Interpretation of Mössbauer experiment in a rotating system: a new proof for general relativity

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Abstract

A historical experiment by Kündig on the transverse Doppler shift in a rotating system measured with the Mössbauer effect (Mössbauer rotor experiment) has been recently first re-analyzed and then replied by an experimental research group. The results of re-analyzing the experiment have shown that a correct re-processing of Kündig's experimental data gives an interesting deviation of a relative redshift between emission and absorption resonant lines from the standard prediction based on the relativistic dilatation of time. That prediction gives a redshift $\frac{\nabla E}{E} \simeq -\frac{1}{2}\frac{v^2}{c^2}$ where v is the tangential velocity of the absorber of resonant radiation, c is the velocity of light in vacuum and the result is given to the accuracy of first-order in $\frac{v^2}{c^2}$. Data re-processing gave $\frac{\nabla E}{E} \simeq -k\frac{v^2}{c^2}$ with $k = 0.596 \pm 0.006$. Subsequent new experimental results by the reply of Kündig experiment have shown a redshift with $k = 0.68 \pm 0.03$ instead.

By using Einstein Equivalence Principle, which states the equivalence between the gravitational "force" and the *pseudo-force* experienced by an observer in a non-inertial frame of reference (included a rotating frame of reference) here we re-analyze the theoretical framework of Mössbauer rotor experiments directly in the rotating frame of reference by using a general relativistic treatment. It will be shown that previous analyses missed an important effect of clock synchronization and that the correct general relativistic prevision in the rotating frame gives $k \simeq \frac{2}{3}$ in perfect agreement with the new experimental results. Such an effect of clock synchronization has been missed in various papers in the literature with some subsequent claim of invalidity of relativity theory and/or some attempts to explain the experimental results through "exotic" effects. Our general relativistic interpretation shows, instead, that the new experimental results of the Mössbauer rotor experiment are a new, strong and independent, proof of Einstein general relativity.

In a couple of recent papers [1, 2], the authors first re-anayzed in [1] the data of a known experiment of Kündig on the transverse Doppler shift in a rotating system measured with the Mössbauer effect [3], and second, they carried out their own experiment on the time dilation effect in a rotating system [2]. In [1] they found that the original experiment by Kündig [3] contained errors in the data processing. A puzzling fact is that, after correction of the errors of Kündig, the experimental data gave the value [1]

$$\frac{\nabla E}{E} \simeq -k\frac{v^2}{c^2}, \tag{1}$$

where $k = 0.596 \pm 0.006$, instead of the standard relativistic prediction $k = 0.5$ due to time dilatation. The authors of [1] stressed that the deviation of the coefficient k in equation (1) from 0.5 exceeds by almost 20 times the measuring error and that the revealed deviation cannot be attributed to the influence of rotor vibrations and other disturbing factors. All these potential disturbing factors have been indeed excluded by a perfect methodological trick applied by Kündig [3], i.e. a first-order Doppler modulation of the energy of $\gamma-$quanta on a rotor at each fixed rotation frequency. In that way, Kündig's experiment can be considered as the most precise among other experiments of the same kind [4–8], where the experimenters measured only the count rate of detected $\gamma-$quanta as a function of rotation frequency. The authors of [1] have also shown that the experiment [8], which contains much more data than the ones in [4–7], also confirms the supposition $k > 0.5$. Motived by their results in [1] the authors carried out their own experiment [2]. They decided to repeat neither the scheme of the Kündig experiment [3] nor the schemes of other known experiments on the subject previously mentioned above [4–8]. In that way, they got independent information on the value of k in equation (1). In particular, they refrained from the first-order Doppler modulation of the energy of $\gamma-$quanta, in order to exclude the uncertainties in the realization of this method [2]. They followed the standard scheme [4–8], where the count rate of detected $\gamma-$quanta N as a function of the rotation frequency ν is measured. On the other hand, differently from the experiments [4–8], they evaluated the influence of chaotic vibrations on the measured value of k [2]. Their developed method involved a joint processing of the data collected for two selected resonant absorbers with the specified difference of resonant line positions in the Mössbauer spectra [2]. The result obtained in [2] is $k = 0.68 \pm 0.03$, confirming that the coefficient k in equation (1) substantially exceeds 0.5. The scheme of the new Mössbauer rotor experiment is in Figure 1, while technical details on it can be found in [2].

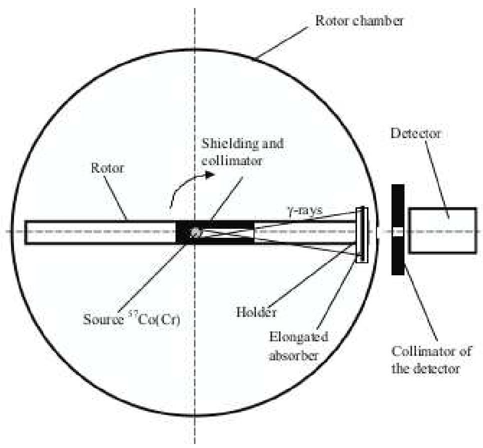

Figure 1: Scheme of the new Mössbauer rotor experiment, adapted from ref. [2]

In this letter, Einstein Equivalence Principle, which states the equivalence between the gravitational "force" and the *pseudo-force* experienced by an observer in a non-inertial frame of reference (included a rotating frame of reference) will be used to re-analyze the theoretical framework of Mössbauer rotor experiments directly in the rotating frame of reference by using a general relativistic treatment. The results will show that previous analyses missed an important effect of clock synchronization and that the correct general relativistic prevision gives $k \simeq \frac{2}{3}$ in perfect agreement with the new experimental results of [2]. In that way, the general relativistic interpretation of this letter shows that the new experimental results of the Mössbauer rotor experiment are a new, strong and independent proof of Einstein general relativity. We stress that various papers in the literature missed the effect of clock synchronization[1]-[8], [11]-[13] with some subsequent claim of invalidity of relativity theory and/or some attempts to explain the experimental results through "exotic" effects [1, 2, 11, 12, 13].

As we consider light propagating in the radial direction, the line element in the rotating frame (Langevin metric) reduces to [9]

$$ds^2 = \left(1 - \frac{r^2\omega^2}{c^2}\right) c^2 dt^2 - dr^2, \qquad (2)$$

where ω is the constant rotation frequency. Einstein Equivalence Principle permits to interpret the line element (2) in terms of a curved spacetime in presence of a static gravitational field. Setting the origin of the rotating frame in the source of the emitting radiation, we have a first contribution which arises from the "gravitational redshift" that can be directly computed using eq. (25.26) in [10] and eq. (3)

$$z_1 \equiv \frac{\nabla\tau_{10}-\nabla\tau_{11}}{\tau} = 1 - |g_{00}(r_1)|^{-\frac{1}{2}} = 1 - \frac{1}{\sqrt{1-\frac{r_1^2\omega^2}{c^2}}}$$
$$= 1 - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \simeq -\frac{1}{2}\frac{v^2}{c^2}, \tag{3}$$

where $\nabla\tau_{10}$ is the delay of the emitted radiation, $\nabla\tau_{11}$ is the delay of the received radiation, $r_1 \simeq c\tau$ is the radial distance between the source and the detector and $v = r_1\omega$ is the tangential velocity of the detector. Thus, we find a first contribution, say k_1, to k. Notice that eq. (3) is slightly different with respect to eq. (25.26) in [10] because here we are considering a gravitational field which increases with increasing radial coordinate while eq. (25.26) in [10] concerns a gravitational field which decreases with increasing radial coordinate.

On the other hand, we calculated the variations of proper time $\nabla\tau_{10}$ and $\nabla\tau_{11}$ in the origin of the rotating frame which is located in the source of the radiation. But the detector is moving with respect to the origin in the rotating frame. Thus, the clock in the detector must be synchronized with the clock in the origin, and this gives a second contribution to the redshift. To compute this second contribution we use eq. (10) of [9] which represents the proper time increment $d\tau$ on the moving clock having radial coordinate r for values $v \ll c$

$$d\tau = dt\left(1 - \frac{r^2\omega^2}{c^2}\right). \tag{4}$$

Inserting the condition of null geodesics $ds = 0$ in eq. (2) one gets

$$cdt = \frac{dr}{\sqrt{1-\frac{r^2\omega^2}{c^2}}}, \tag{5}$$

where we take the positive sign in the square root because the radiation is propagating in the positive r direction. Combining eqs. (4) and (5) one obtains

$$cd\tau = \sqrt{1-\frac{r^2\omega^2}{c^2}}dr. \tag{6}$$

Eq. (6) is well approximated by

$$cd\tau \simeq \left(1 - \frac{1}{2}\frac{r^2\omega^2}{c^2} + ...\right)dr, \tag{7}$$

which permits to find the second contribution of order $\frac{v^2}{c^2}$ to the variation of proper time as

$$c\nabla\tau_2 = \int_0^{r_1}\left(1 - \frac{1}{2}\frac{r^2\omega^2}{c^2} + ...\right)dr - r_1 = -\frac{1}{6}\frac{r_1^3\omega^2}{c^2} = -\frac{1}{6}r^1\frac{v^2}{c^2}. \tag{8}$$

Thus, as $r_1 \simeq c\tau$ is the radial distance between the source and the detector, we get the second contribution of order $\frac{v^2}{c^2}$ to the redshift as

$$z_2 \equiv \frac{\nabla\tau_2}{\tau} = -k_2 \frac{v}{c^2}^2 = -\frac{1}{6}\frac{v^2}{c^2}. \tag{9}$$

Then, we obtain $k_2 = \frac{1}{6}$ and using eqs. (3) and (9) the total redshift is

$$\begin{aligned} z \equiv z_1 + z_2 = \tfrac{\nabla\tau_{10} - \nabla\tau_{11} + \nabla\tau_2}{\tau} = -\left(k_1 + k_2\right)\tfrac{v^2}{c^2} \\ = -\left(\tfrac{1}{2} + \tfrac{1}{6}\right)\tfrac{v^2}{c^2} = -k\tfrac{v^2}{c^2} = -\tfrac{2}{3}\tfrac{v^2}{c^2} = 0.\bar{6}\tfrac{v^2}{c^2}, \end{aligned} \tag{10}$$

which is completely consistent with the result $k = 0.68 \pm 0.03$ in [2].

Resuming, in this letter Einstein Equivalence Principle, stating the equivalence between the gravitational "force" and the *pseudo-force* experienced by an observer in a non-inertial frame of reference (included a rotating frame of reference) has been used to re-analyze the theoretical framework of the new Mössbauer rotor experiment in [2] directly in the rotating frame of reference by using a general relativistic treatment. The results have shown that previous analyses missed an important effect of clock synchronization and that the correct general relativistic prevision gives $k \simeq \frac{2}{3}$ in perfect agreement with the new experimental results in [2]. Thus, the general relativistic interpretation in this letter shows that the new experimental results of the Mössbauer rotor experiment are a new, strong and independent proof of Einstein general relativity. The importance of our results is stressed by the issue that various papers in the literature missed the effect of clock synchronization[1]-[8], [11]-[13] with some subsequent claim of invalidity of relativity theory and/or some attempts to explain the experimental results through "exotic" effects [1, 2, 11, 12, 13].

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