}(r'_1)|^{-\frac{1}{2}} = 1 - \frac{1}{\sqrt{1-\frac{{r'_1}^2\omega^2}{c^2}}}$$
$$= 1 - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \simeq -\frac{1}{2}\frac{v^2}{c^2}, \tag{7}$$

where $\nabla\tau_{10}$ is the delay of the emitted radiation, $\nabla\tau_{11}$ is the delay of the received radiation, $r'_1 \simeq c\tau$ is the radial distance between the source and the detector and $v = r'_1\omega$ is the tangential velocity of the detector. Hence, we find a first contribution, say $k_1 = \frac{1}{2}$, to k.

3 Clock synchronization

We stress that we calculated the variations of proper time $\nabla\tau_{10}$ and $\nabla\tau_{11}$ in the origin of the rotating frame which is located in the source of the radiation. But the detector is moving with respect to the origin in the rotating frame. Thus, the clock in the detector must be synchronized with the clock in the origin, and this gives a second contribution to the redshift. To compute this second contribution we use eq. (10) of [9] which represents the proper time increment $d\tau$ on the moving clock having radial coordinate r' for values $v \ll c$

$$d\tau = dt'\left(1 - \frac{r'^2\omega^2}{c^2}\right). \tag{8}$$

Inserting the condition of null geodesics $ds = 0$ in eq. (5) one gets

$$cdt' = \frac{dr'}{\sqrt{1 - \frac{r'^2\omega^2}{c^2}}}, \tag{9}$$

where we take the positive sign in the square root because the radiation is propagating in the positive r direction. Combining eqs. (8) and (9) one obtains

$$cd\tau = \sqrt{1 - \frac{r'^2\omega^2}{c^2}}dr'. \tag{10}$$

Eq. (10) is well approximated by

$$cd\tau \simeq \left(1 - \frac{1}{2}\frac{r'^2\omega^2}{c^2} + ...\right)dr', \tag{11}$$

which permits to find the second contribution of order $\frac{v^2}{c^2}$ to the variation of proper time as

$$c\nabla\tau_2 = \int_0^{r'_1}\left(1 - \frac{1}{2}\frac{{r'_1}^2\omega^2}{c^2}\right)dr' - r'_1 = -\frac{1}{6}\frac{{r'_1}^3\omega^2}{c^2} = -\frac{1}{6}r'_1\frac{v^2}{c^2}. \tag{12}$$

Thus, as $r_1' \simeq c\tau$ is the radial distance between the source and the detector, we get the second contribution of order $\frac{v^2}{c^2}$ to the redshift as

$$z_2 \equiv \frac{\nabla \tau_2}{\tau} = -k_2 \frac{v}{c^2}^2 = -\frac{1}{6}\frac{v^2}{c^2}. \tag{13}$$

Then, we obtain $k_2 = \frac{1}{6}$ and using eqs. (7) and (13) the total redshift is

$$\begin{aligned} z \equiv z_1 + z_2 &= \tfrac{\nabla\tau_{10} - \nabla\tau_{11} + \nabla\tau_2}{\tau} = -\left(k_1 + k_2\right)\tfrac{v^2}{c^2} \\ &= -\left(\tfrac{1}{2} + \tfrac{1}{6}\right)\tfrac{v^2}{c^2} = -k\tfrac{v^2}{c^2} = -\tfrac{2}{3}\tfrac{v^2}{c^2} = 0.\bar{6}\tfrac{v^2}{c^2}, \end{aligned} \tag{14}$$

which is completely consistent with the result $k = 0.68 \pm 0.03$ in [2].

4 Discussion of the correction due to clock synchronization and analogy with the use of General Relativity in Global Positioning Systems

We stress that the additional factor $-\frac{1}{6}$ in eq. (13) comes from clock synchronization [15]. In other words, its theoretical absence in the works [1]-[8], [11]-[13] reflected the incorrect comparison of clock rates between a clock at the origin and one at the detector [15]. This generated wrong claims of invalidity of relativity theory and/or some attempts to explain the experimental results through "exotic" effects [1, 2, 11, 12, 13] which, instead, must be rejected.

We evoked the appropriate reference [9] for a discussion of the Langevin metric. This is dedicated to the use of General Relativity in Global Positioning Systems (GPS), which leads to the following interesting realisation [15]: the correction of $-\frac{1}{6}$ in eq. (13) is analog to the correction that one must consider in GPS when accounting for the difference between the time measured in a frame co-rotating with the Earth geoid and the time measured in a non-rotating (locally inertial) Earth centered frame (and also the difference between the proper time of an observer at the surface of the Earth and at infinity). Indeed, if one simply considers the Schwarzschild gravitational redshift but neglects the effect of the Earth's rotation, GPS would not work [15]! In fact, following Chapters 3 and 4 of [9] in order to address the problem of clock synchronization within the GPS one starts from an approximate solution of Einstein's field equations in isotropic coordinates in a locally inertial, non-rotating, freely falling coordinate system with origin at the earth's center of mass, i.e. eq. (12) in [9] which is

$$ds^2 = \left(1 + \frac{2V}{c^2}\right)(cdt)^2 - \left(1 - \frac{2V}{c^2}\right)\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) \tag{15}$$

where V is the Newtonian gravitational potential of the earth and r, θ, ϕ are spherical polar coordinates. V is given approximately by [9]:

$$V \simeq \frac{-GM_E}{r}\left[1 - J_2\left(\frac{a_1}{r}\right)^2 P_2 \cos\theta\right], \tag{16}$$

where M_E is the earth's mass, J_2 the earth's quadrupole moment coefficient, a_1 the earth's equatorial radius and P_2 the Legendre polynomial of degree 2 [9]. Usually, one retains only terms of first order in the small quantity $\frac{V}{c^2}$ in eq. (15), while higher multipole moment contributions to eq. (16) have negligible effect for relativity in GPS [9]. The equivalent transformations of eqs. (3) for spherical polar coordinates are [9]:

$$t = t' \ .r = r' \ \theta = \theta' \ \phi' + \omega_E t' \tag{17}$$

where ω_E is the earth's uniform angular rate. Applying the transformations (17) to the line element (15) and retaining only terms of order $1/c^2$ one gets the line element for the so called earth-centered, earth-fixed, reference frame (ECEF frame) [9]:

$$\begin{aligned} ds^2 = & \left[1 + \tfrac{2V}{c^2} - \left(\tfrac{\omega_E r' \sin\theta'}{c}\right)^2\right](cdt')^2 + 2\omega_E r'^2 \sin^2\theta' d\phi' dt' \\ & - \left(1 - \tfrac{2V}{c^2}\right)\left(dr'^2 + r'^2 d\theta'^2 + r'^2 \sin^2\theta' d\phi'^2\right). \end{aligned} \tag{18}$$

The rate of coordinate time in eq. (15 is defined by standard clocks at rest at infinity [9]. We prefer to consider the rate of coordinate time by standard clocks at rest on the earth's surface [9]. Then, a new coordinate time t'' can be defined through a constant rate change [9]:

$$t'' = \left(1 + \frac{\Phi_0}{c^2}\right) t' = \left(1 + \frac{\Phi_0}{c^2}\right) t, \tag{19}$$

where [9]

$$\Phi_0 \equiv -\frac{GM_E}{a_1} - \frac{GM_E J_2}{2a_1} - \frac{1}{2}\left(\omega_E a_1\right)^2, \tag{20}$$

and the correction (19) is order seven parts in 10^{10} [9]. Applying this time scale change in the ECEF line element (18) and reteining only terms of order $\frac{1}{c^2}$ we obtain [9]

$$\begin{aligned} ds^2 = & \left[1 + \tfrac{2(\Phi - \Phi_0)}{c^2}\right](cdt'')^2 + 2\omega_E r'^2 \sin^2\theta' d\phi' dt'' \\ & - \left(1 - \tfrac{2V}{c^2}\right)\left(dr'^2 + r'^2 d\theta'^2 + r'^2 \sin^2\theta' d\phi'^2\right), \end{aligned} \tag{21}$$

where [9]

$$\Phi \equiv \frac{V}{c^2} - \frac{1}{2}\left(\frac{\omega_E r' \sin\theta'}{c}\right)^2. \tag{22}$$

is the effective gravitational potential in the rotating frame. Applying the time scale change (19) in the non-rotating line element (15) and dropping the primes on t'' in order to just use the symbol t one gets [9]

$$ds^2 = \left[1 + \frac{2\left(V - \Phi_0\right)}{c^2}\right](cdt)^2 - \left(1 - \frac{2V}{c^2}\right)\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right). \tag{23}$$

The coordinate time t in eq. (23) is valid in a very large coordinate patch which covers both the earth and the GPS satellite constellation [9]. Thus, such a coordinate time is used as a basis for synchronization in the earth's neighborhood [9]. The difference $(V - \Phi_0)$ in the first term of eq. (23) is due to the issue that in the underlying earth-centered locally inertial frame in which the line element (23) is expressed, the unit of time is determined by moving clocks in a spatially-dependent gravitational field [9]. We observe that eq. (23) contains both the effects of apparent slowing of moving clocks and frequency shifts due to gravitation [9]. This implies that the proper time elapsing on the orbiting GPS clocks cannot be simply used to transfer time from one transmission event to another because path-dependent effects must be taken into due account, exactly like in the discussion of clock synchronization in Section 3.

The discussion in this Section has been inserted in order to highlight that the obtained correction $-\frac{1}{6}$ in eq. (13) is not an obscure mathematical or physical detail, but a fundamental ingredient that must be taken into due account [15].

5 Conclusion remarks

In this paper Einstein Equivalence Principle, stating the equivalence between the gravitational "force" and the *pseudo-force* experienced by an observer in a non-inertial frame of reference (included a rotating frame of reference) has been used to re-analyze the theoretical framework of the new Mössbauer rotor experiment in [2] directly in the rotating frame of reference by using a general relativistic treatment. The results have shown that previous analyses missed an important effect of clock synchronization and that the correct general relativistic prevision gives $k \simeq \frac{2}{3}$ in perfect agreement with the new experimental results in [2]. Thus, the general relativistic interpretation in this letter shows that the new experimental results of the Mössbauer rotor experiment are a new, strong and independent proof of Einstein general relativity. The importance of our results is stressed by the issue that various papers in the literature missed the effect of clock synchronization[1]-[8], [11]-[13] with some subsequent claim of invalidity of relativity theory and/or some attempts to explain the experimental results through "exotic" effects [1, 2, 11, 12, 13].

An analogy with the use of General Relativity in Global Positioning Systems has been highlight in the final Section of the paper.

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