Recent View about Kähler Geometry and Spin Structure of "World of Classical Worlds"

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Abstract

The construction of Kähler geometry of WCW ("world of classical worlds") is fundamental to TGD program. I ended up with the idea about physics as WCW geometry around 1985 and made a breakthrough around 1990, when I realized that Kähler function for WCW could correspond to Kähler action for its preferred extremals defining the analogs of Bohr orbits so that classical theory with Bohr rules would become an exact part of quantum theory and path integral would be replaced with genuine integral over WCW. The motivating construction was that for loop spaces leading to a unique Kähler geometry. The geometry for the space of 3-D objects is even more complex than that for loops and the vision still is that the geometry of WCW is unique from the mere existence of Riemann connection.

This chapter represents the updated version of the construction providing a solution to the problems of the previous construction. The basic formulas remain as such but the expressions for WCW super-Hamiltonians defining WCW Hamiltonians (and matrix elements of WCW metric) as their anticommutator are replaced with those following from the dynamics of the modified Dirac action.

1 Introduction

The construction of Kähler geometry of WCW ("world of classical worlds") is fundamental to TGD program. I ended up with the idea about physics as WCW geometry around 1985 and made a breakthrough around 1990, when I realized that Kähler function for WCW could correspond to Kähler action for its preferred extremals defining the analogs of Bohr orbits so that classical theory with Bohr rules would become an exact part of quantum theory and path integral would be replaced with genuine integral over WCW. The motivating construction was that for loop spaces leading to a unique Kähler geometry [A7]. The geometry for the space of 3-D objects is even more complex than that for loops and the vision still is that the geometry of WCW is unique from the mere existence of Riemann connection.

The basic idea is that WCW is union of symmetric spaces G/H labelled by zero modes which do not contribute to the WCW metric. There have been many open questions and it seems the details of the ealier approach [?]ust be modified at the level of detailed identifications and interpretations.

1. A longstanding question has been whether one could assign Equivalence Principle (EP) with the coset representation formed by the super-Virasoro representation assigned to G and H in such a manner that the four-momenta associated with the representations and identified as inertial and gravitational four-momenta would be identical. This does not seem to be the case. The recent view will be that EP reduces to the view that the classical four-momentum associated with Kähler action is equivalent with that assignable to modified Dirac action supersymmetrically related to Kähler action: quantum classical correspondence (QCC) would be in question. Also strong form of general coordinate invariance implying strong form of holography in turn implying

that the super-symplectic representations assignable to space-like and light-like 3-surfaces are equivalent could imply EP with gravitational and inertial four-momenta assigned to these two representations.

At classical level EP follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least.

2. The detailed identification of groups G and H and corresponding algebras has been a longstanding problem. Symplectic algebra associated with$\delta M^4_\pm \times CP2$ ($\delta M^4_\pm$ is light-cone boundary - or more precisely, with the boundary of causal diamond (CD) defined as Cartesian product of CP_2 with intersection of future and past direct light cones of M^4 has Kac-Moody type structure with light-like radial coordinate replacing complex coordinate z. Virasoro algebra would correspond to radial diffeomorphisms. I have also introduced Kac-Moody algebra assigned to the isometries and localized with respect to internal coordinates of the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian and which serve as natural correlates for elementary particles (in very general sense!). This kind of localization by force could be however argued to be rather ad hoc as opposed to the inherent localization of the symplectic algebra containing the symplectic algebra of isometries as sub-algebra. It turns out that one obtains direct sum of representations of symplectic algebra and Kac-Moody algebra of isometries naturally as required by the success of p-adic mass calculations.

3. The dynamics of Kähler action is not visible in the earlier construction. The construction also expressed WCW Hamiltonians as 2-D integrals over partonic 2-surfaces. Although strong form of general coordinate invariance (GCI) implies strong form of holography meaning that partonic 2-surfaces and their 4-D tangent space data should code for quantum physics, this kind of outcome seems too strong. The progress in the understanding of the solutions of modified Dirac equation led however to the conclusion that spinor modes other than right-handed neutrino are localized at string world sheets with strings connecting different partonic 2-surfaces. This leads to a modification of earlier construction in which WCW super-Hamiltonians are essentially integrals with integrand identified as a Noether super current for the deformations in G Each spinor mode gives rise to super current and the modes of right-handed neutrino and other fermions differ in an essential manner. Right-handed neutrino would correspond to symplectic algebra and other modes to the Kac-Moody algebra and one obtains the crucial 5 tensor factors of Super Virasoro required by p-adic mass calculations.

The matrix elements of WCW metric between Killing vectors are expressible as anti-commutators of super-Hamiltonians identifiable as contractions of WCW gamma matrices with these vectors and give Poisson brackets of corresponding Hamiltonians. The anti-commutation relates of induced spinor fields are dictated by this condition. Everything is 3-dimensional although one expects that symplectic transformations localized within interior of X^3 act as gauge symmetries so that in this sense effective 2-dimensionality is achieved. The components of WCW metric are labelled by standard model quantum numbers so that the connection with physics is extremely intimate.

4. An open question in the earlier visions was whether finite measurement resolution is realized as discretization at the level of fundamental dynamics. This would mean that only certain string world sheets from the slicing by string world sheets and partonic 2-surfaces are possible. The requirement that anti-commutations are consistent suggests that string world sheets correspond to surfaces for which Kähler magnetic field is constant along string in well defined sense ($J_{\mu\nu}\epsilon^{\mu\nu}g^{1/2}$ remains constant along string). It however turns that by a suitable choice of coordinates of 3-surface one can guarantee that this quantity is constant so that no additional constraint results.

5. Quantum criticality is one of the basic notions of quantum TGD and its relationship to coset construction has remained unclear. In this chapter the concrete realization of criticality in terms of symmetry breaking hierarchy of Super Virasoro algebra acting on symplectic and Kac-Moody algebras. Also a connection with finite measurement resolution - second key notion of TGD - emerges naturally.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [?]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [?]. The topics relevant to this chapter are given by the following list.

- Hierarchy of Planck constants [?]
- Hyperfinite factors and TGD [?]
- Structure of WCW [?]
- TGD as infinite-dimensional geometry [?]
- WCW gamma matrices [?]
- WCW spinor fields [?]
- Weak form of electric-magnetic duality [?]
- Zero Energy Ontology (ZEO) [?]
- 4-D spin glass degeneracy [?]
- Equivalence Principle [?]

- Holography [?]
- Quantum Classical Correspondence [?]
- Quantum criticality [?]
- Symmetries of WCW [?]
- TGD as ATQFT [?]
- Vacuum functional in TGD [?]
- KD equation [?]
- Kaehler-Dirac action [?]

2 WCW as a union of homogenous or symmetric spaces

In the following the vision about WCW as union of coset spaces is discussed in more detail.

2.1 Basic vision

The basic view about coset space construction for WCW has not changed.

1. The idea about WCW as a union of coset spaces G/H labelled by zero modes is extremely attractive. The structure of homogenous space [A1] (`http://en.wikipedia.org/wiki/Homogenous_space`) means at Lie algebra level the decomposition $g = h \oplus t$ to sub-Lie-algebra h and its complement t such that $[h, t] \subset t$ holds true. Homogeneous spaces have G as its isometries. For symmetric space the additional condition $[t, t] \subset h$ holds true and implies the existence of involution changing at the Lie algebra level the sign of elements of t and leaving the elements of h invariant. The assumption about the structure of symmetric space [A6] (`http://en.wikipedia.org/wiki/Symmetric_space`) implying covariantly constant curvature tensor is attractive in infinite-dimensional case since it gives hopes about calculability.

   An important source of intuition is the analogy with the construction of CP_2, which is symmetric space A particular choice of h corresponds to Lie-algebra elements realized as Killing vector fields which vanish at particular point of WCW and thus leave 3-surface invariant. A preferred choice for this point is as maximum or minimum of Kähler function. For this 3-surface the Hamiltonians of h should be stationary. If symmetric space property holds true then commutators of $[t, t]$ also vanish at the minimum/maximum. Note that Euclidian signature for the metric of WCW requires that Kähler function can have only maximum or minimum for given zero modes.

2. The basic objection against TGD is that one cannot use the powerful canonical quantization using the phase space associated with configuration space - now WCW. The reason is the extreme non-linearity of the Kähler action and its huge vacuum degeneracy, which do not allow the construction of Hamiltonian formalism. Symplectic and Kähler structure must be realized at the level of WCW. In particular, Hamiltonians must be represented in completely new manner. The key idea is to construct WCW Hamiltonians as anti-commutators of super-Hamiltonians defining the contractions of WCW gamma matrices with corresponding Killing vector fields and therefore defining the matrix elements of WCW metric in the tangent vector basis defined by Killing vector fields. Super-symmetry therefre gives hopes about constructing quantum theory in which only induced spinor fields are second quantized and imbedding space coordinates are treated purely classically.

3. It is important to understand the difference between symmetries and isometries assigned to the Kähler function. Symmetries of Kähler function do not affect it. The symmetries of Kähler action are also symmetries of Kähler action because Kähler function is Kähler action for a preferred extremal (here there have been a lot of confusion). Isometries leave invariant only the quadratic form defined by Kähler metric $g_{M\overline{N}} = \partial_M \partial_{\overline{L}} K$ but not Kähler function in general. For G/H decomposition G represents isometries and H both isometries and symmetries of Kähler function.

   CP_2 is familiar example: $SU(3)$ represents isometries and $U(2)$ leaves also Kähler function invariant since it depends on the $U(2)$ invariant radial coordinate r of CP_2. The origin $r = 0$ is left invariant by $U(2)$ but for $r > 0$ $U(2)$ performs a rotation at $r = constant$ 3-sphere. This simple picture helps to understand what happens at the level of WCW.

   How to then distinguish between symmetries and isometries? A natural guess is that one obtains also for the isometries Noether charges but the vanishing of boundary terms at spatial infinity crucial in the argument leading to Noether theorem as $\Delta S = \Delta Q = 0$ does not hold true anymore and one obtains charges which are not conserved anymore. The symmetry breaking contributions would now come from effective boundaries defined by wormhole throats at which the induce metric changes its signature from Minkowskian to Euclidian. A more delicate situation is in which first order contribution to ΔS vanishes and therefore also ΔQ and the contribution to ΔS comes from second variation allowing also to define Noether charge which is not conserved.

4. The simple picture about CP_2 as symmetric space helps to understand the general vision if one assumes that WCW has the structure of symmetric space. The decomposition $g = h + t$ corresponds to decomposition of symplectic deformations to those which vanish at 3-surface (h) and those which do not (t).

   For the symmetric space option, the Poisson brackets for super generators associated with t give Hamiltonians of h identifiable as the matrix elements of WCW metric. They would not vanish although they are stationary at 3-surface meaning that Riemann connection vanishes at 3-surface. The Hamiltonians which vanish at 3-surface X^3 would correspond to t and the Hamiltonians for which Killing vectors vanish and which therefore are stationary at X^3 would correspond to h. Outside X^3 the situation would of course be different. The metric would be obtained by parallel translating the metric from the preferred point of WCW to elsewhere and symplectic transformations would make this parallel translation.

For the homogenous space option the Poisson brackets for super generators of t would still give Hamiltonians identifiable as matrix elements of WCW metric but now they would be necessary those of h. In particular, the Hamiltonians of t do not in general vanish at X^3.

2.2 Equivalence Principle and WCW

2.3 EP at quantum and classical level

Quite recently I returned to an old question concerning the meaning of Equivalence Principle (EP) in TGD framework.

Heretic would of course ask whether the question about whether EP is true or not is a pseudo problem due to uncritical assumption there really are two different four-momenta which must be identified. If even the identification of these two different momenta is difficult, the pondering of this kind of problem might be waste of time.

At operational level EP means that the scattering amplitudes mediated by graviton exchange are proportional to the product of four-momenta of particles and that the proportionality constant does not depend on any other parameters characterizing particle (except spin). The are excellent reasons to expect that the stringy picture for interactions predicts this.

1. The old idea is that EP reduces to the coset construction for Super Virasoro algebra using the algebras associated with G and H. The four-momenta assignable to these algebras would be identical from the condition that the differences of the generators annihilate physical states and identifiable as inertial and gravitational momenta. The objection is that for the preferred 3-surface H by definition acts trivially so that time-like translations leading out from the boundary of CD cannot be contained by H unlike G. Hence four-momentum is not associated with the Super-Virasoro representations assignable to H and the idea about assigning EP to coset representations does not look promising.

2. Another possibility is that EP corresponds to quantum classical correspondence (QCC) stating that the classical momentum assignable to Kähler action is identical with gravitational momentum assignable to Super Virasoro representations. This forced to reconsider the questions about the precise identification of the Kac-Moody algebra and about how to obtain the magic five tensor factors required by p-adic mass calculations [K9].

   A more precise formulation for EP as QCC comes from the observation that one indeed obtains two four-momenta in TGD approach. The classical four-momentum assignable to the Kähler action and that assignable to the modified Dirac action. This four-momentum is an operator and QCC would state that given eigenvalue of this operator must be equal to the value of classical four-momentum for the space-time surfaces assignable to the zero energy state in question. In this form EP would be highly non-trivial. It would be justified by the Abelian character of four-momentum so that all momentum components are well-defined also quantum mechanically. One can also consider the splitting of four-momentum to longitudinal and transversal parts as done in the parton model for hadrons: this kind of splitting would be very natural at the boundary of CD. The objection is that this correspondence is nothing more than QCC.

3. A further possibility is that duality of light-like 3-surfaces and space-like 3-surfaces holds true. This is the case if the action of symplectic algebra can be defined at light-like 3-surfaces or even for the entire space-time surfaces. This could be achieved by parallel translation of light-cone boundary providing slicing of CD. The four-momenta associated with the two representations of super-symplectic algebra would be naturally identical and the interpretation would be in terms of EP.

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets.

2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

4. The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore [K16].

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since the induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

2.4 Criticism of the earlier construction

The earlier detailed realization of super-Hamiltonians and Hamiltonians can be criticized.

1. Even after these more than twenty years it looks strange that the Hamiltonians should reduce to flux integrals over partonic 2-surfaces. The interpretation has been in terms of effective 2-dimensionality suggested strongly by strong form of general coordinate invariance stating that the descriptions based on light-like orbits of partonic 2-surfaces and space-like three surfaces at the ends of causal diamonds are dual so that only partonic 2-surfaces and 4-D tangent space data at them would matter. Strong form of holography implies effective 2-dimensionality but this should correspond gauge character for the action of symplectic generators in the interior the space-like 3-surfaces at the ends of CDs, which is something much milder.

   One expects that the strings connecting partonic 2-surfaces could bring something new to the earlier simplistic picture. The guess is that imbedding space Hamiltonian assignable to a point of partonic 2-surface should be replaced with that defined as integral over string attached to the point. Therefore the earlier picture would suffer no modification at the level of general formulas.

2. The fact that the dynamics of Kähler action and modified Dirac action are not directly involved with the earlier construction raises suspicions. I have proposed that Kähler function could allow identification as Dirac determinant [K6] but one would expect more intimate connection. Here the natural question is whether super-Hamiltonians for the modified Dirac action could correspond to Kähler charges constructible using Noether's theorem for corresponding deformations of the space-time surface and would also be identifiable as WCW gamma matrices.

2.5 Is WCW homogenous or symmetric space?

A key question is whether WCW can be symmetric space [A6] (`http://en.wikipedia.org/wiki/Riemannian_symmetric_space`) or whether only homogenous structure is needed. The lack of covariant constancy of curvature tensor might produce problems in infinite-dimensional context.

The algebraic conditions for symmetric space are $g = h + t$, $[h,t] \subset t$, $[t,t] \subset h$. The latter condition is the difficult one.

1. δCD Hamiltonians should induce diffeomorphisms of X^3 indeed leaving it invariant. The symplectic vector fields would be parallel to X^3. A stronger condition is that they induce symplectic transformations for which all points of X^3 remain invariant. Now symplectic vector fields vanish at preferred 3-surface (note that the symplectic transformations are r_M local symplectic transformations of $S^2 \times CP_2$).

2. For Kac-Moody algebra inclusion $H \subset G$ for the finite-dimensional Lie-algebra induces the structure of symmetric space. If entire algebra is involved this does not look physically very attractive idea unless one believes on symmetry breaking for both $SU(3)$, $U(2)_{ew}$, and $SO(3)$ and E_2 (here complex conjugation corresponds to the involution). If one assumes only Kac-Moody algebra as critical symmetries, the number of tensor factors is 4 instead of five, and it is not clear whether one can obtain consistency with p-adic mass calculations.

   Examples of 3-surfaces remaining invariant under $U(2)$ are 3-spheres of CP_2. They could correspond to intersections of deformations of CP_2 type vacuum extremals with the boundary of CD. Also geodesic spheres S^2 of CP_2 are invariant under $U(2)$ subgroup and would relate naturally to cosmic strings. The corresponding 3-surface would be $L \times S^2$, where L is a piece of light-like radial geodesic.

3. In the case of symplectic algebra one can construct the imbedding space Hamiltonians inducing WCW Hamiltonians as products of elements of the isometry algebra of $S^2 \times CP_2$ for with parity under involution is well-defined. This would give a decomposition of the symplectic algebra satisfying the symmetric space property at the level imbedding space. This decomposition does not however look natural at the level of WCW since the only single point of CP_2 and light-like geodesic of δM^4_+ can be fixed by $SO(2) \times U(2)$ so that the 3-surfaces would reduce to pieces of light rays.

4. A more promising involution is the inversion $r_M \to 1/r_M$ of the radial coordinate mapping the radial conformal weights to their negatives. This corresponds to the inversion in Super Virasoro algebra. t would correspond to functions which are odd functions of $u \equiv log(r_M/r_0)$ and h to even function of u. Stationary 3-surfaces would correspond to $u = 1$ surfaces for which $log(u) = 0$ holds true. This would assign criticality with Virasoro algebra as one expects on general grounds.

   $r_M = constant$ surface would most naturally correspond to a maximum of Kähler function which could indeed be highly symmetric. The elements with even u-parity should define Hamiltonians, which are stationary at the maximum of Kähler function. For other 3-surfaces obtained by $/r_M$-local) symplectic transformations the situation is different: now H is replaced with its symplectic conjugate hHg^{-1} of H is acceptable and if the conjecture is true one would obtained 3-surfaces assignable to perturbation theory around given maximum as symplectic conjugates of the maximum. The condition that H leaves X^3 invariant in poin-twise manner is certainly too strong and imply that the 3-surface has single point as CP_2 projection.

5. One can also consider the possibility that critical deformations correspond to h and non-critical ones to t for the preferred 3-surface. Criticality for given h would hold only for a preferred 3-surface so that this picture would be very similar that above. Symplectic conjugates of h would define criticality for other 3-surfaces. WCW would decompose to a union corresponding to different criticalities perhaps assignable to the hierarchy of sub-algebras of conformal algebra labelled by integer whose multiples give the allowed conformal weights. Hierarchy of breakings of conformal symmetries would characterize this hierarchy of sectors of WCW.

   For sub-algebras of the conformal algebras (Kac-Moody and symplectic algebra) the condition $[t, t] \subset h$ cannot hold true so that one would obtain only the structure of homogenous space.

2.6 Symplectic and Kac-Moody algebras as basic building bricks

The basic building bricks are symplectic algebra of δCD (this includes CP_2 besides light-cone boundary) and Kac-Moody algebra assignable to the isometries of δCD [K4]. It seems however that the longheld view about the role of Kac-Moody algebra must be modified. Also the earlier realization of super-Hamiltonians and Hamiltonians seems too naive.

1. I have been accustomed to think that Kac-Moody algebra could be regarded as a sub-algebra of symplectic algebra. p-Adic mass calculations however requires five tensor factors for the coset representation of Super Virasoro algebra naturally assigned to the coset structure G/H of a sector of WCW with fixed zero modes. Therefore Kac-Moody algebra cannot be regarded as a sub-algebra of symplectic algebra giving only single tensor factor and thus inconsistent with interpretation of p-adic mass calculations.

2. The localization of Kac-Moody algebra generators with respect to the internal coordinates of light-like 3-surface taking the role of complex coordinate z in conformal field theory is also questionable: the most economical option relies on localization with respect to light-like radial coordinate of light-cone boundary as in the case of symplectic algebra. Kac-Moody algebra cannot be however sub-algebra of the symplectic algebra assigned with covariantly constant right-handed neutrino in the earlier approach.

3. Right-handed covariantly constant neutrino as a generator of super symmetries plays a key role in the earlier construction of symplectic super-Hamiltonians. What raises doubts is that other spinor modes - both those of right-handed neutrino and electro-weakly charged spinor modes - are absent. All spinor

modes should be present and thus provide direct mapping from WCW geometry to WCW spinor fields in accordance with super-symmetry and the general idea that WCW geometry is coded by WCW spinor fields.

Hence it seems that Kac-Moody algebra must be assigned with the modes of the induced spinor field which carry electroweak quantum numbers. If would be natural that the modes of right-handed neutrino having no weak and color interactions would generate the huge symplectic algebra of symmetries and that the modes of fermions with electroweak charges generate much smaller Kac-Moody algebra.

4. The dynamics of Kähler action and modified Dirac action action are invisible in the earlier construction. This suggests that the definition of WCW Hamiltonians is too simplistic. The proposal is that the conserved super charges derivable as Noether charges and identifiable as super-Hamiltonians define WCW metric and Hamiltonians as their anti-commutators. Spinor modes would become labels of Hamiltonians and WCW geometry relates directly to the dynamics of elementary particles.

5. Note that light-cone boundary $\delta M^4_+ = S^2 \times R_+$ allows infinite-dimensional group of isometries consisting of conformal transformation of the sphere S^2 with conformal scaling compensated by an S^2 local scaling or the light-like radial coordinate of R_+. These isometries contain as a subgroup symplectic isometries and could act as gauge symmetries of the theory.

3 Preferred extremals of Kähler action, solutions of the modified Dirac operator, and quantum criticality

Perhaps due to my natural laziness I have not bothered to go through the basic construction [K4, K3] although several new ideas have emerged during last years [K13].

1. The new view about preferred extremals of Kähler action involves the slicing of space-time surface to string world sheets labelled by points of any partonic two-surface or vice versa. I have called this structure Hamilton-Jacobi structure [K2]. A number theoretic interpretation based on the octonionic representation of imbedding space gamma matrices. A gauge theoretic interpretation in terms of two orthogonal 2-D spaces assignable to polarization and momentum of massless field mode is also possible. The slicing suggests duality between string world sheets and conformal field theory at partonic 2-surfaces analogous to AdS/CFT. Strong form of holography implied by strong form of GCI would be behind the duality.

2. The new view about the solutions of modified Dirac equation involves localization of the modes at string world sheets: this emerges from the condition that electric charge is well defined quantum number for the modes. The effective 2-dimensionality of the space of the modified gamma matrices is crucial

for the localization. This leads to a concrete model of elementary particles as string like objects involving two space-time sheets and flux tubes carrying Kähler magnetic monopole flux. Holomorphy and complexification of modified gamma matrices are absolutely essential consequences of the localization and is expected to be crucial also in the construction of WCW geometry. The weakest interpretation is that the general solution of modified Dirac is superposition of these localized modes parametrized by the points of partonic 2-surface and integer labelling the modes themselves as in string theory. One has the same general picture as in ordinary quantum theory.

One can wonder whether finite measurement resolution is realized dynamically in the sense that a discrete set of stringy world sheets are possible. It will be found that quantization of induced spinor fields leads to a concrete proposal realizing this: strings would be identified as curves along which Kähler magnetic field has constant value.

3. Quantum criticality is central notion in TGD framework: Kähler coupling strength is the only coupling parameter appearing in Kähler action and is analogous to temperature. The idea of quantum criticality is that TGD Universe is quantum critical so that Kähler coupling strength is analogous to critical temperature. The hope is that this could make the theory unique. I have not however been able to really understand it and relate it to the coset space construction of WCW and to coset representations of Super Virasoro.

3.1 What criticality is?

The basic technical problem has been characterization of it quantitatively [K6]. Here there is still a lot of fuzzy thinking and unanswered questions. What is the precise definition of criticality and what is its relation to G/H decomposition of WCW? Could H correspond to critical deformations so that it would have purely group theoretical characterization, and one would have nice unification of two approaches to quantum TGD?

1. Does criticality correspond to the failure of classical determinism?

The intuitive guess is that quantum criticality corresponds classically to the criticality of Kähler action implying non-determinism. The preferred extremal associated with given 3-surface at the boundary of CD is not unique. There are several deformations of space-time surface *vanishing* at X^3 and leaving the Kähler action and thus Kähler function invariant.

Some nitpicking before continuing is in order.

1. The key word is *"vanishing"* in the above definition of criticality relying on classical non-determinism. Could one allow also non-vanishing deformations of X^3 with the property that Kähler function and Kähler action are not changed? This would correspond to the idea that critical directions correspond to flat directions for the potential in quadratic approximation: now it would be Kähler function in quadratic approximation. The flat direction would not contribute to Kähler metric $G_{K\overline{L}} = \partial_K \partial_{\overline{L}}$.

Clearly, the subalgebra h associated with H would satisfy criticality in this sense for all 3-surfaces except the one for which it acts as isotropy group: in this case one would have criticality in the strong sense.

This identification of criticality is consistent with that based on non-determinism *only if* the deformations in H leaving X^3 fixed do not leave $X^4(X^3)$ fixed. This would apply also to h. One would have bundle like structure: 3-surface would represent base point of the bundle and space-time surfaces associated with it would correspond to the points in the fiber permuted by h.

2. What about zero modes, which appear only in the conformal scaling factor of WCW metric but not in the differentials appearing the line element? Are the critical modes zero modes but only up to second order in functional Taylor expansion?

Returning to the definition of criticality relying on classical non-determinism. One can try to fix $X^4(X^3)$ uniquely by fixing 3-surface at the second end of CD but even this need not be enough? One expects non-uniqueness in smaller scales in accordance with approximate scaling invariance and fractality assignable to criticality.

A possible interpretation would be in terms of dynamical symmetry analogous to gauge symmetry assignable to H and having interpretation in terms of measurement resolution. Increasing the resolution would mean fixing X^3 at upper and lower boundaries in shorter scale. Finite measurement resolution would give rise to dynamical gauge symmetry. This conforms with the idea that TGD Universe is analogous to a Turing machine able to mimic any gauge dynamics. The hierarchy of inclusions for hyper-finite factors of type II_1 supports this view too [K10].

Criticality would be a space-time correlate for quantum non-determinism. I have assigned this nondeterminism to multi-furcations of space-time sheets giving rise to the hierarchy of Planck constants. This involves however something new: namely the idea that several alternative paths are selected in the multi-furcation simultaneously [K5, K12].

2. Further aspects of criticality

1. Mathematically the situation at criticality of Kähler action for $X^4(X^3)$ (as distinguished from Kähler function for X^3) is analogous to that at the extremum of potential when the Hessian defined by second derivatives has vanishing determinant and there are zero modes. Now one would have an infinite number of deformations leaving Kähler action invariant in second order. What is important that critical deformations leave X^3 invariant so that they cannot correspond to the sub-algebra h except possibly at point for which H acts as an isotropy group.

2. Criticality would suggest that conserved charges linear in deformation vanish: this because deformation vanishes at X^3. Second variation would give rise to charges to and invariance of the Kähler action in this action would mean that $\Delta S_2 = \Delta Q_2 = 0$ holds true unless effective boundary terms spoil the situation. Second order charges would be quadratic in the variation and it is not at all clear whether there is any hope about having a non-linear analog of

Lie-algebra or super algebra structure. I do not know whether mathematicians have considered this kind of possibility. Yangian algebra represent involving besides Lie algebra generators also generators coming as their multilinears have some formal resemblance with this kind of non-linear structure.

3. Supersymmetry would suggest that criticality for the Kähler action implies criticality for the modified Dirac action. The first order charges for Dirac action involve the partial derivatives of the canonical momentum currents T_k^α with respect to partial derivatives $\partial_\beta h^l$ of imbedding space coordinates just as the second order charges for Kähler action do. First order Noether charges vanish if criticality means that variation vanishes at X^3 but not at $X^4(X^3)$ since they involve linearly δh^k vanishing at X^3. Second order charges for modified Dirac action get second contribution from the modification of the induced spinor field by a term involving spin rotation and from the second variation of the modified gamma matrices. Here it is essential that derivatives of $\partial_k \delta h^l$, which need not vanish, are involved.

   Note: I use the notation ∂_α for space-time partial derivatives and ∂_k for imbedding space partial derivatives).

3.2 Do critical deformations correspond to Super Virasoro algebra?

One can try to formulate criticality a in terms of super-conformal algebras and their sub-algebras $h_{c,m}$ for which conformal weights are integer multiples of integer m. Now I mean with super-conformal algebra also symplectic and super Kac-Moody algebras. These decompositions - call them just $g_c = t_c \oplus h_c$ need *not* correspond to $g + h$ associated with G/H although it could do so. For instance, if g_c corresponds to Super Virasoro algebra then the decomposition $g_c = t_c \oplus h_c$ does not correspond to $g = t \oplus h$.

1. There would be a hierarchy of included sub-algebras $h_{c,m}$, which corresponds to hierarchy of conformal algebras assignable to the light-like radial coordinate of the boundary of light-cone and criticalities could form hierarchy in this sense. The algebras form inclusion hierarchies $h_{m_1} \supset h_{m_2} \supset ...$ labelled by sequences consisting of integers such that given integer is divisible by the previous integer in the sequence: $m_n \ mod \ m_{n-1} = 0$.

   Critical deformations assignable to $h_{c,m}$ would vanish at preferred X^3 for which H is isotropy group and leave Kähler action invariant and would not therefore contribute to Kähler metric at X^3. They could however affect $X^4(X^3)$.

   Non-critical deformation would correspond to the complement of this sub-algebra affecting both $X^4(X^3)$ and X^3. This hierarchy would correspond to an infinite hierarchy of conformal symmetry breakings and would be manifested at the level of WCW geometry. Also a connection with the inclusion hierarchy for hyper-finite factors of type II_1 [K10] having interpretation in terms of finite measurement resolution is suggested by this hierarchy. Super Virasoro generators with conformal weight coming as a multiple of m would annihilate physical states so that effectively the criticality correspond to finite-D

Hilbert space. This is something new as compared to the ordinary view about criticality for which all Super Virasoro generators annihilate the states.

2. A priori $g = t + h$ decomposition need not have anything to do with the decomposition of deformations to non-critical and critical ones. Critical deformations could indeed appear as sub-algebra of $g = t + h$ and be present for both t and h in the same manner: that is as sub-algebras of super- Virasoro algebras: Super Virasoro would represent the non-determinism and criticality and in 2-D conformal theories describing criticality this is indeed the case. In this case the actions of G and H identified as super-symplectic and super Kac-Moody algebras could be unique and non-deterministic aspect would not be present. This corresponds to the physical intuition.

   If criticality corresponds to G/H structure, symmetric space property $[t, t] \subset h$ would not hold true as is clear from the additivity of super-conformal weights in the commutators of conformal algebras. The reduction of G/H structure to criticality would be very nice but personally I would give up covariant constancy of curvature tensor in infinite-dimensional context only with heavy heart.

3. The super-symmetric relation between Kähler action and corresponding modified Dirac action suggests that the criticality of Kähler action implies vanishing conserved charges also for the modified Dirac action (both ordinary and super charges so that one has super-symmetry). The reason is that conserved charge is linear in deformation. Conservation in turn means that Kähler action is not changed: $\Delta S = \Delta Q = 0$. For non-critical deformations the boundary terms at the orbits wormhole throats imply non-conservation so that ΔQ (the difference of charges at space-like ends of space-time surface) is non-vanishing although local conservation law holds strue. This in terms implies that the contribution to the Kähler metric is non-trivial.

   At criticality both bosonic and fermionic conserved currents can be assigned to the second variation and are thus quadratic in deformation just like that associated with Kähler action. If effective boundary terms vanish the criticality for Kähler action implies the conservation of second order charges by $\Delta_2 S = \Delta_2 Q = 0$.

3.3 Connection with the vanishing of second variation for Kähler action

There are three general conjectures related to modified Dirac equation and the conserved currents associated with the vanishing second variation of Kähler action at critical points analogous to extrema of potential function at which flat directions appear and the determinant defined by second derivatives of the potential function does not have maximal rank.

1. Quantum criticality has as a correlate the vanishing of the second variation of Kähler action for critical deformations. The conjecture is that the number of these directions is infinite and corresponds to sub-algebras of Super Virasorol

algebra corresponding to conformal weights coming as integer multiples of integer. Super Virasoro hypothesis implies that preferred extremals have same algebra of critical deformations at all points.

Noether theorem applied to critical variations gives rise to conserved currents and charges which are quadratic in deformation. For non-critical deformations one obtains linearity in deformation and this charges define the super-conformal algebras.

Super Virasoro algebra indeed has a standard representation in which generators are indeed quadratic in Kac-Moody (and symplectic generators in the recent case). This quadratic character would have interpretation in terms of criticality not allowing linear representation.

2. Modified Dirac operator is assumed to have a solution spectrum for which both non-critical and critical deformations act as symmetries. The critical currents vanish in the first order. Second variation involving first variation for the modified gamma matrices and first variation for spinors (spinor rotation term) gives and second variation for canonical momentum currents gives conserved current. The general form of the current is very similar to the corresponding current associated with Kähler action.

3. The currents associated with the modified Dirac action and Kähler action have same origin. In other words: the conservation of Kähler currents implies the conservation of the currents associated with the modes of the modified Dirac operator. A question inspired by quantum classical correspondence is whether the eigen values of the fermionic charges correspond to the values of corresponding classical conserved charges for Kähler action in the Cartan algebra. This would imply that all space-time surfaces in superposition representing momentum eigen state have the same value of classical four-momentum. A stronger statement of QCC would be that classical correlation functions are same as the quantal ones.

4 Quantization of the modified Dirac action

The quantization of the modified Dirac action follows standard rules.

1. The general solution is written as a superposition of modes, which are for other fermions than ν_R localized to string world sheets and parametrized by a point of partonic 2-surface which can be chosen to be the intersection of light-like 3-surface at which induced metric changes signature with the boundary of CD.

2. The anti-commutations for the induced spinor fields are dictated from the condition that the anti-commutators of the super-Hamiltonians identified as WCW gamma matrices give WCW Hamiltonians as matrix elements of WCW metric. Super Hamiltonians are identified as Noether charges for the modified Dirac action assignable to the symplectic algebra of δCD being labelled also by the quantum numbers labelling the modes of the induced spinor field.

3. Consistency conditions for the modified Dirac operator require that the modified gamma matrices have vanishing divergence: this is true for the extremals of Kähler action.

4. The guess for the critical algebra is as sub-algebra of Super Virasoro algebra affecting on the radial light-like coordinate of δCD as diffeomorphisms. The deformations of the modified Dirac operator should annihilate spinor modes. This requires that the deformation corresponds to a gauge transformation for the induced gauge fields. Furthermore, the deformation for the modified gamma matrices determined by the deformation of the canonical momentum densities contracted covariant derivatives should annihilate the spinor modes. The situation is analogous to that for massless Dirac operator: Dirac equation for momentum eigenstate does not imply vanishing of the momentum but only that of mass. The condition that the divergence for the deformation of the modified gamma matrices vanishes as does also the divergence of the modified gamma matrices implies that the second variation of Kähler action vanishes. One obtains classical Kähler charges and Dirac charges: the latter act as operators. The equivalence of the two definitions of of four-momenta would corresponds to EP and QCC.

5. An interesting question of principle is what the almost topological QFT property meaning that Kähler action reduces to Chern-Simons form integrated over boundary of space-time and over the light-like 3-surfaces means. Could one write the currents in terms of Chern-Simons form alone? Could one use also Chern-Simons analog of modified Dirac action. What looks like problem at the first glance is that only the charges associated with the symplectic group of CP_2 would be non-vanishing. Here the weak form of electric-magnetic duality [K6, K13] however introduce constraint terms to the action implying that all charges can be non-vanishing.

The challenge is to construct explicit representations of super charges and demonstrate that suitably defined anti-commutations for spinor fields reproduce the anti-commutations of the super-symplectic algebra.

4.1 Integration measure in the superposition over modes

One can express Ψ as a superposition over modes as usually. Except for ν_R, the modes are localized at string world sheets and can be labelled by a point of X^2, integer characterizing the mode and analogous to conformal weight, and quantum numbers characterizing spin, electroweak quantum numbers, and M^4 handedness. The de-localization of the modes of ν_R decouple from left-handed neutrino if the modified gamma matrices involved only M^4 or CP_2 gamma matrices. It might be possible to choose the string coordinate to be light-like radial coordinate of δCD but this is by no means necessary.

The integration measure $d\mu$ in the superposition of modes has nothing to do with the metric determinants assignable to 3-surface X^3 or with the corresponding space-time surface at X^3. $d\mu$ at partonic 1-surface X^2 must be taken to be such that its square multiplied by transversal delta function resulting in anti-commutation of two modes gives a measure defined by the Kähler form $J_{\mu\nu}$ and given by $d\mu = J_{\mu\nu}dx^\mu dx^\nu = J\sqrt{g_2}dx^1 \wedge dx^2$, $J = J_{\mu\nu}\epsilon^{\mu\nu}$ (note that permutation tensor is inversely proportional $\sqrt{g_2}$). This measure appears in the earlier definition of WCW Hamiltonian as the analog of flux integral $\oint H_A J dx^1 \wedge dx^2$, where H_A is Hamiltonian to be replaced with its integral over string.

There are two manners to get J to the measure for Hamiltonian flux.

- Option I: One uses for super charges has "half integration measure" given by $d\mu_{1/2} = \sqrt{J\sqrt{g_2}}dx^1 \times dx^2$. Note that $\sqrt{J}$ is imaginary for $J < 0$ and also the unique choice of sign of the square root might produce problems.

- Option II: The integration measure is $d\mu = J(x, end)\sqrt{g_2}dx^1 \wedge dx^2$ for the super charge and anti-commutations of Ψ at string are proportional to $1/J(x, end)\sqrt{g_2}$ so that anti-commutator of supercharges would be proportional to $J(x, end)\sqrt{g_2}$ and metric determinant disappears from the integration measure. Note that the vanishing of $J(x, end)$ does not produce any problems in anti-commutators.

  $J(x, end)$ means a non-locality in the anti-commutator. If the string is interpreted as beginning from the partonic surface at its second end, one obtains two different anti-commutation relations unless strings are $J(x, y)\sqrt{g_2} = constant$ curves. This could make sense for flux tubes which are indeed assumed to carry the Kähler flux. Note also that partonic 2-surface decomposes naturally into regions with fixed sign of J forming flux tubes.

  $J(x, y)\sqrt{g_2} = constant$ condition seems actually trivial. The reason is that by a suitable coordinate transformations $(x, y) \rightarrow (f(x, y)$ leaving string coordinate invariant the $\sqrt{g_2}$ gains a factor equal to the Jacobian of the transformation which reduces to 2-D Jacobian for the transformation for the coordinates of partonic 2-surface. By a suitable choice of this transformation $J(x, y)\sqrt{g_2} = constant$ condition is satisfied along string world sheets. This transformation is determined only modulo an area preserving - thus symplectic - transformation for each partonic 2-surface in the slicing. One obtains space-time analog of symplectic invariance as an additional symmetry having identification as a remnant of 3-D GCI. Since also string parameterizations $t \rightarrow f(t)$ are allowed so that 3-D GCI reduces to 1-D Diff and 2-D Sympl. Natural 4-D extension of string reparameterizations would be to the analogs of conformal transformations associated with the effective metric defined by modified gamma matrices so that 4-D Diff would reduce to a product of 2-D conformal and symplectic groups.

  The physical state is specified by a finite number of fermion number carrying string world sheets (one can of course have a superposition of these states with different locations of string world sheets). One can ask whether QCC forces the space-time surface to code this state in its geometry in the sense that only these string world sheets are possible. $J(x, y)\sqrt{g_2} = constant$ condition does not force this.

- Option III: If one assumes slicing by partonic 2-surfaces with common coordinates $x = (x^1, x^2)$ and that $J(x, y)\sqrt{g}$ is included to current density at the point of string and that $1/J(x, y)\sqrt{g_2}$ in the anti-commutations is evaluated at the point x of the partonic surface intersecting the string at y, the flux is replaced with the superposition of local fluxes from all points in the slicing by partonic 2-surfaces and $J(x,y)$. For $J\sqrt{g_2}$= constant along strings Options II an III are equivalent.

On basis of physical picture Option II with $J\sqrt{g_2}$= constant achieved by a proper choice of partonic coordinates for the slicing looks very attractive.

4.2 Fermionic supra currents as Noether currents

Fermionic supra currents can be taken as Noether currents assignable to the modified Dirac action. Charges are obtained by integrating over string. Here possible technical problems relate to the correct identification of the integration measure. In the normal situation the integration measure is $\sqrt{g_4}$ but now 2-D delta function restricts the charge density for a given mode to the string world sheet and might produce additional factors.

The general form of the super current at given string world sheet corresponding to a given string world sheet is given by

$$\begin{aligned} J^\alpha &= \left[\overline{\Psi_n} O^\alpha_{\beta,k}\delta h^k D_\alpha \Psi + \overline{\Psi_n}\Gamma^\alpha \delta\Psi\right]\sqrt{g_4} \ , \\ O^\alpha_{\beta,k} &= \frac{\partial \Gamma^\alpha}{\partial(\partial_\beta h^k)} \ . \end{aligned} \tag{4.1}$$

The covariant divergence of J^α vanishes. Modified gamma matrices appearing in the equation are defined as contractions of the canonical momentum densities T^α_k of Kähler action with imbedding space gamma matrices Γ^k as

$$\begin{aligned} \Gamma^\alpha &= T^\alpha_k \Gamma^k \ , \\ T^\alpha_k &= \frac{L_K}{\partial(\partial_\beta h^k)} \ , \end{aligned} \tag{4.2}$$

Ψ_n is the mode of induced spinor field considered. $\delta\Psi$ is the change of Ψ in spin rotation given by

$$\delta\Psi = \partial_l j_k \Sigma^{kl} \ . \tag{4.3}$$

The corresponding current is obtained by replacing Ψ_n with Ψ and integrating over the modes.

The current could quite well vanish. The reason is that holography means that one half of modified gamma matrices whose number is effectively 2 annihilates the spinor modes. Also the covariant derivative D_z or $D_{\overline{z}}$ annihilates it. One obtains vanishing result if the quantity $O^z_{\beta,k}$ is proportional to Γ^z. This can be circumvented if it is superposition of gamma matrices which are not parallel to the string world sheet or if is superposition of Γ^z and $\Gamma^{\overline{z}}$: this could have interpretation as breaking of conformal invariance.

For critical deformations vanishing at X^3 δh^k appearing in the formula of current vanishes so that one obtains non-vanishing charge only for second variation.

Note that the quantity $O^{\alpha}_{\beta,k}$ involves terms $J^{\alpha k}J^l_{\beta}$ and can be non-vanishing even when J vanishes. The replacement of ordinary γ^0 in fermionic anti-commutation relations with the modified gamma matrix Γ^0 helps here since modified gamma matrices vanish when J vanishes.

Note that for option II favoured by the existing physical picture J is constant along the strings and anti-commutation relations are non-singular for $J \neq 0$.

4.3 Anti-commutators of super-charges

The anti-commutators for fermionic fields- or more generally, quantities related to them - should be such that the anti-commutator of fermionic super-Hamiltonians defines WCW Hamiltonian with correct group theoretical properties. To obtain the correct anti-commutator requires that one obtains Poisson bracket of δCD Hamiltonians appearing in the super-Hamiltonians. This is the case if the anti-commutator involved is proportional to iJ_{kl} since this gives the desired Poisson bracket

$$J_{kl}j^k_Aj^l_B = \{H_A, H_B\} \ . \tag{4.4}$$

This is achieved if one replaces the anti-commutators of Ψ and $\overline{\Psi}$ with anti-commutator of $A_k \equiv O^0_k\Psi$ and $\overline{A_l} \equiv \overline{\Psi}O^0_l$ (O^{α}_k was defined in Eq. 4.1) and assumes

$$\{A_k, \overline{A}_l\} = iJ_{kl}\Gamma^0\delta_2(x_2, y_2)\delta_1(y_1, y_2)\frac{X}{g_4^{1/2}} \ . \tag{4.5}$$

Here Γ^0 is modified gamma matrix and δ_2 is delta function assignable to the partonic 2-surface and δ_1 is delta function assignable with the string. Depending on whether one assumes option I, II, or III one has $X = 1$, $X = 1/J_{x,end}$ or $1/J(x_1, x_2, y)$.

The modified anti-commutation relations do not make sense in higher imbedding space dimensions since the number of spinor components exceeds imbedding space dimension. For $D = 8$ the dimension of H and the number of independent spinor components with given H-chirality are indeed same (leptons and quarks have opposite H-chirality). This makes the dimension $D = 8$ unique in TGD framework.

4.4 Strong form of General Coordinate Invariance and strong form of holography

Strong form of general coordinate invariance (GCI) suggests a duality between descriptions using light-like 3-surfaces X^3_l at which the signature of the induced metric changes and space-like 3-surface X^3 at the ends of the space-time surface. Also the translates of these surfaces along slicing might define the theory but with a Kähler function to which real part of a holomorphic function defined in WCW is added.

In order to define the formalism for light-like 3-surfaces, one should be able to define the symplectic algebra. This requires the translation of the boundaries of the light-cone along the line connecting the tips of the CD so that the Hamiltonians of δM^4_+ or δM^4_- make sense at X^3_l. Depending on whether the the state function reduction has occurred on upper or lower boundary of CD one must use translates of δM^4_+ or δM^4_-: this would be one particular manifestation for the arrow of time.

4.5 Radon, Penrose ja TGD

The construction of the induced spinor field as a superposition of modes restricted to string world sheets to have well-defined em charge (except in the case of right-handed neutrino) brings in mind Radon transform [A4] (`http://en.wikipedia.org/wiki/Radon_transform`) and Penrose transform [A3] (`http://en.wikipedia.org/wiki/Penrose_transform`). In Radon transform the function defined in Euclidian space E^n is coded by its integrals over $n-1$ dimensional hyper-planes. All planes are allowed and are characterized by their normal whose direction corresponds to a point of $n-1$-dimensional sphere S^{n-1} and by the orthogonal distance of the plane from the origin. Note that the space of hyper-planes is n-dimensional as it should be if it is to carry same information as the function itself. One can easily demonstrate that n-dimensional Fourier transform is composite of 1-dimensional Fourier transform in the direction normal vector parallel to wave vector obtained integrating over the distance parameter associated with $n-$dimensional Radon transform defined by function multplied by the plane wave.

In the case of Penrose transform [A3] (`http://en.wikipedia.org/wiki/Penrose_transform`) one has 6-dimensional twistor space CP_3 and the space of complex two - planes- topologically spheres in CP_3 - one for each point of in CP_3 - defines 4-D compactified Minkowski space. A massless field in M^4 has a representation in CP_3 with field value at given point of M^4 represented as an integral over S^3 of holomorphic field in CP_3.

In the recent case the situation resembles very much that for Penrose transform. In the case of space-like 3-surface CP_3 is replaced with the space of strings emanating from the partonic 2-surface and its points are labelled by points of partonic 2-surface and points of string so that dimension is still $D = 3$. The transform describes second quantize spinor field as a collection of "Fourier components" along stringy curves. In 4-D case one has 4-D space-time surface and collection of "Fourier components" along string world sheets. One could say that charge densities assignable to partonic 2-surfaces replace the massless fields in M^4. Now however the decomposition into strings and string world sheets takes place at the level of physics rather than only mathematically.

5 About the notion of four-momentum in TGD framework

The starting point of TGD was the energy problem of General Relativity [K9]. The solution of the problem was proposed in terms of sub-manifold gravity and based on the lifting of the isometries of space-time surface to those of $M^4 \times CP_2$ in which space-times are realized as 4-surfaces so that Poincare transformations act on space-time surface as an 4-D analog of rigid body rather than moving points at space-time surface. It however turned out that the situation is not at all so simple.

There are several conceptual hurdles and I have considered several solutions for them. The basic source of problems has been Equivalence Principle (EP): what does EP mean in TGD framework [K9, K16]? A related problem has been the interpretation of gravitational and inertial masses, or more generally the corresponding 4-momenta. In General Relativity based cosmology gravitational mass is not conserved and this seems to be in conflict with the conservation of Noether charges. The resolution is in terms of zero energy ontology (ZEO), which however forces to modify slightly the original view about the action of Poincare transformations.

A further problem has been quantum classical correspondence (QCC): are quantal four-momenta associated with super conformal representations and classical four-momenta associated as Noether charges with Kähler action for preferred extremals identical? Could inertial-gravitational duality - that is EP - be actually equivalent with QCC? Or are EP and QCC independent dualities. A powerful experimental input comes p-adic mass calculations [K15] giving excellent predictions provided the number of tensor factors of super-Virasoro representations is five, and this input together with Occam's razor strongly favors QCC=EP identification.

There is also the question about classical realization of EP and more generally, TGD-GRT correspondence.

Twistor Grassmannian approach has meant a technical revolution in quantum field theory (for attempts to understand and generalize the approach in TGD framework see [K11, K7]. This approach seems to be extremely well suited to TGD and I have considered a generalization of this approach from $\mathcal{N}=4$ SUSY to TGD framework by replacing point like particles with string world sheets in TGD sense and super-conformal algebra with its TGD version: the fundamental objects are now massless fermions which can be regarded as on mass shell particles also in internal lines (but with unphysical helicity). The approach solves old problems related to the realization of stringy amplitudes in TGD framework, and avoids some problems of twistorial QFT (IR divergences and the problems due to non-planar diagrams). The Yangian variant of 4-D conformal symmetry is crucial for the approach in $\mathcal{N}=4$ SUSY, and implies the recently introduced notion of amplituhedron [**?**]. A Yangian generalization of various super-conformal algebras seems more or less a "must" in TGD framework. As a consequence, four-momentum is expected to have characteristic multilocal contributions identifiable as multipart on contributions now and possibly relevant for the understanding of bound states such as hadrons.

5.1 Scale dependent notion of four-momentum in zero energy ontology

Quite generally, General Relativity does not allow to identify four-momentum as Noether charges but in GRT based cosmology one can speak of non-conserved mass [K8], which seems to be in conflict with the conservation of four-momentum in TGD framework. The solution of the problem comes in terms of zero energy ontology (ZEO) [K1, K14], which transforms four-momentum to a scale dependent notion: to each causal diamond (CD) one can assign four-momentum assigned with say positive energy part of the quantum state defined as a quantum superposition of 4-surfaces inside CD.

ZEO is necessary also for the fusion of real and various p-adic physics to single coherent whole. ZEO also allows maximal "free will" in quantum jump since every zero energy state can be created from vacuum and at the same time allows consistency with the conservation laws. ZEO has rather dramatic implications: in particular the arrow of thermodynamical time is predicted to vary so that second law must be generalized. This has especially important implications in living matter, where this kind of variation is observed.

More precisely, this superposition corresponds to a spinor field in the "world of classical worlds" (WCW) [K14]: its components - WCW spinors - correspond to elements of fermionic Fock basis for a given 4-surface - or by holography implied by general coordinate invariance (GCI) - for 3-surface having components at both ends of CD. Strong form of GGI implies strong form of holography (SH) so that partonic 2-surfaces at the ends of space-time surface plus their 4-D tangent space data are enough to fix the quantum state. The classical dynamics in the interior is necessary for the translation of the outcomes of quantum measurements to the language of physics based on classical fields, which in turn is reduced to sub-manifold geometry in the extension of the geometrization program of physics provided by TGD.

Holography is very much reminiscent of QCC suggesting trinity: GCI-holography-QCC. Strong form of holography has strongly stringy flavor: string world sheets connecting the wormhole throats appearing as basic building bricks of particles emerge from the dynamics of induced spinor fields if one requires that the fermionic mode carries well-defined electromagnetic charge [K13].

5.2 Are the classical and quantal four-momenta identical?

One key question concerns the classical and quantum counterparts of four-momentum. In TGD framework classical theory is an exact part of quantum theory. Classical four-momentum corresponds to Noether charge for preferred extremals of Kähler action. Quantal four-momentum in turn is assigned with the quantum superposition of space-time sheets assigned with CD - actually WCW spinor field analogous to ordinary spinor field carrying fermionic degrees of freedom as analogs of spin. Quantal four-momentum emerges just as it does in super string models - that is as a parameter associated with the representations of super-conformal algebras. The precise action of translations in the representation remains poorly specified. Note that quantal four-momentum does not emerge as Noether charge: at at least it is not at all obvious that this could be the case.

Are these classical and quantal four-momenta identical as QCC would suggest? If so, the Noether four-momentum should be same for all space-time surfaces in the superposition. QCC suggests that also the classical correlation functions for various general coordinate invariant local quantities are same as corresponding quantal correlation functions and thus same for all 4-surfaces in quantum superposition - this at least in the measurement resolution used. This would be an extremely powerful constraint on the quantum states and to a high extend could determined the U-, M-, and S-matrices.

QCC seems to be more or less equivalent with SH stating that in some respects the descriptions based on classical physics defined by Kähler action in the interior of space-time surface and the quantal description in terms of quantum states assignable to the intersections of space-like 3-surfaces at the boundaries of CD and light-like

3-surfaces at which the signature of induced metric changes. SH means effective 2-dimensionality since the four-dimensional tangent space data at partonic 2-surfaces matters. SH could be interpreted as Kac-Mody and symplectic symmetries meaning that apart from central extension they act almost like gauge symmetries in the interiors of space-like 3-surfaces at the ends of CD and in the interiors of light-like 3-surfaces representing orbits of partonic 2-surfaces. Gauge conditions are replaced with Super Virasoro conditions. The word "almost" is of course extremely important.

5.3 What Equivalence Principle (EP) means in quantum TGD?

EP states the equivalence of gravitational and inertial masses in Newtonian theory. A possible generalization would be equivalence of gravitational and inertial four-momenta. In GRT this correspondence cannot be realized in mathematically rigorous manner since these notions are poorly defined and EP reduces to a purely local statement in terms of Einstein's equations.

What about TGD? What could EP mean in TGD framework?

1. Is EP realized at both quantum and space-time level? This option requires the identification of inertial and gravitational four-momenta at both quantum and classical level. It is now clear that at classical level EP follows from very simple assumption that GRT space-time is obtained by lumping together the space-time sheets of the many-sheeted space-time and by the identification the effective metric as sum of M^4 metric and deviations of the induced metrics of space-time sheets from M^2 metric: the deviations indeed define the gravitational field defined by multiply topologically condensed test particle. Similar description applies to gauge fields. EP as expressed by Einstein's equations would follow from Poincare invariance at microscopic level defined by TGD space-time. The effective fields have as sources the energy momentum tensor and YM currents defined by topological inhomogenities smaller than the resolution scale.

2. QCC would require the identification of quantal and classical counterparts of both gravitational and inertial four-momenta. This would give three independent equivalences, say $P_{I,class} = P_{I,quant}$, $P_{gr,class} = P_{gr,quant}$, $P_{gr,class} = P_{I,quant}$, which imply the remaining ones.

   Consider the condition $P_{gr,class} = P_{I,class}$. At classical level the condition that the standard energy momentum tensor associated with Kähler action has a vanishing divergence is guaranteed if Einstein's equations with cosmological term are satisfied. If preferred extremals satisfy this condition they are constant curvature spaces for non-vanishing cosmological constant. A more general solution ansatz involves several functions analogous to cosmological constant corresponding to the decomposition of energy momentum tensor to terms proportional to Einstein tensor and several lower-dimensional projection operators [K16]. It must be emphasized that field equations are extremely non-linear and one must also consider preferred extremals (which could be identified in terms of space-time regions having so called Hamilton-Jacobi structure): hence these proposals are guesses motivated by what is known about exact solutions of field equations.

Consider next $P_{gr,class} = P_{I,class}$. At quantum level I have proposed coset representations for the pair of super conformal algebras g and $h \subset g$ which correspond to the coset space decomposition of a given sector of WCW with constant values of zero modes. The coset construction would state that the differences of super-Virasoro generators associated with g resp. h annhilate physical states.

The identification of the algebras g and h is not straightforward. The algebra g could be formed by the direct sum of super-symplectic and super Kac-Moody algebras and its sub-algebra h for which the generators vanish at partonic 2-surface considered. This would correspond to the idea about WCW as a coset space G/H of corresponding groups (consider as a model $CP_2 = SU(3)/U(2)$ with $U(2)$ leaving preferred point invariant). The sub-algebra h in question includes or equals to the algebra of Kac-Moody generators vanishing at the partonic 2-surface. A natural choice for the preferred WCW point would be as maximum of Kähler function in Euclidian regions: positive definiteness of Kähler function allows only single maximum for fixed values of zero modes). Coset construction states that differences of super Virasoro generators associated with g and h annihilate physical states. This implies that corresponding four-momenta are identical that is Equivalence Principle.

3. Does EP at quantum level reduce to one aspect of QCC? This would require that classical Noether four-momentum identified as inertial momentum equals to the quantal four-momentum assignable to the states of super-conformal representations and identifiable as gravitational four-momentum. There would be only one independent condition: $P_{class} \equiv P_{I,class} = P_{gr,quant} \equiv P_{quant}$.

   Holography realized as AdS/CFT correspondence states the equivalence of descriptions in terms of gravitation realized in terms of strings in 10-D space-time and gauge fields at the boundary of AdS. What is disturbing is that this picture is not completely equivalent with the proposed one. In this case the super-conformal algebra would be direct sum of super-symplectic and super Kac-Moody parts.

Which of the options looks more plausible? The success of p-adic mass calculations [K15] have motivated the use of them as a guideline in attempts to understand TGD. The basic outcome was that elementary particle spectrum can be understood if Super Virasoro algebra has five tensor factors. Can one decide the fate of the two approaches to EP using this number as an input?

This is not the case. For both options the number of tensor factors is five as required. Four tensor factors come from Super Kac-Moody and correspond to translational Kac-Moody type degrees of freedom in M^4, to color degrees of freedom and to electroweak degrees of freedom ($SU(2) \times U(1)$). One tensor factor comes from the symplectic degrees of freedom in $\Delta CD \times CP_2$ (note that Hamiltonians include also products of δCD and CP_2 Hamiltonians so that one does not have direct sum!).

The reduction of EP to the coset structure of WCW sectors is extremely beautiful property. But also the reduction of EP to QCC looks very nice and deep. It is of course possible that the two realizations of EP are equivalent and the natural conjecture is that this is the case.

For QCC option the GRT inspired interpretation of Equivalence Principle at space-time level remains to be understood. Is it needed at all? The condition that the energy momentum tensor of Kähler action has a vanishing divergence leads in General Relativity to Einstein equations with cosmological term. In TGD framework preferred extremals satisfying the analogs of Einstein's equations with several cosmological constant like parameters can be considered.

Should one give up this idea, which indeed might be wrong? Could the divergence of of energy momentum tensor vanish only asymptotically as was the original proposal? Or should one try to generalize the interpretation? QCC states that quantum physics has classical correlate at space-time level and implies EP. Could also quantum classical correspondence itself have a correlate at space-time level. If so, space-time surface would able to represent abstractions as statements about statements about.... as the many-sheeted structure and the vision about TGD physics as analog of Turing machine able to mimic any other Turing machine suggest.

5.4 TGD-GRT correspondence and Equivalence Principle

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see fig. `http://www.tgdtheory.fi/appfigures/fieldsuperpose.jpg` or fig. 11 in the appendix of this book).

2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

4. The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore [K16].

5.5 How translations are represented at the level of WCW?

The four-momentum components appearing in the formulas of super conformal generators correspond to infinitesimal translations. In TGD framework one must be able to identify these infinitesimal translations precisely. As a matter of fact, finite measurement resolution implies that it is probably too much to assume infinitesimal translations. Rather, finite exponentials of translation generators are involved and translations are discretized. This does not have practical signficance since for optimal resolution the discretization step is about CP_2 length scale.

Where and how do these translations act at the level of WCW? ZEO provides a possible answer to this question.

5.5.1 Discrete Lorentz transformations and time translations act in the space of CDs: inertial four-momentum

Quantum state corresponds also to wave function in moduli space of CDs. The moduli space is obtained from given CD by making all boosts for its non-fixed boundary: boosts correspond to a discrete subgroup of Lorentz group and define a lattice-like structure at the hyperboloid for which proper time distance from the second tip of CD is fixed to $T_n = n \times T(CP_2)$. The quantization of cosmic redshift for which there is evidence, could relate to this lattice generalizing ordinary 3-D lattices from Euclidian to hyperbolic space by replacing translations with boosts (velocities).

The additional degree of freedom comes from the fact that the integer $n > 0$ obtains all positive values. One has wave functions in the moduli space defined as a pile of these lattices defined at the hyperboloid with constant value of $T(CP_2)$: one can say that the points of this pile of lattices correspond to Lorentz boosts and scalings of CDs defining sub-WCW:s.

The interpretation in terms of group which is product of the group of shifts $T_n(CP_2) \rightarrow T_{n+m}(CP_2)$ and discrete Lorentz boosts is natural. This group has same Cartesian product structure as Galilean group of Newtonian mechanics. This would give a discrete rest energy and by Lorentz boosts discrete set of four-momenta giving a contribution to the four-momentum appearing in the super-conformal representation.

What is important that each state function reduction would mean localisation of either boundary of CD (that is its tip). This localization is analogous to the localization of particle in position measurement in E^3 but now discrete Lorentz boosts and discrete translations $T_n - - > T_{n+m}$ replace translations. Since the second end of CD is necessary del-ocalized in moduli space, one has kind of flip-flop: localization at second end implies de-localization at the second end. Could the localization of the second end (tip) of CD in moduli space correspond to our experience that momentum and position can be measured simultaneously? This apparent classicality would be an illusion made possible by ZEO.

The flip-flop character of state function reduction process implies also the alternation of the direction of the thermodynamical time: the asymmetry between the two ends of CDs would induce the quantum arrow of time. This picture also allows to understand what the experience growth of geometric time means in terms of CDs.

5.5.2 The action of translations at space-time sheets

The action of imbedding space translations on space-time surfaces possibly becoming trivial at partonic 2-surfaces or reducing to action at δCD induces action on space-time sheet which becomes ordinary translation far enough from end end of space-time surface. The four-momentum in question is very naturally that associated with Kähler action and would therefore correspond to inertial momentum for $P_{I,class} = P_{quant,gr}$ option. Indeed, one cannot assign quantal four-momentum to Kähler action as an operator since canonical quantization badly fails. In finite measurement infinitesimal translations are replaced with their exponentials for $P_{I,class} = P_{quant,gr}$ option.

What looks like a problem is that ordinary translations in the general case lead out from given CD near its boundaries. In the interior one expects that the translation acts like ordinary translation. The Lie-algebra structure of Poincare algebra including sums of translation generators with positive coefficient for time translation is preserved if only time-like superpositions if generators are allowed also the commutators of time-like translation generators with boost generators give time like translations. This defines a Lie-algebraic formulation for the arrow of geometric time. The action of time translation on preferred extremal would be ordinary translation plus continuation of the translated preferred extremal backwards in time to the boundary of CD. The transversal space-like translations could be made Kac-Moody algebra by multiplying them with functions which vanish at δCD.

A possible interpretation would be that $P_{quant,gr}$ corresponds to the momentum assignable to the moduli degrees of freedom and $P_{cl,I}$ to that assignable to the time like translations. $P_{quant,gr} = P_{cl,I}$ would code for QCC. Geometrically quantum classical correspondence would state that time-like translation shift both the interior of space-time surface and second boundary of CD to the geometric future/past while keeping the second boundary of space-time surface and CD fixed.

5.6 Yangian and four-momentum

Yangian symmetry implies the marvellous results of twistor Grassmannian approach to $\mathcal{N} = 4$ SUSY culminating in the notion of amplituhedron which promises to give a nice projective geometry interpretation for the scattering amplitudes [?]. Yangian symmetry is a multilocal generalization of ordinary symmetry based on the notion of co-product and implies that Lie algebra generates receive also multilocal contributions. I have discussed these topics from slightly different point of view in [K11], where also references to the work of pioneers can be found.

5.6.1 Yangian symmetry

The notion equivalent to that of Yangian was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras. The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [K11] . Besides ordinary product in the enveloping algebra there is co-product Δ which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant and and concrete manner in the article *Yangian Symmetry in D=4 superconformal Yang-Mills theory* [?] . Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with discrete index n replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N} = 4$ SUSY). One of the conditions conditions is that the tensor product $R \otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n = 0$ and $n = 1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n = 1$ generators with themselves are however something different for a non-vanishing deformation parameter h. Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h = 0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n > 0$ are $n + 1$-local in the sense that they involve $n + 1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

5.6.2 How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, it is not much to say. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

1. The first thing to notice is that the Yangian symmetry of $\mathcal{N} = 4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [A2] and Virasoro algebras [A5] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras. In TGD framework conformal algebra of Minkowski space reduces to Poincare algebra and its extension to Kac-Moody allows to have also massive states.
2. The formal generalization looks surprisingly straightforward at the formal level. In zero energy ontology one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($CD \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones. The polygon with light-like momenta is naturally replaced with a polygon with more general momenta in zero energy ontology and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.
3. This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $CD \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely (if this word can be used in this kind of context)?

1. At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups. This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro

algebras contains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N} = 4$ SUSY would allow only the adjoint.

2. Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of the partonic 2-surface. This picture also justifies p-adic thermodynamics applied to either symplectic or isometry Super-Virasoro and giving thermal contribution to the vacuum conformal and thus to mass squared.

3. The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.

4. Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

5.6.3 Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute. Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

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