

Gene H. Barbee  
October, 2014

## Black Holes and Quantum Gravity

### Abstract

Black holes are interesting to the scientific community. Observations indicate that remnants of stars that become black holes range from about 4 solar masses through about 20 solar masses. A supermassive black hole with mass  $4.2 \times 10^6$  solar masses has also been observed. Astronomers use x-ray emission and the bending of light around massive objects to study them and observations to date challenge physicists to extend their knowledge.

A layer by layer model of pressure, temperature, density and fusion power (MeV/sec) of the sun was developed and compared to a supermassive black hole with similar density and fusion kinetics. To simulate smaller black holes, the author developed a method to determine the pressure and density as a function of radius. Conditions at the core of small black holes are extreme enough to cause degenerate (relativistic) behavior of known forces. It was unknown however, whether collapse actually occurs. The author uses a cellular model of gravity, space, time, expansion, kinetic and potential energy at the quantum level [6]. Quantum gravity indicates that black holes do not involve a singularity. Gravitational kinetic energy resists the pressure at the core of black holes similar to the way electromagnetic kinetic energy controls the thermodynamics of gases.

### The Schwarzschild radius

Schwarzschild solved the metric equations for general relativity. His solution defining the radius  $S$  of a black hole was  $S = 2GM/C^2$  where:  $G = 6.67 \times 10^{-11} \text{ NT m}^2/\text{kg}^2$ ,  $M$  is the mass of the central body and  $C$  is the speed of light.  $S'$  is half the  $S$  value defined and agrees with the geodesic radius. The geodesic is defined by the gravitational constant  $G = S'C^2/M$ . The author defines a cell as the volume surrounding a single proton. Using  $S'$  to determine the radius of a black hole (BH), the average density, cell radius, cell kinetic energy and temperature can be determined for a series of BH's with increasing mass.  $\text{Volume BH} = \frac{4}{3}\pi S'^3$ .  $\text{Volume cell} = (\text{Volume BH}/N \text{ cells})$  with  $N \text{ cells} = M \text{ black hole}/1.67 \times 10^{-27} \text{ kg}$ . This gives the critical parameter discussed below:  $\text{cell radius} = (\text{Volume cell}/(\frac{4}{3}\pi))^{1/3}$ . Radius can also be calculated from density  $\rho$  assuming hydrogen.  $\text{Cell radius} = (1/\rho * 1.67 \times 10^{-27}/(\frac{4}{3}\pi))^{1/3}$ .

|                             |             |                 |           |           |           |
|-----------------------------|-------------|-----------------|-----------|-----------|-----------|
| S=2GM/c^2=1.48e-27*m        |             |                 |           |           |           |
| My Geodesic at C and High M |             |                 |           |           |           |
| S=2Geodesic                 |             |                 |           |           |           |
|                             |             |                 | Minimum   | 20x solar | smassive  |
| G nt m^2/kg^2               | 6.674E-11   |                 |           |           |           |
| C m/sec                     | 299792458   | Kg              | 9.94E+30  | 4.00E+31  | 4.20E+40  |
| S'=MC^2/2                   |             | S' meters       | 7.38E+03  | 2.97E+04  | 3.12E+13  |
| Density                     | M/vol       | kg/m^3          | 5.90E+18  | 3.64E+17  | 3.31E-01  |
|                             |             | N cells         | 5.95E+57  | 2.40E+58  | 2.51E+67  |
|                             |             | Vol cell (m^3)  | 2.83E-46  | 4.58E-45  | 5.05E-27  |
| Average Cell Radius         |             | Cell radius (m) | 4.087E-16 | 1.034E-15 | 1.067E-09 |
| S'/m=G/C^2                  | 7.42583E-28 |                 |           |           |           |
| proton m (Kg)               | 1.67E-27    |                 |           |           |           |

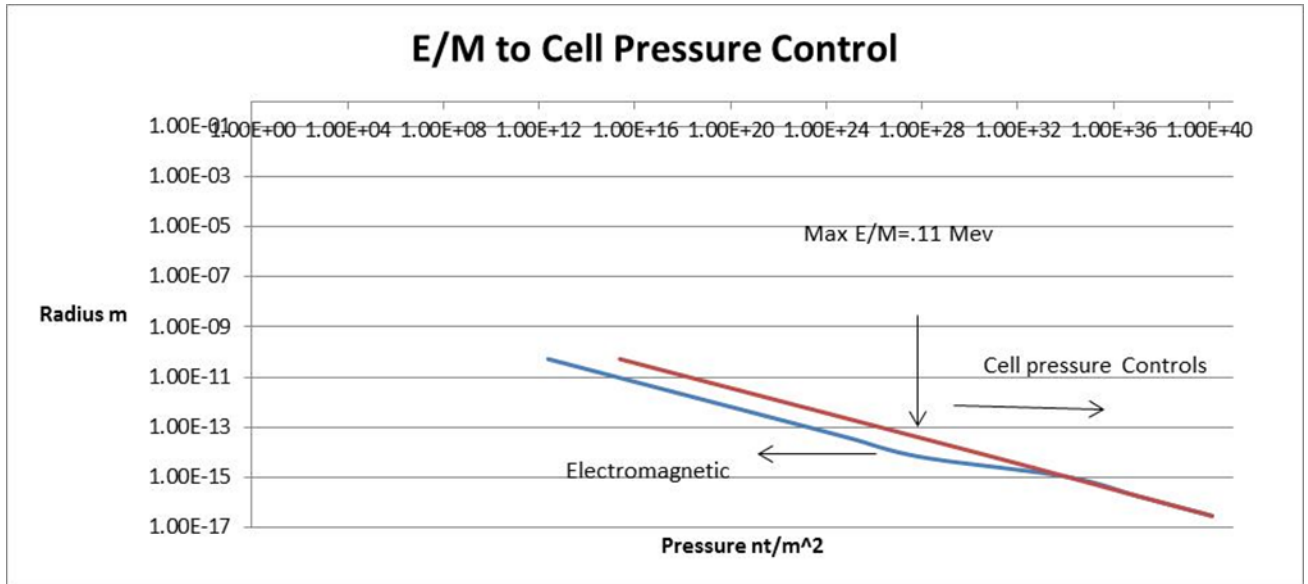
## What resists collapse?

For gases at low pressure, the electron is circling the nucleus at a radius of approximately 5.29e-11 meters. Electromagnetic kinetic energy 13.6e-6 MeV is associated with temperature and pressure that resist compression. The origin of gas thermodynamics for hydrogen is shown below:

|                  |                                     |  |                             |  |  |
|------------------|-------------------------------------|--|-----------------------------|--|--|
| <b>0.511</b>     | Electron                            |  | mass MeV                    |  |  |
| <b>9.11E-31</b>  | .511*1.78e-30 kg/MeV                |  | mass kg                     |  |  |
| <b>2.72E-05</b>  | E/M field                           |  | Field MeV                   |  |  |
| <b>1.36E-05</b>  | E/M field/2                         |  | Kinetic Energy MeV          |  |  |
| <b>1.00E+00</b>  | 0.511/(0.511+ke)                    |  | gamma                       |  |  |
| <b>7.30E-03</b>  | V/C=(1-(g)^2)^0.5                   |  | V/C                         |  |  |
| <b>5.290E-11</b> | r                                   |  | Radius (meters)             |  |  |
| <b>8.24E-08</b>  | F=(9.11E-31/g)*(v/C)^2/R            |  | Force Nt                    |  |  |
| <b>3.52E-20</b>  | area=4*pi*r^2                       |  | Surface area m^2            |  |  |
| <b>2.34E+12</b>  | P=F/area                            |  | Pressure nt/m^2             |  |  |
| <b>1.05E+05</b>  | T=ke/1.5/8.6e-11                    |  | Temperature based on ke (K) |  |  |
| <b>2.69E+03</b>  | rho=(1/r^3*1.67E-27/(4/3*PI()))     |  | Density=P/(RT)              |  |  |
| <b>8251.03</b>   | R=P/(rhoT) Nt-K/(m-kg)              |  | Gas constant                |  |  |
|                  |                                     |  |                             |  |  |
| <b>5.29E-11</b>  | r=(1/rho*1.67E-27/(4/3*PI()))^(1/3) |  |                             |  |  |

In the model above rho is calculated from radius r and the gas constant is derived from R=P/(rhoT). The gas constant derived agrees with literature. As radius r decreases with compression of the gas, ke=13.6e-6\*5.29e-11/radius MeV and this increases the pressure and temperature. When the radius decreases to the point where compression kinetic energy =0.11 MeV the electron is no longer sustained in the shell and transition of the proton to a neutron occurs (the Proton model in Appendix 1 justifies the use of 0.11

MeV. Also read Wiki concerning White Dwarfs and Neutron Stars). At pressures higher than  $9.6e27 \text{ nt/m}^2$  cell it will be proposed that cell pressure resists collapse and a chart similar to the one above is required. It is developed below under the heading “Pressure inside cells prevents black hole collapse” but results are shown below:



We can understand quantum gravity as follows:

## Fundamentals of space and time

Reference 6 identifies the source of the gravitational constant at the quantum level. The gravitational field energy 2.683 MeV from the Proton mass model (Appendix 1) underlies the quantum mechanics for a fundamental radius  $r$  and a fundamental time  $t$ . In the equation below, the value  $1.93e-13$  meters-MeV is  $HC/(2*\pi)$  where  $H$  is Heisenberg’s constant  $4.136e-21$  MeV-sec and  $C$  is light speed,  $3e8$  meters/sec. The radius  $r$  is the radius of a quantum circle for gravity with 2.68 MeV field energy.

### Identify the radius and time for the gravitational orbit described above

$$\text{Fundamental radius} = 1.93e-13 / (2.68 * 2.68)^{.5} = 7.354e-14 \text{ meters}$$

$$\text{Fundamental time} = 7.354e-14 * 2 * \pi / (3e8) = h/E = 4.13e-21 / 2.68$$

$$\text{Fundamental time} \quad 1.541E-21 \text{ seconds}$$

## Gravitation

If radius  $r$  for the conventional physics (Wiki) force calculation is  $7.35e-14$  meters, as proposed above, the force in Newtons (NT) is:

|  |                   |            |          |
|--|-------------------|------------|----------|
| $F=(5.9068e-39)*hC/R^2$                            |                   |            |          |
|  | hbar              | 6.5821E-22 | mev-sec  |
|  | hbar in NT-m-sec  | 1.05E-34   | NT m sec |
|  | hbarC in NT-m^2=K | 3.16E-26   | NT m^2   |
| $F=(5.9068e-39)*K/R^2$                             |                   |            |          |
| $F=(5.9068e-39)*3.16e-26/(7.35e-14)^2=3.39e-38$ NT |                   |            |          |
| <b>3.453E-38</b> NT                                |                   |            |          |

This result agrees with the simple Newtonian force for particles separated by 7.35e-14 meters.

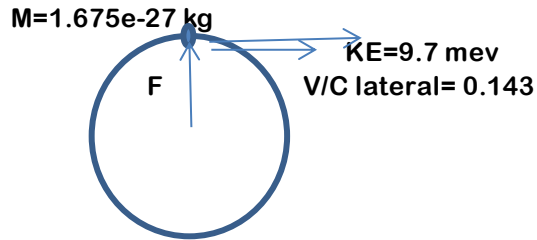
$F=Gm^2/R^2$  (NT)= $6.67428e-11*1.6726e-27^2/7.35e-14^2=3.452e-38$  NT where m is proton mass and R is meters.

Using values for the proton mass model that the author believes unify nature's forces (6), the gravitational constant is calculated below and agrees with the published constant,  $G=6.674e-11$  NT meters<sup>2</sup>/kg<sup>2</sup>.

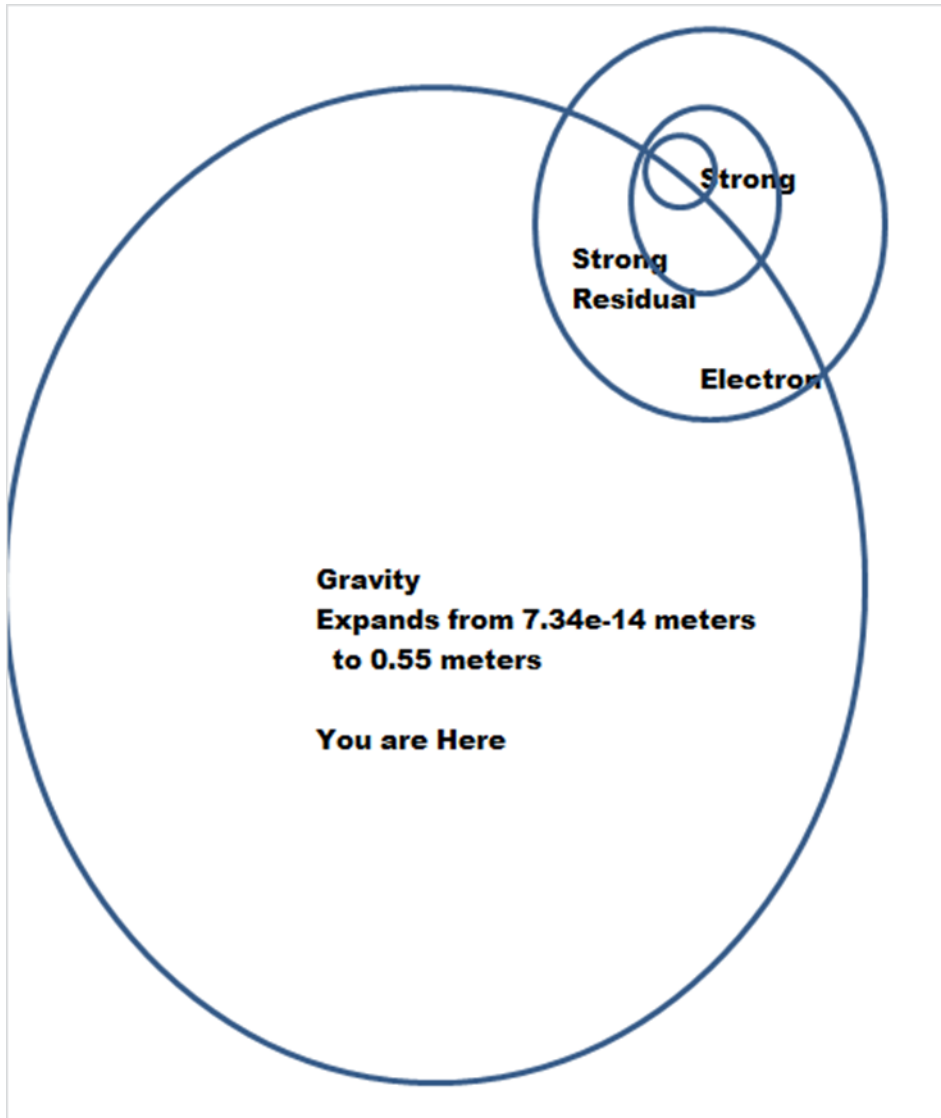
|   |  |                   |
|---|--|-------------------|
| <b>GRAVITY</b>                            |  |                   |
|   |  | <b>proton</b>     |
| Proton Mass (mev)                         |  | <b>938.272</b>    |
| Proton Mass M (kg)                        |  | <b>1.673E-27</b>  |
| Field Energy E (mev)                      |  | <b>2.683</b>      |
| Kinetic Energy ke (mev)                   |  | <b>9.720</b>      |
| Gamma (g)=M/(M+ke)                        |  | <b>0.9897</b>     |
| Velocity Ratio v/C=(1-g^2)^0.5            |  | 0.1428            |
| <b>R (meters) =(HC/(2pi))/(E*E)^0.5</b>   |  | <b>7.354E-14</b>  |
| F (NT)=M/g*(v/C*C)^2/R/exp(90)            |  | <b>3.452E-38</b>  |
| HC/(2pi)=1.97e-13 mev-m                   |  |                   |
| Calculation of gravitational constant G   |  |                   |
| Inertial Force=(M/g*C^2/R)*1/EXP(90)      |  | <b>3.452E-38</b>  |
| Radius R (Meters)                         |  | <b>7.354E-14</b>  |
| Mass M (kg)                               |  | <b>1.673E-27</b>  |
| <b>G=F*R^2/M^2=NT m^2/kg^2</b>            |  | <b>6.674E-11</b>  |
| Published by Partical Data Group (PDC)    |  | <b>6.674E-11</b>  |
| PE fall MeV                               |  | <b>19.34</b>      |
| Ke fall MeV                               |  | <b>9.720</b>      |
| F =PE/R *1.6022e-13 NT                    |  | <b>3.4524E-38</b> |
| PE/R=(19.34*1.603e-13/7.3543e-14/exp(90)) |  |                   |

The gravitation constant G is calculated above from fundamentals. Appendix 1 contains a review and origin of the coupling constant 1/exp(90) that scales the quantum level to the large scale we observe around us. It has the effect of dramatically reducing the force between protons and makes gravity very long range compared to the other forces. It also

unifies special relativity and general relativity [6][13] and allows nature to use the general theory of relativity at the quantum level. The author calls this cellular cosmology. A cell is diagrammed below.



The cell diagram shows an *outward* force  $F$ , identified with the gravitational inertial force.  $V/C$  is the velocity of a proton around radius,  $r$ .  $V/C$  originally is the gravitational kinetic energy 9.7 MeV from the proton model and  $G=rV^2/m$  is the geodesic. As the universe expands, kinetic energy  $KE$  decreases directly with radius. As it does so, the radius of each cell expands and  $\exp(180)$  expanding cells define the space of the universe.



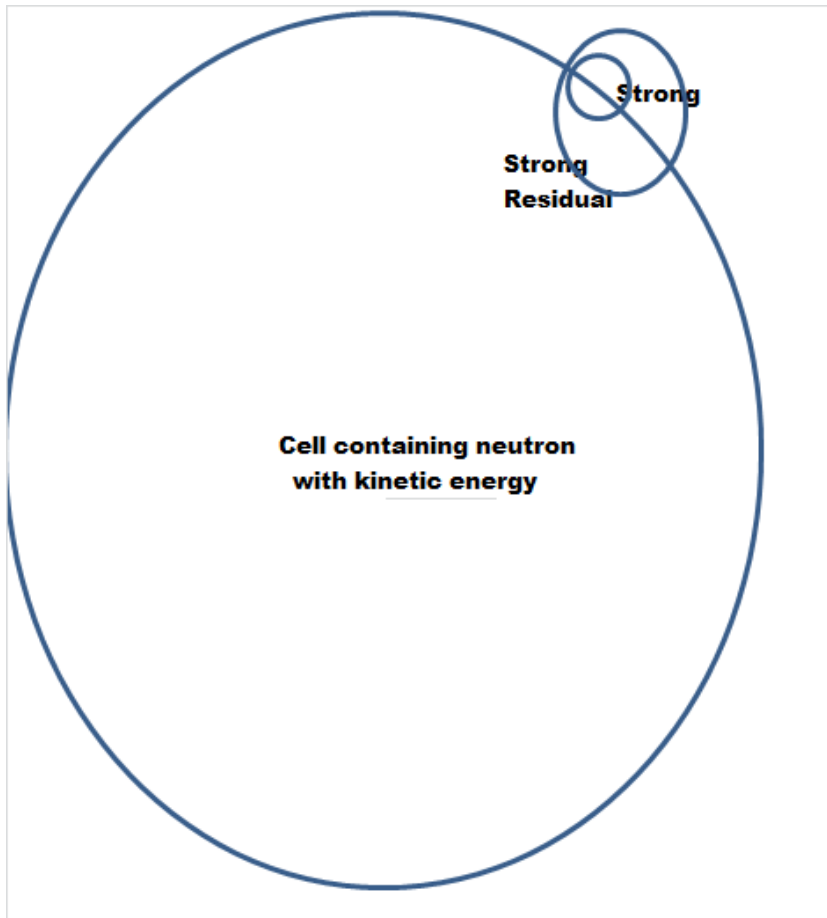
The above diagram emphasizes the fact that the space that protons occupy is identically the space that we occupy. It is expanded quantum gravity. The protons have kinetic energy and as the radius of the gravitationally determined cell changes, it either expands the universe or compresses. When cells are squeezed inside a black hole kinetic energy of the protons resist collapse.

Reference 13 analyzes expansion in terms of pressure and temperature. It concludes that pressure expands the universe and shows that 9.7 MeV of kinetic energy has been converted to potential energy integral  $PdV = 9.7 \text{ MeV}$  after expansion. The kinetic energy after expansion is very low and  $r$  for the diagram above is 0.55 meters. This gives the current Radius of the universe  $= 0.55 * \exp(60) = 6.3e25$  meters. Further, reference 13 shows that kinetic KE in the cell diagram can be considered temperature. Using 9.7 MeV as the beginning energy and temperature  $T = ke/1.5/B$  (and adjusting it upward for primordial He4 fusion), the kinetic energy is now associated with the cosmic microwave background temperature (CMB)  $= 2.73 \text{ K}$ . Using  $KE = 1.5BT$  (B is Boltzmann's constant

8.6e-11 MeV/K) the kinetic energy is now 3.5e-10 MeV. Knowing that there is pressure inside each cell is a key understanding used below.

## **Pressure inside cells prevents black hole collapse**

Literature states that strong forces resist compression after the electron no longer exists. At extremely high pressure, the proton plus the electron (and a neutrino) become a neutron. Neutrons have no charge and de-generate electrons no longer exist. It is believed by some that the Pauli exclusion principle prevents collapse. Others believe that collapse occurs and a singularity forms. The author believes that cells have internal pressure and temperature high enough to resist collapse of a black hole. The quantum level determines and maintains the gravitational constant  $G$  but gravitational kinetic energy energizes particles. As indicated above, cells transition and are no longer quantum as they expand the universe. Proof that they have internal pressure is that integral  $Pdv=9.9$  MeV was shown to be the energy that expands the universe and the related temperature is presently the cosmic background temperature that we measure. The pressure inside the cell no longer causes inertial force because it is not a neutron orbiting a field like it was at the quantum level. The neutron has gravitational kinetic energy and velocity but it is disorganized into temperature and pressure. There is no place for the internal energy to go. (Recall that the gravitational space is the *only* space outside the neutron). The cell diagram is repeated below that used to be labeled "you are here". Thankfully your cells weren't among the cells that fell into the black hole.

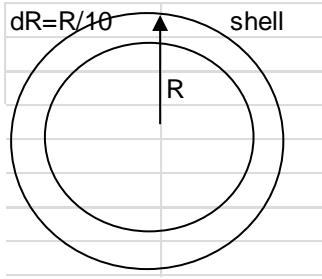


The neutrons bounce off of each other because of strong force exclusion. As the radius becomes small, the kinetic energy of the neutrons increases the pressure in the cell. Because the neutron has only random velocity it has net velocity zero, i.e.  $V_n=0$  and is not on the geodesic. The velocity would have to be high to be on the geodesic. In fact the required geodesic velocity approaches  $C$  if the neutrons fall into a black hole. The equation  $F=mc^2/r-mV_n^2/r$  is very large because  $C$  is large and  $r$  is small. The pressure in the cells has to make up for the inertia that the neutrons need to be on the geodesic. Again, the gravitational constant  $G$  is maintained by fundamental relationships. Cells resist compression because the neutrons have temperature and pressure.

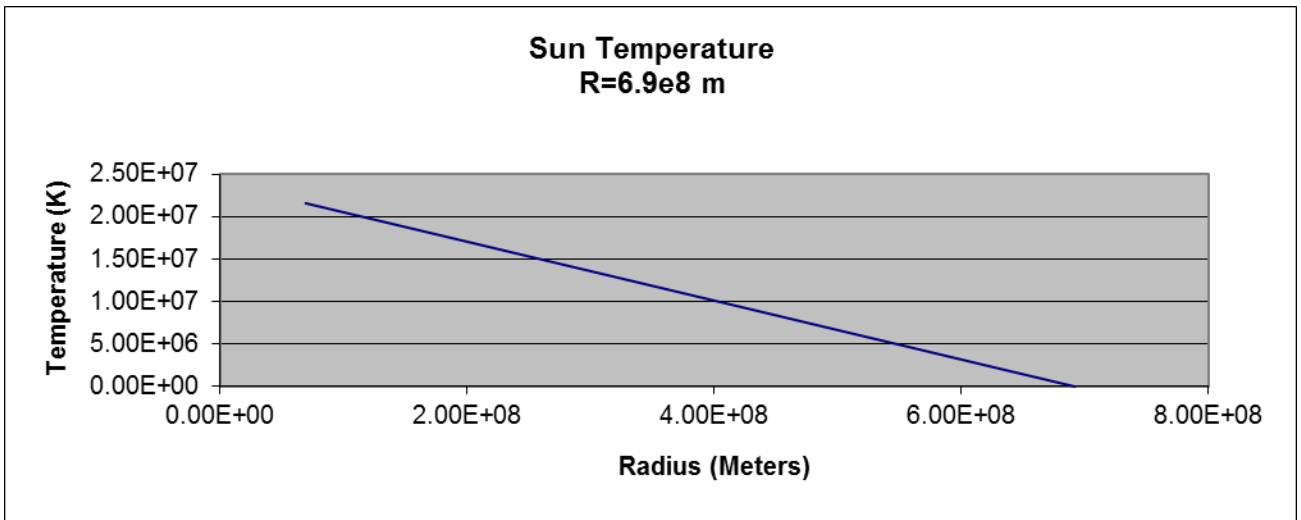
### **Approximate Solar Model**

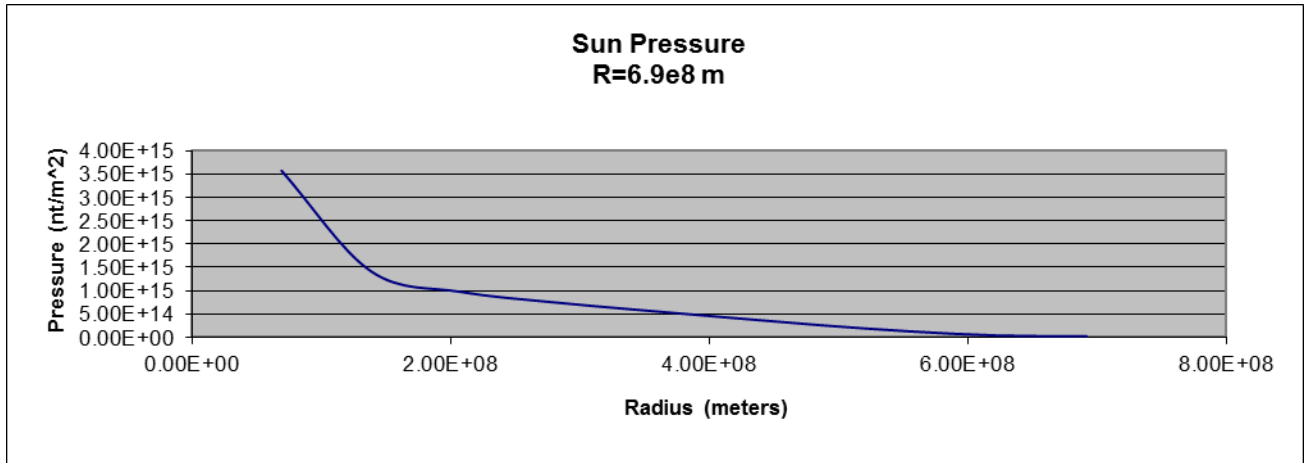
An analysis of black holes will be presented below. To develop the calculation procedure we will start with the sun since it can be compared with data. An approximate model of the pressure and temperature of the sun as a function of radius is shown below. It is based on breaking the sun into 10 radial shells each  $1/10$  of the full radius.





The mass above a shell surface is gravitationally attracted to the mass below the shell yielding acceleration  $a = G \cdot M_{\text{central}} / R^2$ . At the bottom of each shell, the pressure is calculated as  $P_{\text{shell}} = \text{acceleration} \cdot \text{density} \cdot dR$ . Pressure adds down through the 10 shells until a central pressure is determined. Because the sun is about 4B years old, most of its early history has dissipated and temperature increases from the outside to the core due to nuclear power in the core. A linear relationship was assumed even though the radiation and convection zones conduct heat differently. The surface temperature is known ( $6e3$  K) and the interior temperature is approximately  $2e7$  K. Density of each shell is determined by the equation  $\rho = P / (R \cdot T)$ , where  $R$  is the gas constant for hydrogen  $7980$  nt-m/(Kg-K). Our sun has an interior density of about  $1.2e5$  nt/m<sup>2</sup> and the temperature and pressure profiles below produce this density in the interior. Here are plots for the model of the  $2e30$  Kg sun.





The core of the 10 shell model is in the chart below along with the equations. All 10 layers are in the Appendix 3. The top of the core (h9) is at radius 6.92e7 meters and R is the sun's overall radius 5.9e8 meters.

|                           |   |             |
|---------------------------|---|-------------|
| P9 (nt/m <sup>2</sup> )   | $P9=P8+\rho8*a8*R/10$                   | 1.41E+15    |
| rho9 kg/m <sup>3</sup>    | $\rho9=P9/(R*T8)$                       | 3.946E+04   |
| h9 (m)                    | $h9=h8-R/10$                            | 6.92E+07    |
| T (K)                     | $T9=T8+dT$                              | 2.160E+07   |
| a10 (m/sec <sup>2</sup> ) | $a9=6.673e-11*(M-cumm10)/(h9)^2$        | 7.90E+02    |
| m10 shell Kg              | $m10=4/3*PI()*(h9^3-(h9-R/10)^3)*\rho9$ | 1.94E+27    |
| cum mass10                | $cumm10=cumm9+m10$                      | 1.943E+30   |
| P10 (nt/m <sup>2</sup> )  | $P10=P9+\rho9*a9*R/10$                  | 3.57E+15    |
| rho10 kg/m <sup>3</sup>   | $\rho10=P10/(R*T9)$                     | 8.869E+04   |
| h10 core                  | $h10=h9-R/10$                           | 0           |
| r cell (m)                | $r_{cell}=(1/\rho10*1.67E-27/)$         | 1.65038E-11 |

Again, the cell radius is the radius around each proton and is a function of density. The sun is dense enough at the core to make the electron degenerate because the cell radius is lower than the electron radius 5.29e-11 meters. The author uses a probability based model [9] [14] for fusion diagramed briefly in Appendix 1. The following table [9] gives the fusion kinetics based on the pressure and temperature model above:

| <b>Solar example</b>                            |   | <b>B=8.62e-11</b> |                              |
|---|---|-------------------|------------------------------|
| <b>Temp deg K</b>                               |   | <b>2.39E+07</b>   | <b>Dmax kg/m<sup>3</sup></b> |
| <b>Density kg/m<sup>3</sup></b>                 |   | <b>1.22E+05</b>   | <b>5.020E+11</b>             |
| <b>KE temp 1.5*B*T</b>                          |   | <b>3.084E-03</b>  |                              |
| <b>degeneracy</b>                               |   | <b>3.56E+00</b>   |                              |
| <b>Degenerate radius (D)</b>                    |   | <b>1.485E-11</b>  |                              |
| <b>v/c</b>                                      |   | <b>0.109</b>      |                              |
| <b>Barrier</b>                                  |   | <b>-0.0139</b>    |                              |
| <b>Example calculation for above conditions</b> |   |                   |                              |
| <b>rate</b>                                     | <b>Pbarrier</b>                                 | <b>Pd=(dens/m</b> | <b>Preaction rate R/sec</b>  |
| <b>Probability/sec</b>                          | <b>exp(-.0139/. (1.2e5/5e11 v/r/exp(62.87))</b> |                   |                              |
| <b>8.19E-18</b>                                 | <b>0.0109429</b>                                | <b>2.43E-07</b>   | <b>1.09667E-09</b>           |
| <b>burn time (Byrs)</b>                         | <b>3.9</b>                                      |                   | <b>3.078E-09</b>             |
| <b>sun N</b>                                    | <b>1.198E+57</b>                                |                   |                              |
| <b>fract burning</b>                            | <b>0.15</b>                                     |                   |                              |
| <b>burn rate N/sec*mev/</b>                     | <b>1.47E+39</b>                                 |                   |                              |

It has a burn time of about 10 Billion years and generates about 4e39 MeV/sec.

### Analysis of BH with 4.7 solar mass (9.9e30 Kg)

A black hole (BH) with 4.7 solar masses is quite different than the sun. To achieve BH status, the Schwarzschild radius  $S' = GM/C^2$  for this 9.9e30Kg black hole was 7800 m, not 6.9e8 meters like the sun. This BH has a density of approximately 1.8e19 kg/m<sup>3</sup> and much higher temperature and pressure. Power generation in the sun increases the core temperature and lowers its density. But BH's of this size are burned out. The lack of nuclear power allowed the density to increase and it became a black hole.

Literature indicates the density of a BH is a function of pressure and suggests [15] the relationship:  $\rho = (\text{pressure}/k)^{5/3}$ . Wiki also indicates that temperature is not important. Pressure at the bottom of each shell is acceleration\* $\rho$ \*dR and accumulates down through the shells.  $\rho$  is the density of the layer, dR is the thickness of the shell and acceleration is  $G*M_{\text{central}}/R^2$  where R is the radius to the center of the black hole and G is the gravitational constant. The core (10<sup>th</sup> and inner most volume of the model) analysis is shown below.

Density starts with low density at the first shell interface and increases exponentially down through the layers. This puts almost the entire mass of the black hole in the core. The core pressure becomes 5.3e39 nt/m<sup>2</sup>. The cell radius would be on the order of 5.5e-23 meters. This radius suggests to most physicists that there must be degenerate nuclear forces to prevent collapse and a singularity.

|                           |  |          |
|---------------------------|--|----------|
| P9 (nt/m <sup>2</sup> )   | P9=P8+rho8*a8*h0/10  | 1.38E+29 |
| rho9 kg/m <sup>3</sup>    | rho9=(P9/1e16) <sup>(5/3)</sup>                              | 5.90E+21 |
| h9 (m)                    | h9=h8-h0/10  | 7.38E+02 |
| T (K)                     | T9=T8+dT   | 1.60E+13 |
| a10 (m/sec <sup>2</sup> ) | a9=6.673e-11*(M-m8)/(h9) <sup>2</sup>                        | 1.22E+15 |
| m10 shell Kg              | ms10=4/3*PI()*h9 <sup>3</sup> -(h9-h0/10) <sup>3</sup> *rho8 | 9.94E+30 |
| cum mass10                | cumm10=cumm9+ms10  | 9.94E+30 |
| P10 (nt/m <sup>2</sup> )  | P10=P9+rho9*a9*h0/10   | 5.30E+39 |
| rho10 kg/m <sup>3</sup>   | rho10=(P10/1e16) <sup>(5/3)</sup>                            | 2.59E+39 |
| h10 core                  | 0  | 0.00E+00 |
| r cell (m)                | rcell=(1/rho10*1.67E-27/4)                                   | 5.36E-23 |
|                           | C=(a*R9) <sup>.5</sup>                                       | 9.48E+08 |
|                           | smallest   |          |

k in the equation  $\rho=(P/k)^{(5/3)}$  was  $1e16$  since this allowed the mass of the layers to total  $9.9e31$  Kg.

### Can $\rho=(P/k)^{(5/3)}$ be correct?

The author finds fault with the density equation ( $\rho=(P/k)^{(5/3)}$ ). The reason follows: Schwarzschild  $S' = GM/C^2$ . Turning this equation around,  $C=(GM/S')^{0.5}$ . Substituting in acceleration  $a=GM/R^2$  and  $S'=R$ , we have  $C=(a*S')^{0.5}$ . Using the values above  $a=1.22e15$  m/sec<sup>2</sup> and  $S'=738$  m,  $(a*S')^{0.5}=9.48e8$  m/sec. Acceleration is too high and  $(a*S')^{0.5}$  exceeds C. One can see that this implies R lower than S' since  $R=GM/9.48e8^2$ . The radius of each cell in the core would be extremely low at  $5.3e-23$  meters.

If we insist that C is not exceeded, the acceleration in each shell is  $a= C^2/R$ , where R is radius to the core. This means R is the Schwarzschild radius S' for each layer. In other words, each layer of the model is like the surface of the black hole and the velocity is C, i.e.  $(a*S')^{0.5}=C$ . Consider the alternatives. If we let velocity at the layer's surface be less than C, the mass will not fit inside S' and it won't be a black hole. The other alternative is to let velocity exceed C. In the author's work, C is just the ratio between space and time and can't be violated. Another argument against the Wiki density relationship involves basic tenants of gravity. In special relativity gravity is the geometry of space-time and gravitational force is inertial force. The reason we feel force upward on the bottom of our feet is that our velocity is too low to be on a geodesic defined by the radius of the earth. We can calculate V to be on the geodesic. First we calculate our acceleration at the surface of the earth,  $a=G*Mearth/Rearth^2=9.75$  m/sec<sup>2</sup>. Next we calculate  $V=(a*R)^{0.5}=7897$  m/sec. The force upward on us is  $F=mass*a=m(V)^2/R=mass*7897^2/6.38e6$  nt. Force upward from the earth is making up for the outward inertial force we are missing because our velocity is too low. The equation could also be written  $F=mass*(7897^2-Vlow^2)/R$ . In this equation Vlow is fixed by us being on earth. Inside a black hole,  $V=C=(a*S')^{0.5}$  and  $Vlow=zero$ ; fixed by the fact that particles are inside the black hole. The equation becomes  $F=mass*(C^2-Vlow^2)/S'$ . The force is associated with velocity C and only light travels at C. The particles in the BH have no velocity, just temperature.

When cell radius  $r$  becomes less than  $6.5e-15$  meters the electromagnetic force is overwhelmed (see chart above in the section entitled “What resists collapse?”) and the resisting pressure is based on cell pressure. Neutrons collide and the change in momentum is force. Pressure is determined by dividing the force by the cell surface area. Originally the star that collapsed into a black hole was hot and when the nuclear fuel was expended, it contracted creating compression energy. Temperatures on the order of  $2e13$  K occur in the bottom shell of the  $9.9e30$  Kg black hole. The kinetic energy associated with this temperature is  $1.5*8.4e-11*2e13=2730$  MeV. This becomes the kinetic energy in the cell diagram above. The gravitational model for the core pressure based on insisting that  $(a*S')^{0.5}=C$  is shown below. The full model results are in the Appendix 3 but the radius of a cell at the core of a  $9.9e30$  Kg black hole is  $2.93e-16$  meters.

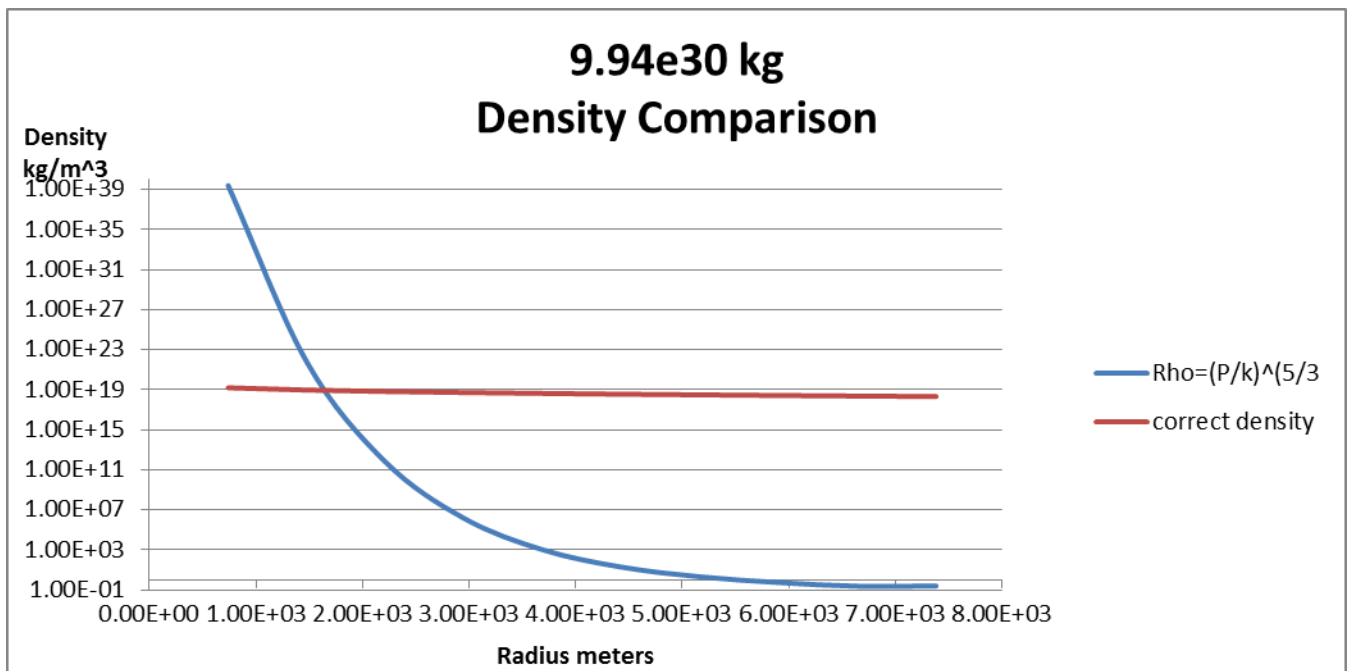
|                                      |  |  |  |                 |
|--------------------------------------|--|--|--|-----------------|
| P9 (nt/m <sup>2</sup> )              |  | $P9=P8+\rho_8*a_8*h_0/10$  |  | 8.82E+35        |
| $\rho_9$ kg/m <sup>3</sup>           |  | $\rho_9=1.67E-27/(4/3*PI()*(((4.9e-25)/(P9)))^{0.2565})^3$       |  | 9.42E+18        |
| $h_9$ (m)                            |  | $h_9=h_8-h_0/10$   |  | 7.38E+02        |
| T (K)                                |  | $T_9=(9.7*7.34e-14/rcell)/1.5/8.6e-11$                           |  | 1.58E+13        |
| $a_{10}$ (m/sec <sup>2</sup> )       |  | $a_9=C^2/h_9$  |  | 1.22E+14        |
| $m_{10}$ shell Kg                    |  | $m_{s10}=4/3*PI()*h_9^3-(h_9-S/10)^3*\rho_9$                     |  | 1.59E+28        |
| cum mass <sub>10</sub>               |  | $cumm_{10}=a_9*h_9^2/G$  |  | 9.96E+29        |
| P <sub>10</sub> (nt/m <sup>2</sup> ) |  | $P_{10}=P_9+\rho_9*a_9*h_0/10$                                   |  | 1.73E+36        |
| $\rho_{10}$ kg/m <sup>3</sup>        |  | $\rho_{10}=1.67E-27/(4/3*PI()*(((4.9e-25)/(P_{10}))^{0.2565})^3$ |  | 1.58E+19        |
| $h_{10}$ core                        |  | 0  |  | 0               |
| $r$ cell (m)                         |  | $rcell=(1/\rho_{10}*1.67E-27/(4/3*PI()))^{1/3}$                  |  | 2.93E-16        |
|                                      |  | $C=(a*R_9)^{.5}$   |  | 3.00E+08        |
|                                      |  |  |  | small $c_{max}$ |
|                                      |  |  |  | <b>proton</b>   |
|                                      |  | mass MeV   | Proton                                 | 938.2720        |
|                                      |  | mass kg  | $938.27*1.78e-30$ kg/MeV               | 1.673E-27       |
|                                      |  | Field MeV  | (See Proton Model)                     | 2.683           |
|                                      |  | ke   | $KE=9.7*7.34e-14/2.93e-16$             | 2432.950        |
|                                      |  | $g=m/(m+ke)$   | $g=938.27/(938.27+ke)$                 | 0.2783          |
|                                      |  | V/C  | $V/C=(1-(g)^2)^{0.5}$                  | 0.9605          |
|                                      |  | $r$ cell (meters)  | $r$                                    | 2.932E-16       |
|                                      |  | $F=m*V^2/R/exp(90)$  | $F=(1.67e-27/g)*(v/C)^2/R$             | 1.392E-33       |
|                                      |  |  |  | 1               |
|                                      |  | $area=4*pi*R^2$  | $area=4*pi*r^2$                        | 1.081E-30       |
|                                      |  | Pressure (nt/m <sup>2</sup> )                                    | $P=F/area$                             | 1.573E+36       |
|                                      |  | Temperature (K)  | $T=ke/1.5/8.6e-11$                     | 2.38524E+13     |
|                                      |  | $\rho$   | $\rho=(1/r^3*1.67E-27/(4/3*PI()))$     | 1.58E+19        |
|                                      |  | "gas constant" R   | $R=P/(\rho*T)$ Nt-K/(m-kg)             | 4.17E+03        |
|                                      |  |  | $r=(1/\rho*1.67E-27/(4/3*PI()))^{1/3}$ | 2.93E-16        |

Note: the equation for density is developed in Appendix 1.

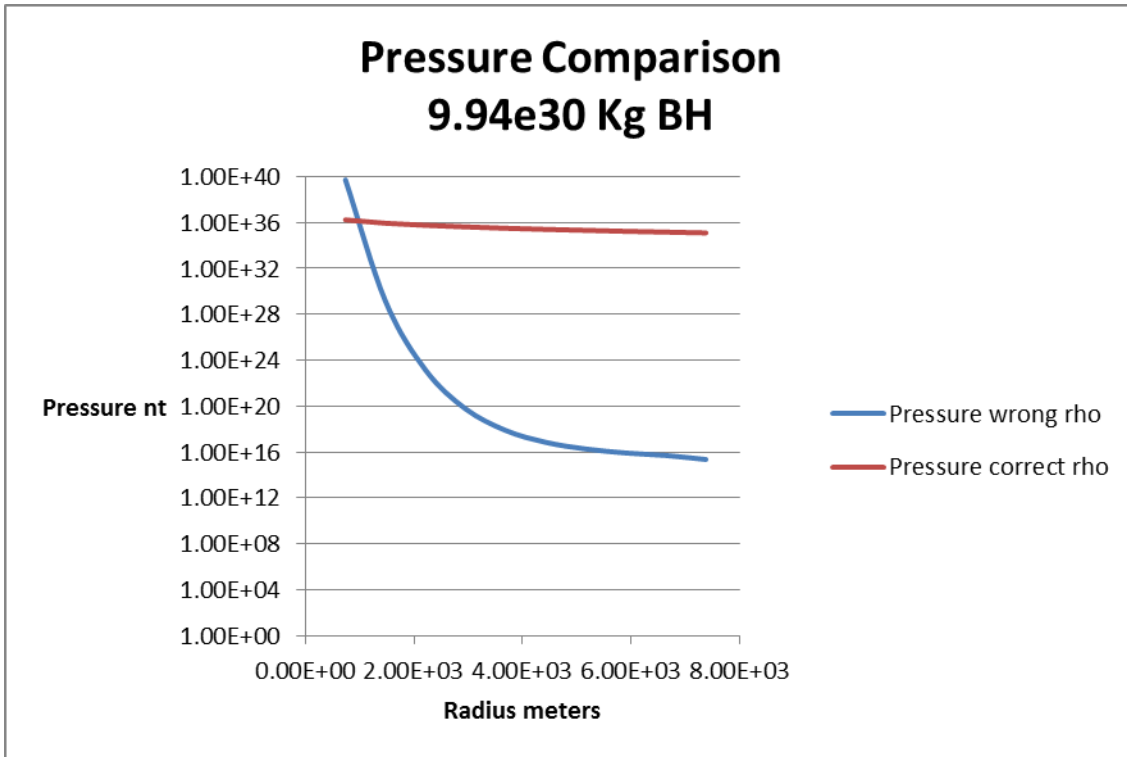
Recognize the bottom part of the above diagram as the quantum gravity calculation except radius has been reduced. It follows the same form as the basis of low pressure gas thermodynamics presented above in the section entitled “What resists collapse?” Since cellular quantum gravity kinetic energy varies with radius, the kinetic energy has increased to  $9.7 \times 7.34 \times 10^{-14} / 2.93 \times 10^{-16} = 2430$  MeV and this is associated with compression temperature of  $2430 / 1.5 / B = 2.14 \times 10^{13}$ . Pressure in the cell is calculated by taking inertial gravitational force/cell surface area. The outward pressure  $1.6 \times 10^{36}$  nt/m<sup>2</sup> exerted by the cell resists the pressure near the center of the black hole ( $1.7 \times 10^{36}$  nt/m<sup>2</sup>) and prevents it from collapsing. The energy is a natural outcome of the temperature resulting from compression and does not need to be associated with degeneracy. Gravity resisted pressure involves temperature but temperature, pressure and density are intimately linked. The density increases until the pressure is resisted similar to the way gases resist compression. This allowed the author to develop the equation below for rho. It is based on solving the gravity equation for cell radius and determining rho from cell radius. It is based on known equations but fits a computationally friendly relationship (Appendix 1) within 8% over small black hole pressures investigated.

$$\rho = 1.67 \times 10^{-27} / (4/3 \times \pi) \times ((4.8 \times 10^{-25} / P)^{0.2565})^3$$

Below is a comparison of the two density relationships.



Below, the pressure is compared for the two density relationships. Note that the central pressure is lower using the correct rho.



The question might be asked, does compressing the radius cause the gravitational constant to change? Actually, the fundamental radius is  $r=9.17e-13 \text{ MeV-sec}/(2.683*2.683)^{.5}=7.34e-14 \text{ meters}$ . This radius does not change because the field energy 2.683 MeV is set by the Proton mass model. Gravitational constant G is maintained by the relationships in the proton mass model. The cell relationships maintain an approximate G but  $G=F*R^2/(M/g)^2$  may decrease slightly if we consider that  $M/\gamma$  increases because of temperature related velocity.

### More massive black holes

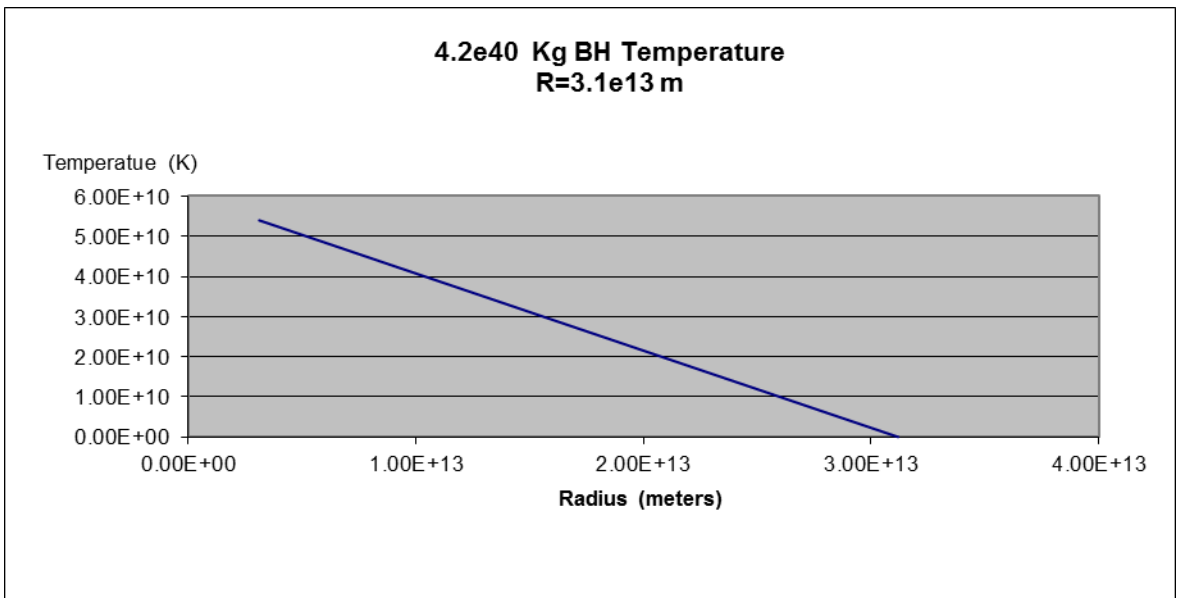
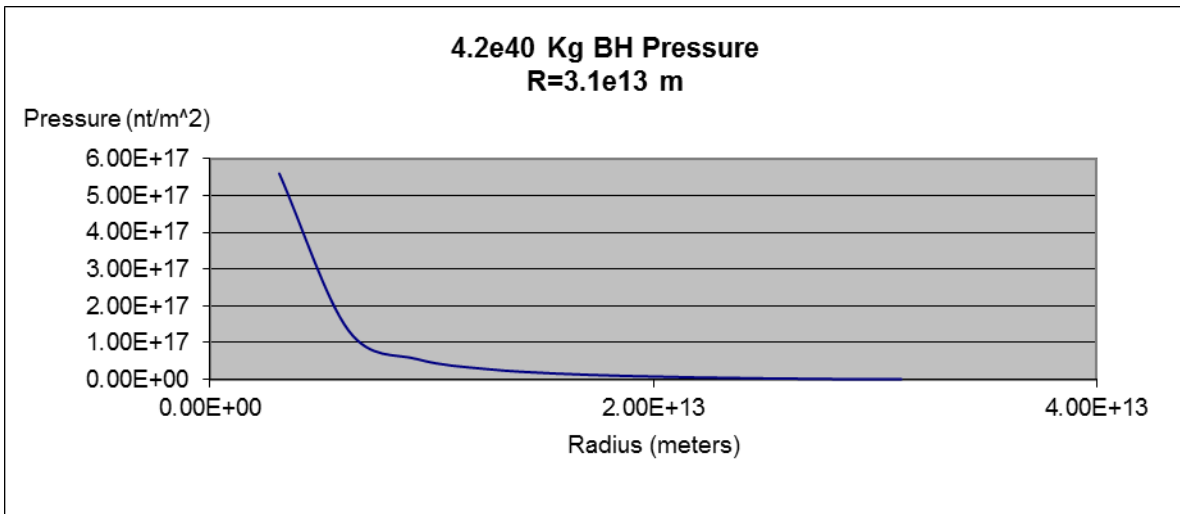
Stars that complete their aging cycle can be about 20 solar masses but not much greater according to Wiki. The analysis above was repeated for this size black hole.

|                           |  |                                     |                  |                            |
|---------------------------|--|-------------------------------------|------------------|----------------------------|
| P9 (nt/m <sup>2</sup> )   | P9=P8+rho8*a8*h0/10                                    |                                     | 7.47E+33         | <b>7.42E+33</b>            |
| rho9 kg/m <sup>3</sup>    | rho9=1.67E-27/(4/3*PI()*(((4.9e-25)/(P9)))^0.2565)^3   |                                     | 2.39E+17         | 1.18524E-15                |
| h9 (m)                    | h9=h8-h0/10  |                                     | 4.22E+04         | 1.01E+00                   |
| T (K)                     | T9=(9.7*7.34e-14/rcell)/1.5/8.6e-11                    |                                     | 4.66E+12         | 4.66E+12                   |
| a10 (m/sec <sup>2</sup> ) | a9=C <sup>2</sup> /h9                                  |                                     | 2.13E+12         | 9.55E-01                   |
| m10 shell Kg              | ms10=4/3*PI()*h9^3-(h9-S/10)^3)*rho9                   |                                     | 1.48E+31         |                            |
| cum mass10                | cumm10=a9*h9^2/G                                       |                                     | 5.70E+31         | 5.98E-01                   |
| P10 (nt/m <sup>2</sup> )  | P10=P9+rho9*a9*h0/10                                   |                                     | 8.98E+33         | <b>8.90E+33</b>            |
| rho10 kg/m <sup>3</sup>   | rho10=1.67E-27/(4/3*PI()*(((4.9e-25)/(P10)))^0.2565)^3 |                                     | 2.76E+17         | 1.13033E-15                |
| h10 core                  | 0  |                                     | 0                | 1.01E+00                   |
| r cell (m)                | rcell=(1/rho10*1.67E-27/(4/3*PI()))^(1/3)              |                                     | 1.13E-15         | 4.88E+12                   |
|                           | C=(a*R9)^.5  |                                     | 3.00E+08         | 9.54E-01                   |
|                           |  |                                     | 20x sol cmax     |                            |
|                           |  |                                     | <b>proton</b>    |                            |
|                           | mass MeV   | Proton                              | <b>9.38E+02</b>  | mass MeV                   |
|                           | mass kg  | 938.27*1.78e-30 kg/MeV              | <b>1.67E-27</b>  | mass kg                    |
|                           | Field MeV  | (See Proton Model)                  | <b>2.68E+00</b>  | Field MeV                  |
|                           | ke   | KE=9.7*7.34e-14/2.93e-16            | <b>631.183</b>   | Kinetic Energy             |
|                           | g=m/(m+ke)   | g=938.27/(938.27+ke)                | <b>5.98E-01</b>  | gamma                      |
|                           | V/C  | V/C=(1-g)^2)^0.5                    | <b>8.02E-01</b>  | V/C                        |
|                           | rcell (meters)   | r                                   | <b>1.130E-15</b> | Radius                     |
|                           | F=m*V^2/R/exp(90)                                      | F=(1.67e-27/g)*(v/C)^2/R            | <b>1.17E-34</b>  | Force Nt                   |
|                           |  |                                     | 1.00E+00         |                            |
|                           | area=4*pi*R^2  | area=4*pi*r^2                       | <b>1.61E-29</b>  | Cell surface area          |
|                           | Pressure (nt/m <sup>2</sup> )                          | P=F/area                            | <b>8.90E+33</b>  | Pressure nt/m <sup>2</sup> |
|                           | Temperature (K)  | T=ke/1.5/8.6e-11                    | 6.18807E+12      |                            |
|                           | rho  | rho=(1/r^3*1.67E-27/(4/3*PI()))     | 2.76E+17         |                            |
|                           | "gas constant" R                                       | R=P/(rhoT) Nt-K/(m-kg)              | 5.21E+03         |                            |
|                           |  | r=(1/rho*1.67E-27/(4/3*PI()))^(1/3) | 1.13E-15         |                            |

## Supermassive Black Holes

Supermassive black holes are more similar to our sun than small mass black holes because their density is low. In fact, analysis shows that supermassive black holes have not burned out and their fusion kinetics are similar to the sun. The author is aware that there are short comings to applying the solar model to a black hole. Nevertheless, it is worth showing results even if the model is inaccurate because it reveals differences involved. First, we will apply the model to a BH of 4.2e40 Kg. Here is a plot of the pressure and temperature for a BH of 4.2e40 Kg with a horizon 3.1e13 meters. The average density of the BH is quite low, only 6e-2 Kg/m<sup>3</sup>. The pressure at the core is about 6e17 nt/m<sup>2</sup>. The thermodynamics are controlled by electromagnetic kinetic energy since the pressure is well below the electromagnetic to gravity transition 1e28 nt/m<sup>2</sup>.





The hydrogen fusion kinetics are estimated below:

|                              |                   | largest cmax |
|------------------------------|-------------------|--------------|
| <b>4.2e40 Kg Solar Model</b> |                   |              |
|                              | Max Dens          | 5.020E+11    |
| Temp                         | deg K             | 6.00E+10     |
| Density                      | kg/m <sup>3</sup> | 3.31E+01     |
| KE                           | 1.5*B*T           | 7.76E+00     |
| degeneracy                   |                   | 2.31E-01     |
| Degenerate radius (DR)       |                   | 5.290E-11    |
| v/c                          |                   | 0.998        |
| Barrier                      |                   | -0.0139      |
| P barrier                    |                   | 9.98E-01     |
| Pdensity                     |                   | 6.60E-11     |
| Rate/sec                     |                   | 7.888E-09    |
| Probability/sec              |                   | 5.19E-19     |
| burn time (Byrs)             |                   | 61           |
| N=m/1.67e-27                 |                   | 5.988E+67    |
| fract burning                |                   | 0.15         |
| burn rate N/sec*mev/N        |                   | 4.67E+48     |
| power mev/sec                |                   | 3.11E+49     |

The 4.2e40 BH is burning so slowly that it could last 60 billion years. Its heat generation can be compared to our sun (1e39 MeV/sec) but is 2.1e10 more massive. One difference from a star is that heat may not escape as readily. This would increase its overall temperature and lower its density. Density can't be lowered much or the mass will not fit within the Schwarzschild radius. The pressure profile depends on the mass and again can't change much. One might ask if the contained mass becomes a "pressure cooker" since nuclear energy cannot easily escape. According to the proton mass model (Appendix 1), there is only 10 MeV/particle of nuclear energy available per particle. The effect of burning this fuel and containing it in the BH is shown below.

|                               |  |
|-------------------------------|--|
| Nuclear energy release 10 MeV |  |
| T core=6e10K                  |  |
| dQ=10 MeV                     |  |
| dt=10/(1.5 B)                 |  |
| 7.73E+10 K                    |  |
| T core with dQ=6e10+7.8e10    |  |
| 1.373E+11 K                   |  |
| P=7829 T rho                  |  |
| Pressure may double           |  |

The pressure may double if all the nuclear energy per particle is released. However pressure is reasonably low and easily contained.

## The BH surface

What are the conditions at the surface of the black hole? As new mass enters the black hole and collides with the spherical surface it carries about 9.8 MeV of kinetic energy ( $7.6 \times 10^8$  K) with it as a result of falling toward the black hole at high velocity. There is a lot of photon energy produced by a hot surface. One question might be: Where does the photon energy go? The Schwarzschild equation tells us that photons are attracted to the large gravitational pull of the BH and photons and travel in curved paths around the BH horizon. They travel at the speed of light but cannot escape. This means that at the horizon there is a large flux of photons. This flux probably photo-disintegrates bonds (electromagnetic and nuclear) of atoms that cross the horizon.

## Summary

Black holes that range from 4.7 to 20 solar masses involve high core pressures. The author addressed the question “What keeps these black holes from collapsing?” Based on the author’s theory of quantum gravity the answer to this question is gravitationally derived kinetic energy inside cells. Thermodynamic pressure in low pressure gases is based on compression of the electron radius. The density in the gas adjusts in such a way to resist compression. When a radius is compressed beyond  $4.6 \times 10^{-15}$  meters, the electron is overwhelmed since the kinetic energy involved converts a proton back into a neutron. Black holes involve radii as low as  $2.93 \times 10^{-16}$  meters. The model for gravity resisted pressure is similar to standard gas thermodynamic pressure except the gravitational kinetic energy is 9.7 MeV, not the electromagnetic kinetic energy  $13.6 \times 10^{-6}$  MeV. The transition between these two zones is about  $1 \times 10^{28}$  nt/m<sup>2</sup>. The equation given in Wiki for high density objects as a function of pressure cannot be used for black holes.

Supermassive black holes as large as  $4.2 \times 10^{40}$  Kg have been observed. These black holes involve temperature and pressures in their cores that strongly suggest nuclear energy generation. However, analysis shows that they burn very slowly and can contain the power generated. Their pressure is low and well within the electromagnetic controlled thermodynamics zone.

## References

1. Brillouin, L., *Science and Information Theory*, New York: Academic Press, Inc., 1956.
2. Shannon, Claude. *A mathematical Theory of Communication*, 1948.
3. P.J.E. Peebles, *Principles of Physical Cosmology*, Princeton University Press, 1993.
4. Bennett, C.L. et al. *First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Preliminary Maps and Basic Data*, *Astrophysical Journal*, 2001 L. Verde, H. V. Perris, D. N. Spergel, M. R. Nolta, C. L. Bennett, M. Halpern, G. Hinshaw, N. Jarosik, A. Kogut, M. Limon, S. S. Meyer, L. Page, G. S. Tucker, E. Wollack, E. L. Wright

5. Barbee, Gene H., *A Top-Down Approach to Fundamental Interactions*, FQXi essay, June 2012 and vixra:1307.0082 revised Mar 2014. Reference Microsoft® spreadsheet Unifying concepts of nature.xls.
6. Barbee, Gene H., *On the Source of the Gravitational Constant at the Low Energy Scale*, vixra:1307.0085, revised Feb 2014. Prespacetime Journal Vol. 5 No. 3 March 2014.
7. Barbee, Gene H., *Application of Proton Mass Model to Cosmology*, vixra:1307.0090, revised Mar 2014. Reference spreadsheet simple1c.xls.
8. Barbee, Gene H., *On Expansion Energy, Dark Energy and Missing Mass*, Prespacetime Journal Vol. 5 No. 5, May 2014. Previously vixra:1307.0089, revised Mar 2014.
9. Barbee, Gene H., *Semi-Fundamental Abundance of the Elements*, vixra:1308.0009, revised Nov 2013.
10. Bergstrom, L. and Goobar, A., *Cosmology and Particle Astrophysics*, 2<sup>nd</sup> Edition, Springer-Praxis Books in Astrophysics and Astronomy, 2004.
11. Barbee, Gene H., *The Effect of He4 Fusion on Primordial Deuterium*, vixra:1404.0465, May 2014.
12. Barbee, Gene H., *The Language of Nature*, Amazon books, May 2014.
13. Barbee, Gene H., *Cosmology, Thermodynamics and Time*, vixra:1407.0187, September 2014.
14. Barbee, Gene H., *A Fundamental Model of Atomic Binding Energy*, vixra:1307.0102, accompanying Microsoft® spreadsheet atom.xls.
15. [http://en.wikipedia.org/wiki/Black\\_hole](http://en.wikipedia.org/wiki/Black_hole)
16. Barbee, Gene H., *Black Holes and Quantum Gravity*, vixra:1409.0242, September 2014.

## Appendix 1 Review of cellular cosmology

Using a small cells of radius  $r$  to simulate a large radius  $R$  (literature would call this the radius of the universe) is critical to understanding cosmology. In this model [5], the universe is filled with the *surface* of many small cells that are equivalent to the *surface* of one large sphere. This is important conceptually because we can be inside the universe (something we all observe), each surface can be identical and the concept that there is no preferred location can be preserved. The model proposed is based on  $\exp(180)$  cells, each associated with a proton like mass.

The derivation of a coupling constant for gravitation from reference 6 is reviewed below: Let small  $r$  represent the radius of a many small spheres and large  $R$  represent the same surface area of one large sphere containing  $\exp(180)$  spheres. There is one proton like mass ( $m$ ) on the surface of each cell. The mass of the universe  $M$  equals  $m \cdot \exp(180)$ . The laws describing each particle are no different than any other particle. Geometrically, many small cells with the same combined surface area offer this feature. General relativity uses the metric tensor  $(ds^2)$ [4]. The surface area of a 2-sphere is broken into

many small spheres with an equal surface area, i.e.  $A=a*\exp(180)$  and  $r=R/\exp(90)$ . The total energy will be that of a proton mass/cell plus a small amount of expansion kinetic energy. Based on geometry, two substitutions are placed in the gravitational constant G below, i.e.  $M=m*\exp(180)$  and  $R=r*\exp(90)$ .

|                                 |                          |                                 |                                |
|---------------------------------|--------------------------|---------------------------------|--------------------------------|
|                                 | Area=4 pi R^2            |                                 |                                |
|                                 | Area=4 pi r^2*exp(180)   |                                 |                                |
|                                 | A/A=1=R^2/(r^2*exp(180)) |                                 |                                |
|                                 | R^2=r^2*exp(180)         |                                 |                                |
|                                 | R=r*exp(90)              |                                 |                                |
|                                 | M=m*exp(180)             |                                 |                                |
| Large space G                   |                          | cellular size G                 |                                |
| $RV^2/M$                        | $G=G$                    | $r^2/m$                         | r is the cell radius           |
| $R^2V^2/M$                      | $G=G$                    | $r^2/m$                         | r' is the proton size geodesic |
| $R'=r*(v/V)^2*(M/m)*1/\exp(90)$ |                          |                                 |                                |
|                                 | $RV^2/M=$                | $r*\exp(90) *v^2/(m*\exp(180))$ |                                |
|                                 |                          | $G=(r *v^2/m)*1/\exp(90)$       |                                |

For G to be equivalent between many small cells and one large sphere the geodesics (the combination of r,v and m that give G) of cells must be multiplied by the small factor  $1/\exp(90)$ . This value is the gravitational coupling constant [6] for a cell that has cosmological properties, i.e. the force is shared with  $\exp(180)$  particles on a surface that is  $1/\exp(90)$  of the total surface. The author documented a quantum theory of gravity [6]. The quantum scale was identified as radius  $7.34e-14$  meters.

The proton is thought to be a primary manifestation of the underlying laws and as such contains information that determines many aspects of nature. The Proton Mass model is the source of constants for unification of forces in the table above.

### Neutron and Proton mass models

First an abbreviated model is shown for the neutron. The abbreviated model adds the three quark masses together. They total 130.163 MeV. Then, we add together the three quark kinetic energies. They total 799.25 MeV. The three quarks are imbedded in a strong field labelled -957.185 and a gravitational field energy labelled -2.683 MeV. The total field energy for the entire table is -959.968 MeV. This is lower than the known Neutron mass 939.565 MeV and the “missing energy” is a potential energy field -20.3 MeV. The quarks with their kinetic energy fall into the field and gain 10.15 MeV from their fall. The neutron subsequently falls to a radius  $7.34e-14$  meters. It originally had kinetic energy 20.3 MeV but when the gravitational orbit was established, the ke was about 10 and the potential energy was about 10. The neutron plus its associated gravitational energy balances the total table energy 959.87 MeV.

| Mass and Kinetic Energy |        |               | Field energy |               |
|-------------------------|--------|---------------|--------------|---------------|
| Mass                    | ke     | Strong        | Strong       | Gravitational |
| Quarks                  |        | Residual      | field energy | Energy        |
| mev                     | mev    | Field         | mev          | mev           |
| Strong                  | 130.16 | 799.25        | -957.18      | -2.68         |
| Strong Residual KE      |        | 10.15         |              |               |
| Neutron                 |        | 939.57        | -20.30       | -959.87       |
| Gravitational ke        |        | 10.07         |              |               |
| Gravitational pe        |        | 10.24         |              |               |
| <b>Total</b>            |        | <b>959.87</b> |              |               |

The more complete model is shown below.

| Mass, Kinetic Energy and Fields for Neutron |            |             |           |            |                |                     |
|---|------------|-------------|-----------|------------|----------------|---------------------|
| Mass  | Difference | Residual ke | Expansion | KE         | Strong field   | Gravitational Field |
| mev   | mev        | mev         | mev       | KE         | MeV            | MeV                 |
| 101.95                                      | 641.88     |             |           |            | -753.29        |                     |
|   |            |             |           |            |                | -0.69               |
| 13.80                                       | 78.69      |             |           |            | -101.95        |                     |
|   |            |             |           |            |                | -0.69               |
| 13.80                                       | 78.69      |             |           |            | -101.95        |                     |
|   |            |             |           |            |                | -0.69               |
|   |            | 10.15       |           | 10.15      |                |                     |
| 0.00  | 0.00       |             |           | 10.15      |                |                     |
|   |            |             |           |            |                |                     |
| 0.62  | 0.00       |             | 0.00      |            | 0.00           |                     |
|   |            |             |           |            |                | -0.62               |
| 130.16                                      | 799.25     | 939.57      | 0.00      | 20.30      | -957.18        | -2.68               |
| <b>NEUTRON MASS</b>                         |            |             |           |            |                |                     |
|   |            |             |           | Total m+ke |                |                     |
|   |            |             |           |            | Total negative |                     |
|   |            |             |           | 959.87     | -959.87        | 0                   |
|   |            |             |           | MeV        | MeV            | Net                 |

The neutron decays into a proton, an electron and a neutrino. This gives the measured proton mass 938.27 MeV below. As the proton and electron split they develop opposite

fields of  $27.2 \times 10^{-6}$  Mev. When the electron falls into the proton field it develops  $13.6 \times 10^{-6}$  Mev.

## Appendix 1 Density for small black holes

Development of equation for rho from pressure for a black hole

|                          |          |  |  |  |
|--------------------------|----------|--|--|--|
| gamma                    | 3.19E-01 | gamma (g)= $938.2/(938.2+9.72 \times 7.34 \times 10^{-14}/r)$  |  |  |
| P (nt/m <sup>2</sup> )   | 7.39E+35 | Pressure= $((1.67 \times 10^{-27}/(g)) \times (((1-(g)^2)^{0.5}) \times C)^2/r)/(4 \times \text{PI}() \times r^2)$ |  |  |
| T (K)                    | 1.55E+13 | T= $(9.7 \times 7.34 \times 10^{-14}/r)/1.5/8.6 \times 10^{-11}$   |  |  |
| rho (kg/m <sup>3</sup> ) | 8.76E+18 | rho= $1/r^3 \times 1.67 \times 10^{-27}/(4/3 \times \text{PI}())$  |  |  |
| R gas constai            | 5.46E+03 | R=P/(rho*T)  |  |  |
|                          |          |  |  |  |
| rcell                    | 3.57E-16 | 3.57E-16   |  |  |

Pressure, temperature and rho are all fixed once the radius of the cell is known. Since r is “buried” in the equations above, an approximate solution was developed and is shown below. Once r was written explicitly, rho was determined from the radius. The equations below give the radius as a function of pressure.

|   |   | $P \cdot r^3$   | $P \cdot r^3$ with $r=1$   | $k = Pr^3/r^{0.988}$        |
|---|---|-----------------|--|-----------------------------|
|   |   |                 |  | $1.86e-26/3.57e-16^{0.988}$ |
| $r^3$   | 4.55E-47  | 4.55E-47        |  |                             |
| gamma   | 3.19E-01  | 3.19E-01        |  |                             |
| P   | <b>7.39E+35</b>   | <b>3.36E-11</b> | <b>1.86E-26</b>  | 3.40E-11                    |
| T   | 1.55E+13  | 1.55E+13        |  |                             |
| rho   | 8.76E+18  | 8.76E+18        |  |                             |
| R   | 5.46E+03  | 5.46E+03        |  |                             |
|   |   | 3.57E-16        |  |                             |
|   | 3.57E-16  | 3.57E-16        |  | 3.57E-16                    |
| $r = (k/P)^{1/3}$                                 |   |                 |  |                             |
| Development of equation for rho                   |   |                 |  |                             |
|   | $3.58E-16 \left( \frac{1.86E-26/3.57e-16^{0.988}}{7.39E+35} \right)^{1/3}$                                    |                 |  |                             |
|   | $3.58E-16 \frac{1.86E-26^{1/3}}{(3.57e-16^{0.988})^{1/3}} \frac{1}{(7.39E+35)^{1/3}}$                         |                 |  |                             |
|   | $3.57e-16 = 1.86e-26^{1/3} / (3.57e-16^{0.988})^{1/3} / (7.39E+35)^{1/3}$                                     |                 |  |                             |
|   |   |                 |  |                             |
| $r \cdot (r^{0.988})^{.33}$                       | $(k/P)^{.333} \cdot (r^{.988})^{.33}$   |                 |  |                             |
|   | $(3.57e-16) \cdot (3.57e-16^{.988})^{1/3} = 1.86e-26^{1/3} / (7.39E+35)^{1/3}$                                |                 |  |                             |
|   |   |                 |  |                             |
|   | $2.92e-21 = (3.57e-16) \cdot (3.57e-16^{.988})^{1/3}$   |                 | $2.93e-21 = 1.86E-26^{1/3} / (7.39E+35)^{1/3}$                     |                             |
|   | $(3.57e-16^{3.988/3})$  |                 |  |                             |
|   |   |                 | $\left( \frac{1.86E-26}{7.39E+35} \right)^{1/3} \frac{1}{(3.988)}$ |                             |
|   | $3/(3 \cdot 3.988) = 0.2508$  |                 |  |                             |
| r   |   |                 |  |                             |
|   | 3.57E-16  |                 | $\left( \frac{1.86E-26}{7.39E+35} \right)^{0.25075}$               |                             |
| $r = \left( \frac{1.86E-26}{P} \right)^{0.25075}$ |   |                 |  |                             |
| Density equation                                  | $\rho = \text{kg/m}^3 = 1.67e-27 / (4/3 \cdot \pi) \cdot r^3$   |                 |  |                             |
| put r into density equation                       |   |                 |  |                             |
|   | <b><math>\rho = 1.67E-27 / (4/3 \cdot \pi) \cdot \left( \frac{4.8E-25}{P} \right)^{0.2565 \cdot 3}</math></b> |                 |  |                             |
|   | (the constants changed slightly as the equation was optimized)  |                 |  |                             |

The above rho equation is within 8% of actual rho over the BH pressure range.

## Appendix 1 Model Results



| Central mass (Kg) | 2.00E+30  | 9.94E+30    | 4.20E+40   | 4.00E+31     |
|-------------------|-----------|-------------|------------|--------------|
|                   | sun model | smallest BH | Largest BH | 20x sol cmax |
| Avg Density       | 1.44E+03  | 5.90E+18    | 3.31E-01   | 3.64E+17     |
|                   | 6.000E+03 | 8.90E+12    | 5.000E+06  | 3.80E+12     |
|                   | 2.399E+06 | 7.90E+11    | 3.000E+12  | 7.2E+11      |
|                   | 2.400E+07 | 1.68E+13    | 3.000E+13  | 1.10E+13     |
|                   |           | 1.40E+19    |            | 4.70E+15     |
| h0 (m)            | 6.92E+08  | 7.38E+03    | 3.12E+13   | 2.97E+04     |
| T0 (K)            | 6.000E+03 | 9.000E+12   | 5.307E+06  | 1.04E+12     |
| a (m/sec^2)       | 2.05E+02  | 1.22E+13    | 2.10E+03   | 3.03E+12     |
| m1 (Kg)           | 5.27E+29  | 6.39E+30    | 1.14E+40   | 1.40E+29     |
| cum mass1         | 5.266E+29 | 9.96E+30    | 1.136E+40  | 4.01E+31     |
| P1 (Nt/m^2)       | 1.99E+13  | 1.26E+35    | 2.16E+15   | 4.23E+31     |
| rho1 (kg/m^3)     | 1.400E+03 | 2.11E+18    | 3.300E-01  | 4.47E+15     |
| h1                | 6.23E+08  | 6.64E+03    | 2.81E+13   | 2.67E+04     |
| T                 | 2.405E+06 | 9.61E+12    | 3.000E+12  | 1.24E+12     |
| a                 | 1.81E+02  | 1.35E+13    | 1.82E+03   | 3.37E+12     |
| mass shell        | 4.22E+29  | 7.70E+29    | 9.10E+39   | 1.06E+29     |
| cum mass2         | 9.483E+29 | 8.96E+30    | 2.046E+40  | 3.61E+31     |
| P2                | 3.74E+13  | 1.47E+35    | 4.04E+15   | 8.70E+31     |
| rhonew            | 8.345E+03 | 2.37E+18    | 1.720E-01  | 7.79E+15     |
| h2                | 5.54E+08  | 5.91E+03    | 2.50E+13   | 2.38E+04     |
| T                 | 4.805E+06 | 1.00E+13    | 6.000E+12  | 1.49E+12     |
| a                 | 1.57E+02  | 1.52E+13    | 1.55E+03   | 3.79E+12     |
| mass shell        | 3.28E+29  | 6.76E+29    | 7.09E+39   | 1.44E+29     |
| cum mass          | 1.277E+30 | 7.97E+30    | 2.755E+40  | 3.21E+31     |
| P                 | 1.28E+14  | 1.74E+35    | 4.87E+15   | 1.75E+32     |
| rhonew            | 1.433E+04 | 2.70E+18    | 1.037E-01  | 1.33E+16     |
| h3                | 4.84E+08  | 5.17E+03    | 2.18E+13   | 2.08E+04     |
| T                 | 7.204E+06 | 1.04E+13    | 9.000E+12  | 1.78E+12     |
| a                 | 1.36E+02  | 1.74E+13    | 1.28E+03   | 4.33E+12     |
| mass shell        | 2.47E+29  | 5.77E+29    | 5.33E+39   | 1.86E+29     |
| cum mass          | 1.524E+30 | 6.97E+30    | 3.288E+40  | 2.81E+31     |
| P                 | 2.63E+14  | 2.08E+35    | 5.28E+15   | 3.46E+32     |
| rhonew            | 1.957E+04 | 3.10E+18    | 7.499E-02  | 2.25E+16     |
| h4                | 4.15E+08  | 4.43E+03    | 1.87E+13   | 1.78E+04     |
| T                 | 9.604E+06 | 1.09E+13    | 1.200E+13  | 2.12E+12     |
| a                 | 1.16E+02  | 2.03E+13    | 1.01E+03   | 5.05E+12     |
| mass shell        | 1.77E+29  | 4.76E+29    | 3.82E+39   | 2.25E+29     |
| cum mass          | 1.700E+30 | 5.98E+30    | 3.669E+40  | 2.40E+31     |
| P                 | 4.20E+14  | 2.55E+35    | 5.52E+15   | 6.83E+32     |
| rhonew            | 2.345E+04 | 3.62E+18    | 5.876E-02  | 3.80E+16     |
| h5                | 3.46E+08  | 3.69E+03    | 1.56E+13   | 1.49E+04     |

| Central mass (Kg)         | 2.00E+30                  | 9.94E+30    | 4.20E+40   | 4.00E+31     |          |
|---------------------------|---------------------------|-------------|------------|--------------|----------|
|                           | sun model                 | smallest BH | Largest BH | 20x sol cmax |          |
| h5 (continued from above) | 3.46E+08                  | 3.69E+03    | 1.56E+13   | 1.49E+04     |          |
| T                         | 1.200E+07                 | 1.15E+13    | 1.500E+13  | 2.52E+12     |          |
| a                         | 1.01E+02                  | 2.44E+13    | 7.54E+02   | 6.06E+12     |          |
| mass shell                | 1.19E+29                  | 3.72E+29    | 2.56E+39   | 2.55E+29     |          |
| cum mass                  | 1.819E+30                 | 4.98E+30    | 3.925E+40  | 2.00E+31     |          |
| P                         | 5.84E+14                  | 3.20E+35    | 5.66E+15   | 1.37E+33     |          |
| rhonew                    | 2.608E+04                 | 4.32E+18    | 4.819E-02  | 6.49E+16     |          |
| h6                        | 2.77E+08                  | 2.95E+03    | 1.25E+13   | 1.19E+04     |          |
| T                         | 1.440E+07                 | 1.22E+13    | 1.800E+13  | 3.01E+12     |          |
| a                         | 9.51E+01                  | 3.05E+13    | 5.13E+02   | 7.57E+12     |          |
| mass shell                | 7.19E+28                  | 2.69E+29    | 1.55E+39   | 2.64E+29     |          |
| cum mass                  | 1.891E+30                 | 3.98E+30    | 4.080E+40  | 1.60E+31     |          |
| P                         | 7.55E+14                  | 4.17E+35    | 5.74E+15   | 2.83E+33     |          |
| rhonew                    | 2.813E+04                 | 5.30E+18    | 4.070E-02  | 1.13E+17     |          |
| h7                        | 2.08E+08                  | 2.21E+03    | 9.36E+12   | 8.91E+03     |          |
| T                         | 1.680E+07                 | 1.31E+13    | 2.100E+13  | 3.63E+12     |          |
| a                         | 1.12E+02                  | 4.06E+13    | 3.05E+02   | 1.01E+13     |          |
| mass shell                | 3.69E+28                  | 1.70E+29    | 7.97E+38   | 2.37E+29     |          |
| cum mass                  | 1.928E+30                 | 2.99E+30    | 4.160E+40  | 1.20E+31     |          |
| P                         | 9.73E+14                  | 5.76E+35    | 5.77E+15   | 6.23E+33     |          |
| rhonew                    | 3.107E+04                 | 6.79E+18    | 3.512E-02  | 2.08E+17     |          |
| h8                        | 1.38E+08                  | 1.48E+03    | 6.24E+12   | 4.52E+04     |          |
| T                         | 1.920E+07                 | 1.42E+13    | 2.400E+13  | 4.45E+12     |          |
| a                         | 2.04E+02                  | 6.10E+13    | 1.82E+02   | 1.99E+12     |          |
| m8 shell Kg               | 1.36E+28                  | 8.00E+28    | 2.94E+38   | 1.49E+31     |          |
| cum mass9 Kg              | 1.941E+30                 | 1.99E+30    | 4.189E+40  | 6.10E+31     |          |
| P9 (nt/m^2)               | 1.41E+15                  | 8.82E+35    | 5.79E+15   | 7.47E+33     |          |
| rho9 kg/m^3               | 3.946E+04                 | 9.42E+18    | 3.084E-02  | 2.39E+17     |          |
| h9 (m)                    | 6.92E+07                  | 7.38E+02    | 3.12E+12   | 4.22E+04     |          |
| T (K)                     | 2.160E+07                 | 1.58E+13    | 2.700E+13  | 4.66E+12     |          |
| a10 (m/sec^2)             | 7.90E+02                  | 1.22E+14    | 4.41E+02   | 2.13E+12     |          |
| m10 shell Kg              | 1.94E+27                  | 1.59E+28    | 4.19E+37   | 1.48E+31     |          |
| cum mass10                | 1.943E+30                 | 9.96E+29    | 4.194E+40  | 5.70E+31     |          |
| P10 (nt/m^2)              | 3.57E+15                  | 1.73E+36    | 5.84E+15   | 8.98E+33     |          |
| rho10 kg/m^3              | 8.869E+04                 | 1.58E+19    | 2.761E-02  | 2.76E+17     |          |
| h10 core                  | 0                         | 0           | 0          | 0            |          |
| r cell (m)                | r <sub>cell</sub> =(1/rho | 1.65E-11    | 2.93E-16   | 2.44E-09     | 1.13E-15 |
|                           |                           |             | 3.00E+08   | 3.71E+07     | 3.00E+08 |