Does Constant Torque Induce a Phase Transition Increasing the Value of Planck Constant?

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1 Introduction

The hierarchy of phases with effective value of Planck constant coming as an integer multiple of the ordinary Planck constant and interpreted as dark matter is crucial in the TGD inspired model of living matter. The challenge is to identify physical mechanisms forcing the increase of effective Planck constant $h_{\text{eff}}$ (whether to call it effective or not is to some extent matter of taste). The work with certain potential applications of TGD led to a discovery of a new mechanism possibly achieving this. The method would be simple: apply constant torque to a rotating system. I will leave it for the reader to rediscover how this can be achieved.

The importance of the result is that it provides strong mathematical motivations for zero energy ontology (ZEO), causal diamonds (CDs), and hierarchy of (effective) Planck constants [K3]. Quite generally, the results apply to systems with external energy feed inducing generalized force acting in some compact degrees of freedom. Living matter represents basic example of this kind of system. Amazingly, ATP synthase enzyme contains generator with a rotating shaft: a possible TGD based interpretation is that the associated torque forces the generation of large $h_{\text{eff}}$ phases. This conforms with the proposal that the basic function of metabolism is to produce large $h_{\text{eff}}$ phases making also possible negentropic entanglement [K1] and generation of ”Akashic records” as negentropically entangled states which are approximately invariant under quantum jumps if they are correspond to interaction free (approximately) measurements for the Akashic records [K2].

2 Could constant torque force the increase of $h_{\text{eff}}$?

Consider a rigid body allowed to rotated around some axes so that its state is characterized by a rotation angle $\phi$. Assume that a constant torque $\tau$ is applied to the system.
2. Could constant torque force the increase of $h_{eff}$?

1. The classical equations of motion are

$$I \frac{d^2 \phi}{dt^2} = \tau .$$

This is true in an idealization as point particle characterized by its moment of inertia around the axis of rotation. Equations of motion are obtained from the variational principle

$$S = \int L dt , \quad L = \frac{I(d\phi/dt)^2}{2} - V(\phi) , \quad V(\phi) = \tau \phi .$$

Here $\phi$ denotes the rotational angle. The mathematical problem is that the potential function $V(\phi)$ is either many-valued or discontinuous at $\phi = 2\pi$.

2. Quantum mechanically the system corresponds to a Schrödinger equation

$$-i \hbar \partial_{\phi}^2 \Psi + \tau \phi \Psi = -i \frac{\partial \Psi}{\partial t} .$$

In stationary situation one has

$$-i \hbar \partial_{\phi}^2 \Psi + \tau \phi \Psi = E \Psi .$$

3. Wave function is expected to be continuous at $\phi = 2\pi$. The discontinuity of potential at $\phi = \phi_0$ poses further strong conditions on the solutions: $\Psi$ should vanish in a region containing the point $\phi_0$. Note that the value of $\phi_0$ can be chosen freely.

The intuitive picture is that the solutions correspond to strongly localized wave packets in accelerating motion. The wavepacket can for some time vanish in the region containing point $\phi_0$. What happens when this condition does not hold anymore?

- Dissipation is present in the system and therefore also state function reductions. Could state function reduction occur when the wave packet contains the point, where $V(\phi)$ is dis-continuous?

- Or are the solutions well-defined only in a space-time region with finite temporal extent $T$? In zero energy ontology (ZEO) this option is automatically realized since space-time sheets are restricted inside causal diamonds (CDs). Wave functions need to be well-defined only inside CD involved and would vanish at $\phi_0$. Therefore the mathematical problems related to the representation of accelerating wave packets in non-compact degrees of freedom could serve as a motivation for both CDs and ZEO.

There is however still a problem. The wave packet cannot be in accelerating motion even for single full turn. More turns are wanted. Should one give up the assumption that wave function is continuous at $\phi = \phi_0 + 2\pi$ and should
one allow wave functions to be multivalued and satisfy the continuity condition
\( \Psi(\phi_0) = \Psi(\phi_0 + n2\pi) \), where \( n \) is some sufficiently large integer. This would mean
the replacement of the configuration space (now circle) with its \( n \)-fold covering.

The introduction of the \( n \)-fold covering leads naturally to the hierarchy of Planck
constants.

1. A natural question is whether constant torque \( \tau \) could affect the system so that
\( \phi = 0 \) ja \( \phi = 2\pi \) do not represent physically equivalent configurations anymore.
Could it however happen that \( \phi = 0 \) ja \( \phi = n2\pi \) for some value of \( n \) are still
equivalent? One would have the analogy of many-sheeted Riemann surface.

2. In TGD framework 3-surfaces can indeed be analogous to \( n \)-sheeted Riemann
surfaces. In other words, a rotation of \( 2\pi \) does not produce the original surface
but one needs \( n2\pi \) rotation to achieve this. In fact, \( h_{eff}/\hbar = n \) corresponds to this
situation geometrically! Space-time itself becomes \( n \)-sheeted covering of itself: this
property must be distinguished from many-sheetedness. Could constant
torque provide a manner to force a situation making space-time \( n \)-sheeted and
thus to create phases with large value of \( h_{eff} \)?

3. Schrödinger amplitude representing accelerated wave packet as a wavefunction
in the \( n \)-fold covering would be \( n \)-valued in the ordinary Minkowski coordinates
and would satisfy the boundary condition
\[
\Psi(\phi) = \Psi(\phi + n2\pi).
\]
Since \( V(\phi) \) is not rotationally invariant this condition is too strong for stationary
solutions.

4. This condition would mean Fourier analysis using the exponentials \( \exp(\im\phi/m) \)
with time dependent coefficients \( c_m(t) \) whose time evolution is dicrated by Schrödinger
equation. For ordinary Planck constant this would mean fractional values of an-
gular momentum
\[
L_z = \frac{m}{n} \hbar.
\]
If one has \( h_{eff} = nh \), the spectrum of \( L_z \) is not affected. It would seem that
constant torque forces the generation of a phase with large value of \( h_{eff} \)! From
the estimate for how many turns the system rotates one can estimate the value
of \( h_{eff} \).

3 What about stationary solutions

Giving up stationary seems the only option on basis of classical intuition. One can
however ask whether also stationary solutions could make sense mathematically and
could make possible completely new quantum phenomena.
4. The connection with WKB approximation and Airy functions

1. In the stationary situation the boundary condition must be weakened to

\[ \Psi(\phi_0) = \Psi(\phi_0 + n2\pi) . \]

Here the choice of \( \phi_0 \) characterizes the solution. This condition quantizes the energy. Normally only the value \( n = 1 \) is possible.

2. The many-valuedness/discontinuity of \( V(\phi) \) does not produce problems if the condition

\[ \Psi(\phi_0, t) = \Psi(\phi_0 + n2\pi, t) = 0 , \quad 0 < t < T . \]

is satisfied. Schrödinger equation would be continuous at \( \phi = \phi_0 + n2\pi \). The values of \( \phi_0 \) would correspond to a continuous state basis.

3. One would have two boundary conditions expected to fix the solution completely for given values of \( n \) and \( \phi_0 \). The solutions corresponding to different values of \( \phi_0 \) are not related by a rotation since \( V(\phi) \) is not invariant under rotations. One obtains infinite number of continuous solution families labelled by \( n \) and they correspond to different phases if \( h_{\text{eff}} \) is different from them.

4 The connection with WKB approximation and Airy functions

Stationary Schrödinger equation with constant force appears in WKB approximation [B2] ([http://en.wikipedia.org/wiki/WKB_approximation](http://en.wikipedia.org/wiki/WKB_approximation)) and follows from a linearization of the potential function at non-stationary point. A good example is Schrödinger equation for a particle in the gravitational field of Earth. The solutions of this equation are Airy functions which appear also in the electrodynamical model for rainbow.

1. The standard form for the Schrödinger equation in stationary case is obtained using the following change of variables

\[ u + e = k\phi , \quad k^3 = \frac{2\hbar}{I} , \quad e = \frac{2IE}{\hbar^3k^2} . \]


\[ \frac{d^2\Psi}{du^2} - u\Psi = 0 . \]

The eigenvalue of energy does not appear explicitly in the equation. Boundary conditions transform to

\[ \Psi(u_0 + n2\pi k) = \Psi(u_0) = 0 . \]
2. In non-stationary case the change of variables is

\[ u = k\phi, \quad k^3 = \frac{2\pi I}{\hbar^2}, \quad v = \frac{\hbar^2 k^2}{2I} \times t. \]

One obtains

\[ \frac{d^2\Psi}{du^2} - u\Psi = i\partial_v \Psi. \]

Boundary conditions are

\[ \Psi(u + kn2\pi, v) = \Psi(u, v), \quad 0 \leq v \leq \frac{\hbar^2 k^2}{2I} \times T. \]

An interesting question is what \( h_{\text{eff}} = nh \) means? Should one replace \( h \) with \( h_{\text{eff}} = nh \) as the condition that the spectrum of angular momentum remains unchanged requires. One would have \( k \propto n^{-2/3} \) and \( e \propto n^{4/3} \). One would obtain boundary conditions non-linear with respect to \( n \).

5 Connection with living matter

The constant torque - or more generally non-oscillatory generalized force in some compact degrees of freedom - requires of a continual energy feed to the system. Continual energy feed serves as a basic condition for self-organization and for the evolution of states studied in non-equilibrium thermodynamics. Biology represents a fundamental example of this kind of situation. The energy feeded to the system represents metabolic energy and ADP-ATP process loads this energy to ATP molecules. Also now constant torque is involved: the ATP synthase molecule [1] (http://en.wikipedia.org/wiki/ATP_synthase) contains the analog of generator having a rotating shaft. Since metabolism and the generation of large \( h_{\text{eff}} \) phases are very closely related in TGD Universe, the natural proposal is that the rotating shaft forces the generation of large \( h_{\text{eff}} \) phases.

REFERENCES

Theoretical Physics


Biology

Books related to TGD

