Further progress concerning the relationship between TGD and GRT and Kähler-Dirac action

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Abstract

The earlier attempts to understand the relationship between TGD and GRT have been in terms of solutions of Einstein's equations imbeddable to $M^4 \times CP_2$ instead of introducing GRT space-time as a fictive notion naturally emerging from TGD as a simplified concept replacing many-sheeted space-time. This resolves also the worries related to Equivalence Principle. TGD can be seen as a "microscopic" theory behind TGD and the understanding of the microscopic elements becomes the main focus of theoretical and hopefully also experimental work some day.

The understanding of Kähler Dirac action has been second long term project. How can one guarantee that em charge is well-defined for the spinor modes when classical W fields are present? How to avoid large parity breaking effects due to classical Z^0 fields? How to avoid the problems due to the fact that color rotations induce vielbein rotation of weak fields? The common answer to these questions is restriction of the modes of induced spinor field to 2-D string world sheets (and possibly also partonic 2-surfaces) such that the induced weak fields vanish. This makes string picture a part of TGD.

1 Introduction

The question how TGD relates to General Relativity Theory (GRT) has been a rich source of problems during last 37 years. In the light of after-wisdom the problems have been due to my too limited perspective. I have tried to understand GRT limit in the TGD framework instead of introducing GRT space-time as a fictive notion naturally emerging from TGD as a simplified concept replacing many-sheeted space-time. This resolves also the worries related to Equivalence Principle.

TGD itself gains the status of "microscopic" theory of gravity and the experimental challenges relate to how make the microscopy of gravitation experimentally visible. This involves questions such as "How to make the presence of Euclidian space-time regions visible?", "How to reveal many-sheeted character of space-time, topological field quantization, and the presence of magnetic flux tubes?,"How to reveal quantum gravity as understood in TGD involving in an essential manner gravitational Planck constant h_{gr} identifiable as h_{eff} inspired by anomalies of bio-electromagnetism?" [?].

More technical questions relate to the Kähler-Dirac action, in particular to how conservation laws are realized. During all these years several questions have been lurking at the boarder of conscious and sub-conscious. How can one guarantee that em charge is well-defined for the spinor modes when classical W fields are present? How to avoid large parity breaking effects due to classical Z^0 fields? How to avoid the problems due to the fact that color rotations induce vielbein rotation of weak fields? The common answer to these questions is restriction of the modes of induced spinor field to 2-D string world sheets (and possibly also partonic 2-surfaces) such that the induced weak fields vanish. This makes string picture a part of TGD.

1.1 TGD and GRT

Concerning GRT limit the basic questions are the following ones.

1. Is it really possible to obtain a realistic theory of gravitation if general space-time metric is replaced with induced metric depending on 8 imbedding space coordinates (actually only 4 by general coordinate invariance?
2. What happens to Einstein equations?
3. What about breaking of Poincare invariance, which seems to be real in cosmological scales? Can TGD cope with it?
4. What about Equivalence Principle (EP)
5. Can one predict the value of gravitational constant?
6. What about TGD counterpart of blackhole, which certainly represents the boundary of realm in which GRT applies?

Consider first possible answers to the first three questions.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets.
2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric in standard coordinates for the space-time sheets. One could replace flat metric of M^4 with effective metric as sum of metric and deviations associated with various space-time sheets "above" the M^4 point. This effective metric of M^4 regarded as independent space would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD. Also standard model gauge potentials can be defined as effective fields in the same manner and one expects that classical electroweak fields vanish in the length scales above weak scale.
3. This picture brings in mind the old intuitive notion of smoothed out quantum average space-time thought to be realized as surface in $M^4 \times CP_2$ rather than in terms of averages metric and gauge potentials in M^4. The problem of this approach was that it was not possible to imagine any quantitative recipe for the averaging and this was essentially dur to the sub-manifold assumption.
4. One could generalize this picture and consider effective metrics for CP_2 and $M^2 \times CP_2$ corresponding to CP_2 type vacuum extremals describing elementary particles and cosmic strings respectively.
5. Einstein's equations could hold true for the effective metric. The vanishing of the covariant divergence of energy momentum tensor would be a remnant of Poincare invariance actually still present in the sense of Zero Energy Ontology (ZEO) but having realization as global conservation laws.
6. The breaking of Poincare invariance at the level of effective metric could have interpretation as effective breaking due to zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

The following considerations are about answers to the fourth and fifth questions.

1. EP at classical level would hold true in local sense if Einstein's equations hold true for the effective metric. Underlying Poincare invariance suggests local covariant conservation laws.

2. The value of gravitational constant is in principle a prediction of theory containing only radius as fundamental scale and Kähler coupling strength as only coupling constant analogous to critical temperature. In GRT inspired quantum theory of gravitation Planck length scale given by $L_P = \sqrt{\hbar_{eff} \times G}$ is the fundamental length scale. In TGD size R defines it and it is independent of h_{eff}. The prediction for gravitational constant is prediction for the TGD counterpart of L_P: $L_P^2 = R^2/n$, n dimensionless constant. The prediction for G would be $G = R^2/(n \times \hbar_{eff})$ or $G = R^2/(n \times \hbar_{eff,min})$. The latter option is the natural one.

Interesting questions relate to the fate of blackholes in TGD framework.

1. Blackhole metric as such is quite possible as effective metric since there is no need to imbed it to imbedding space. One could however argue that blackhole metric is so simple that it must be realizable as single-sheeted space-time surface. This is indeed possible above some radius which can be smaller than Schwarschild radius. This is due to the compactness of CP_2 . A general result is that the embedding carriers non-vanishing gauge charge say em charge. This need not have physical significance if the metric of GRT corresponds to the effective metric obtained by the proposed recipe.

2. TGD forces to challenge the standard view about black holes. For instance, could it be that blackhole interior corresponds microscopically to Euclidian space time regions? For these CP_2 endowed with effective metric would be appropriate GRT type description. Reissner-Nordström metric with cosmological constant indeed allows CP_2 as solution [?]. M^4 region and CP_2 region would be joined along boundaries at which determinant of four-metric vanishes. If the radial component of R-N metric is required to be finite, one indeed obtains metric with vanishing determinant at horizon and it is natural to assume that the metric inside is Euclidian. Similar picture would applied to the cosmic strings as spaces $M^2 \times S^2$ with effective metric.

3. Could holography hold true in the sense that blackhole horizon is replaced with a partonic 2-surface with astrophysical size and having light-like orbit as also black-hole horizon has.

4. The notion of gravitational Planck constant $h_{gr} = GMm/v_0$, where v_0 is typical rotation velocity in the system consisting of masses M and m, has been one of the speculative aspects of TGD. h_{gr} would be assigned with "gravitational" magnetic flux tube connecting the systems in question and it has turned out that the identification $h_{gr} = h_{eff}$ makes sense in particle length scales. The gravitational Compton length is universal and given $\lambda_{gr} = GM/v_0$. This strongly suggests that quantum gravity becomes important already above Schwarschild radius $r_S = 2GMm$. The critical velocity at which gravitational Compton length becomes smaller than r_S is $v_0/c = 1/\sqrt{2}$. All astrophysical objects would be genuinely quantal objects in TGD Universe point and blackholes would lose their unique role. An experimental support for these findings comes from experiments of Tajmar et al [E1, E3] [?].

For few ago entropic gravity [?, ?] was a buzzword in blogs. The idea was that gravity would have a purely thermodynamical origin. I have commented the notion of entropic gravity from the point of view of TGD earlier [?].

The basic objection is standard QM against the entropic gravity is that gravitational interaction of neutrons with Earth's gravitational field is describable by Schrödinger equation and this does not fit with thermodynamical description.

Although the idea as such does not look promising TGD indeed suggests that the correlates for thermodynamical quantities at space-time level make sense in ZEO leading to the view that quantum TGD is square root of thermodynamics.

Th interesting question is whether temperature has space-time correlate.

1. In Zero Energy Ontology quantum theory can be seen as a square root of thermodynamics formally and this raises the question whether ordinary temperature could parametrize wave

functions having interpretation as square roots of thermal distributions in ZEO. The quantum model for cell membrane [?] having the usual thermodynamical model as limit gives support for this idea. If this were the case, temperature would have by quantum classical correspondence direct space-time correlate.

2. A less radical view is that temperature can be assigned with the effective space-time metric only. The effective metric associated with M^4 defining GRT limit of TGD is defined statistically in terms of metric of many-sheeted space-time and would naturally contain in its geometry thermodynamical parameters. The averaging over the WCW spinors fields involving integral over 3-surfaces is also involved.

1.2 Equivalence Principle

Equivalence Principle has several interpretations.

1. The global form form of Equivalence Principle (EP) realized in Newtonian gravity states that inertial mass = gravitational mass (mass is replaces with four-momentum in the possible relativistic generalization). This form does not make sense in general relativity since four-momentum is not well-defined: this problem is the starting point TGD.
2. The local form of EP can be expressed in terms of Einstein's equations. Local covariant conservation law does not imply global conservation law since energy momentum tensor is indeed tensor. One can try to define gravitational mass as something making sense in special cases. The basic problem is that there is no unique identification of empty space Minkowski coordinates. Gravitational mass could be identified as a a parameter appearing in asymptotic expression of solutions of Einstein's equations.

In TGD framework EP need not be problem of principle.

1. In TGD gravitational interaction couples to inertial four-momentum, which is well-defined as classical Noether charge associated with Kähler action. The very close analogy of TGD with string models suggest the same.
2. Only if one assumes that gravitational and inertial exist separately and are forced to be identical, one ends up with potential problems in TGD. This procedure might have sound physical basis in TGD but one should identify it in convincing manner.
3. In cosmology mass is not conserved, which in positive energy ontology would suggests breaking of Poincare invariance. In Zero Energy Ontology (ZEO) this is not the case. The conserved four-momentum assignable to either positive or negative energy part of the states in the basis of zero energy states depends on the scale of causal diamond (CD). Note that in ZEO zero energy states can be also superpositions of states with different four-momenta and even fermion numbers as in case of coherent state formed by Cooper pairs.

Consider now EP in quantum TGD.

1. Inertial momentum is defined as Noether charge for Kähler action.
2. One can assign to Kähler-Dirac action quantal four-momentum (I will use "Kähler-Dirac" instead of "modified" used in earlier work) [?, ?]. Its conservation is however not at all all trivial since imbedding space coordinates appear in KD action like external fields. It however seems that at least for the modes localized at string world sheets the four-momentum conservation could be guaranteed by an assumption motivated by holomorphy [?]. The assumption states that the variation of holomorphic/antiholomorphic Kähler-Dirac gamma matrices induced by isometry is superposition of K-D gamma matrices of same type.
3. Quantum Classical Correspondence (QCC) suggests that the eigenvalues of quantal four-momentum are equal to those of Kähler four-momentum. If this is the case, QCC would imply EP and force conservation of antal four-momenta even if the assumption about variations of gamma matrices fails! This could be realized in terms of Lagrange multiplier terms added to Kähler action and localized at the ends of CD and analogous to constraint terms in ordinary thermodynamics.

4. QCC generalizes to Cartan sub-algebra of symmetries and would give a correlation between geometry of space-time sheet and conserved quantum numbers. One can consider even stronger form of QCC stating that classical correlation functions at space-time surface are same as the quantal once.

The understanding of EP at classical level has been a long standing head-ache in TGD framework. What seems to be the eventual solution looks disappointingly trivial in the sense that its discovery requires only some common sense.

The trivial but important observation is that the GRT limit of TGD does *not* require that the space-times of GRT limit are imbeddable to the imbedding space $M^4 \times CP_2$. The most elegant understanding of EP at classical level relies on following argument suggesting how GRT space-time emerges from TGD as an effective notion.

1. Particle experiences the sum of the effects caused by gravitational forces. The linear superposition for gravitational fields is replaced with the sum of effects describable in terms of effective metric in GRT framework. Hence it is natural to identify the metric of the effective space-time as the sum of M^4 metric and the deviations of various space-time sheets to which particle has topological sum contacts. This metric is defined for the M^4 serving as coordinate space and is not in general expressible as induced metric.

2. Underlying Poincare invariance is not lost but global conservation laws are lost for the effective space-time. A natural assumption is that that global energy-momentum conservation translates to the vanishing of covariant divergence of energy momentum tensor.

3. By standard argument this implies Einstein's equations with cosmological constant Λ: this at least in statistical sense. Λ would parametrize the presence of topologically condensed magnetic flux tubes. Both gravitational constant and cosmological constant would come out as predictions.

This picture is in principle all that is needed. TGD is in this framework a "microscopic" theory of gravitation and GRT describes statistically the many-sheetedness in terms of single sheeted space-time identified as M^4 as manifold. All notions related to many-sheeted space-time - such as cosmic strings, magnetic flux tubes, generalized Feynman diagrams representing deviations from GRT. The theoretical and experimental challenge is discover what these deviations are and how to make them experimentally visible.

One can of course ask whether EP or something akin to it could be realized for preferred extremals of Kähler action.

1. In cosmological and astrophysical models vacuum extremals play a key role. Could small deformations of them provide realistic enough models for astrophysical and cosmological scales in statistical sense?

2. Could preferred extremals satisfy something akin to Einstein's equations? Maybe! The mere condition that the covariant divergence of energy momentum tensor for Kähler action vanishes, is satisfied if Einsteins equations with cosmological terms are satisfied. One can however consider also argue that this condition can be satisfied also in other manners. For instance, four-momentum currents associated with them be given by Einstein's equations involving several cosmological "constants". The vanishing of covariant divergence would however give a justification for why energy momentum tensor is locally conserved for the effective metric and thus gives rise to Einstein's equations.

2 The recent view about Kähler-Dirac action

The the following the definition of Kähler-Dirac (or modified Dirac) action will be discussed with special emphasis on the condition that the electroweak properties of the solutions should conform with intuitive expectations.

2.1 Kähler-Dirac action

Kähler-Dirac action (equivalently modified Dirac action) is super-symmetric concomitant of Kähler action.

1. K-D(or modified) gamma matrices replace induced gamma matrices being defined as contractions of canonical momentum currents associated with Kähler action (or any action) with imbedding space gamma matrices. If the action is taken to be four-volume the K-D gamma matrices are induced gamma matrices. It is good to list basic properties of the K-D gamma matrices.

2. K-D gamma matrices do not in general span four-D space as induced gamma matrices. This is essential for having modes of K-D equation restricted to 2-D string worlds sheets. For instance, K-D gamma matrices vanish for vacuum extremals with vanishing induced Kähler field. For massless extremals the only non-vanishing K-D gamma matrix is light-like.

3. Mixing of M^4 and CP_2 chiralities in K-D gamma matrices cause mixing of M^4 chiralities and is a signature for massivation since the solutions of K-D equation cannot have well-define M^4-chirality anymore.

4. Anti-commutators of K-D gammas define effective metric. An interesting question concerning its interpretation. In particular, what might be the physical interpretation for the analog of light-velocity?

5. K-D equation internally consistent only if an extremal of Kähler action in question. This is a manifestation of supersymmetry: one obtains infinite number of super-charges associated with the modes of the induced spinor field as contractions with second quantized induced spinor field.

Well-definedness of em charge highly non-trivial physical condition on the solutions of K-D Dirac equation and must be posed as an additional condition.

1. Induced W fields mean that the modes of Kähler-Dirac equation do not in general have well-defined em charge.

2. In TGD one cannot locally redefine em charge by going to unitary gauge since the geometry of fixes uniquely the direction of em charge as covariantly constant generator of electro-weak gauge algebra.

3. For spinor modes in CP_2 and imbedding space it is possible to speak about well-defined em charge despite the mixing but it seems that this is not possible for induced spinor fields.

4. The problem disappears if the induced W gauge fields vanish. This does not yet guarantee that couplings to classical gauge fields are physical in long scales. Also classical Z^0 field should vanish so that the couplings would be purely vectorial. Vectoriality might be true in long enough scales only. If W and Z^0 fields vanish in all scales then electroweak forces are due to the exchanges of corresponding gauge bosons described as string like objects/partonic 2-surfaces in TGD and represent non-trivial space-time geometry and topology at microscopic scale.

5. The conditions solves also another long-standing interpretational problem. Color rotations induce rotations in electroweak-holonomy group so that the vanishing of all induced weak fields also guarantees that color rotations do not spoil the property of spinor modes to be eigenstates of em charge.

One can study the conditions quite concretely by using the formulas for the components of spinor curvature [?] (`http://www.tgdtheory.fi/public_html/pdfpool/append.pdf`).

1. The representation of the covariantly constant curvature tensor is given by

$$\begin{array}{lllllll} R_{01} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , & R_{23} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , \\ R_{02} & = & e^0\wedge e^2 - e^3\wedge e^1 \ , & R_{31} & = & -e^0\wedge e^2 + e^3\wedge e^1 \ , \\ R_{03} & = & 4e^0\wedge e^3 + 2e^1\wedge e^2 \ , & R_{12} & = & 2e^0\wedge e^3 + 4e^1\wedge e^2 \ . \end{array} \quad (2.1)$$

$R_{01} = R_{23}$ and $R_{03} = -R_{31}$ combine to form purely left handed classical W boson fields and Z^0 field corresponds to $Z^0 = 2R_{03}$.

Kähler form is given by

$$J = 2(e^{03} + e^1 \wedge e^2) \ . \quad (2.2)$$

2. The vanishing of classical weak fields is guaranteed by the conditions

$$\begin{aligned} e^0 \wedge e^1 - e^2 \wedge e^3 = 0 \ , \\ e^0 \wedge e^2 - e^3 \wedge e^1 \ , \\ 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \ . \end{aligned}$$

3. There are many manners to satisfy these conditions. For instance, the condition $e^1 = a \times e^0$ and $e^2 = -a \times e^3$ with arbitrary a which can depend on position guarantees the vanishing of classical W fields. The CP_2 projection of the tangent space of the region carrying the spinor mode must be 2-D.

   Also classical Z^0 vanishes if $a^2 = 2$ holds true. This guarantees that the couplings of induced gauge potential are purely vectorial. One can consider other alternaties. For instance, one could require that only classical Z^0 field or induced Kähler form is non-vanishing and deduce similar condition.

4. The vanishing of the weak part of induced gauge field implies that the CP_2 projection of the region carrying spinor mode is 2-D. Therefore the condition that the modes of induced spinor field are restricted to 2-surfaces carrying no weak fields sheets guarantees well-definedness of em charge and vanishing of classical weak couplings. This condition does not imply string world sheets in the general case since the CP_2 projection of the space-time sheet can be 2-D.

How string world sheets and partonic 2-surfaces could emerge?

1. Additional consistency condition on string world sheets/partonic 2-surfaces is that Kähler-Dirac gamma matrices have no components orthogonal to the 2-surface in question. Hence various fermionic currents would flow along string world sheet/partonic 2-surface.

2. If the Kähler-Dirac gamma matrices at string world sheet/partonic 2-surface are expressible in terms of two non-vanishing gamma matrices parallel to string world sheet and thus define an integrable distribution of tangent vectors, this is achieved. What is important that modified gamma matrices can indeed span lower than 4-D space and often do so as already described. Induced gamma matrices define always 4-D space so that the restriction of the modes to string world sheets/partonic 2-surfaces is not possible.

3. String models suggest that string world sheets/partonic 2-surafces are minimal surfaces of space-time surface or of imbedding space but it might not be necessary to pose this condition separately.

In the proposed scenario string world sheets/partonic 2-surfaces emerge rather than being postulated from beginning. That partonic 2-surfaces should satisfy the same conditions seems natural since wormhole throats are responsible for the interaction between different space-time sheets and between particles and classical fields.

Also higher-D objects might satisfy the conditions in some situations. Light-like orbits of partonic 2-surfaces comes in mind first but 2-D character of CP_2 projection would require that Chern-Simons action vanishes identically so that this option does not make sense.

4-D space-time surfaces satisfying the condition are however possible and cosmic strings define the basic example. Number-theoretically the reduction of induced gauge fields to em field would correspond to reduction of complexified quaternionic tangent space of space-time surface to complex or hyper-complex sub-space.

1. The vanishing conditions for induced weak fields allow also 4-D spinor modes if they are true for entire spatime surface. This is true if the space-time surface has 2-D CP_2 projection. One can expect that in this case space-time surface has foliation by string world sheets and the general solution of K-D equation is continuous superposition of the 2-D modes in this case and discrete one in the generic case.

2. If the CP_2 projection of space-time surface is homologically non-trivial geodesic sphere S^2, the field equations reduce to those in $M^4 \times S^2$ since the second fundamental form for S^2 is vanishing. Is it possible to have geodesic sphere for which induced gauge field has only em component?

3. If the CP_2 projection is complex manifold as it is for string like objects, the vanishing of weak fields might be also achieved. Number theoretic vision supports that the guess is correct. This has to be checked.

4. Does the phase of cosmic strings assumed to dominate primordial cosmology correspond to this phase with no classical weak fields? During radiation dominated phase 4-D string like objects would transform to string world sheets. Kind of dimensional transmutation would occur [**?**].

Right-handed neutrino has exceptional role in K-D action.

1. Electroweak gauge potentials do not couple to ν_R at all. Therefore em neutrality condition is unnecessary if the induced gamma matrices do not mix right handed neutrino with left-handed one. This is guaranteed if M^4 and CP_2 parts of Kähler-Dirac operator annihilate separately right-handed neutrino spinor mode. Also ν_R modes can be interpreted as continuous superpositions of 2-D modes and this allows to define overlap integrals for them and induced spinor fields needed to define WCW gamma matrices and super-generators.

2. For covariantly constant right-handed neutrino mode defining a generator of super-symmetries is certainly a solution of K-D. Whether more general solutions of K-D exist remains to be checked out.

2.2 Kähler-Dirac equation in the interior of space-time surface

The solution of K-D equation at string world sheets is very much analogous to that in string models and holomorphy (actually, its Minkowskian counterpart) plays a key role. Note however the K-D gamma matrices might not necessarily define effective metric with Minkowskian signature even for string world sheets. Second point to notice is that one can consider also solutions restricted to partonic 2-surfaces. Physical intuition suggests that they are very important because wormhole throats carry particle quantum numbers and because wormhole contacts mediat the interaction between space-time sheets. Whether partonic 2-surfaces are somehow dual to string world sheets remains an open question.

1. Conformal invariance/its Minkowskian variant based on hyper-complex numbers realized at string world sheets suggests a general solution of Kähler-Dirac equation. The solution ansatz is essentially similar to that in string models.

2. Second half of complexified Kähler-Dirac gamma matrices annihilates the spinors which are either holomorphic or anti-holomorphic functions of complex (hyper-complex) coordinate.

3. What about possible modes delocalized into entire 4-D space-time sheet possible if there are preferred extremals for which induced gauge field has only em part. What suggests itself is global slicing by string world sheets and obtain the solutions as integrals over localized modes over the slices.

The understanding of symmetries (isometries of imbedding space) of K-D equation has turned out to be highly non-trivial challenge. The problem is that imbedding space coordinates appear in the role of external fields in K-D equation. One cannot require the vanishing of the variations of the K-D action with respect to the imbedding space-time coordinates since the action itself is second quantized object. Is it possible to have conservation laws associated with the imbedding space isometries?

1. Quantum classical correspondence (QCC) suggests the conserved Noether charges for Kähler action are equal to the eigenvalues of the Noether charges for Kähler-Dirac action. The quantal charge conservation would be forced by hand. This condition would realize also Equivalence Principle.

2. Second possibility is that the current following from the vanishing of second variation of Kähler action and the modification of Kähler gamma matrices defined by the deformation are linear combinations of holomorphic or anti-holomorphic gammas just like the gamma matrix itself so that K-D remains true. Conformal symmetry would therefore play a fundamental role. Isometry currents would be conserved although variations with respect to imbedding space coordinates would not vanish in general.

3. The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book).

2.3 Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naive guess that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This need not however be correct and therefore it is best to carefully consider what one wants.

2.3.1 What one wants?

It is could to make first clear what one really wants.

1. What one wants is generalized Feynman diagrams demanding massless Dirac propagators at the boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that twistor Grassmannian approach emerges at QFT limit. This boils down to the condition

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$$

at the space-like ends of space-time surface. This condition makes sense also at partonic orbits although they are not boundaries in the usual sense of the word. Here however delicacies since g_4 vanishes at them. The localization of induced spinor fields to string world sheets implies that fermionic propagation takes place along their boundaries and one obtains the braid picture.

The general idea is that the space-time geometry near the fermion line would *define* the four-momentum propagating along the line and quantum classical correspondence would be realized. The integral over four-momenta would be included to the functional integral over 3-surfaces.

The basic condition is that $\sqrt{g_4}\Gamma^n$ is constant at the boundaries of string world sheets and depends only on the piece of this boundary representing fermion line rather than on its point. Otherwise the propagator does not exist as a global notion. Constancy allows to write $\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$ since only M^4 gamma matrices are constant.

2. If p^k is light-like one can assume massless Dirac equation and restriction of the induced spinor field inside the Euclidian regions defining the line of generalized Feynman diagram. The interpretation would be as on mass-shell massless fermion. If p^k is not light-like, this is not possible and induced spinor field is delocalized outside the Euclidian portions of the line of generalized Feynman diagram: interactions would be basically due to the dispersion of induced spinor fields to Minkowskian regions. The interpretation would be as a virtual particle. The challenge is to find whether this interpretation makes sense and whether it is possible to articulate this idea

mathematically. The alternative assumption is that also virtual particles can localized inside Euclidian regions.

3. One can wonder what the spectrum of p_k could be. If the identification as virtual momenta is correct, continuous mass spectrum suggests itself. For the incoming lines of generalized Feynman diagram one expects light-like momenta so that Γ^n should be light-like. This assumption is consistent with super-conformal invariance since physical states would correspond to bound states of massless fermions, whose four-momenta need not be parallel. Stringy mass spectrum would be outcome of super-conformal invariance and 2-sheetedness forced by boundary conditions for Kähler action would be essential for massivation. Note however that the string curves along the space-like ends of space-time surface are also internal lines and expected to carry virtual momentum: classical picture suggests that p^k tends to be space-like.

2.3.2 Chern-Simons Dirac action from mathematical consistency

A further natural condition is that the possible boundary term is well-defined. At partonic orbits the boundary term of Kähler-Dirac action need not be well-defined since $\sqrt{g_4}\Gamma^n$ becomes singular. This leaves only Chern-Simons Dirac action

$$\overline{\Psi}\Gamma^{\alpha}_{C-S}D_{\alpha}\Psi$$

under consideration at both sides of the partonic orbits and one can consider continuity of C-S-D action as the boundary condition. Here Γ^{α}_{C-S} denotes the C-S-D gamma matrix, which does not depend on the induced metric and is non-vanishing and well-defined. This picture conforms also with the view about TGD as almost topological QFT.

One could restrict Chern-Simons-Dirac action to partonic orbits since they are special in the sense that they are not genuine boundaries. Also Kähler action would naturally contain Chern-Simons term.

One can require that the action of Chern-Simons Dirac operator is equal to multiplication with $ip^k\gamma_k$ so that massless Dirac propagator is the outcome. Since Chern-Simons term involves only CP_2 gamma matrices this would define the analog of Dirac equation at the level of imbedding space. I have proposed this equation already earlier and introduction this it as generalized eigenvalue equation having pseudomomenta p^k as its solutions.

If space-like ends of space-time surface involve no Chern-Simons term, one obtains the boundary condition

$$\sqrt{g_4}\Gamma^n\Psi = 0 \tag{2.4}$$

at them. Ψ would behave like massless mode locally. The condition $\sqrt{g_4}\Gamma^n\Psi = \gamma^k p_k\Psi = 0$ would state that incoming fermion is massless mode globally. If Chern-Simons term is present one obtains also Chern-Simons term in this condition but also now fermion would be massless in global sense. The physical interpretation would be as incoming massless fermions.

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