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Relations of Energy Density and Its Entropic Density with Temperature concerning Schwarzschild Black Hole and Holographic Dark Energy

Yong Bao

Postbox 777, 100 Renmin South Road, Luoding 527200, Guangdong, China
E-mail: baoyong9803@163.com

We consider the relations of the energy density and its entropic density with the temperature of Schwarzschild black hole (SBH) and holographic dark energy (HDE). On the basis of literature [4], we obtain the energy density of the scalar field being proportional to the square of the Hawking-Unruh temperature near the event horizon, and that its entropic density is directly proportional to the Hawking-Unruh one. Basing on [6], we find the equation of gravitational energy density inside SBH; derive that the gravitational energy density is proportional to the square of the effective temperature far from the event horizon in SBH interior, whether the gravitational fields of SBH are Coulomb-like or wave-like; and their entropic density is directly proportional to the effective temperature. Basing on [7, 8, 9], we gain the HDE density being proportional to square of the Gibbons-Hawking temperature, and that its entropic density is directly proportional to the Gibbons-Hawking one. These equations are similar and have relations with each other. We suggest that these relations are interesting and significant for SBH and HDE.

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I. Introduction

What is the relation of the energy density and its entropic density with the temperature of Schwarzschild black hole (SBH) and holographic dark energy (HDE)? It is an interesting and significant question. At the event horizon of SBH, the Stefan-Boltzmann law (SBL) [1, 2] can be modified as the General SBL by Meng Q M *et al* [3, 4] and A. I. Fisenko *et al* [5]. In SBH interior, T. Clifton *et al* proposed the gravitational energy density formula and the effective temperature expression respectively [6], but they are irrelative. M. Li *et al* proposed the HDE density equation [7, 8, 9] which has no relation with the temperature.

This paper is organized as follows. In Sec. II, we get the energy density of the scalar field being proportional to the square of the Hawking-Unruh temperature [10, 11] near the horizon of SBH, and that its entropic density is directly proportional to the Hawking-Unruh one. In Sec. III, we find that the gravitational energy density is proportional to the square of the effective temperature far from the horizon inside SBH, and its entropic density is directly proportional to the effective temperature. In Sec. IV, we obtain the HDE density being proportional to square of the Gibbons-Hawking temperature [12, 13, 14, 15], and that its entropic density is directly proportional to the Gibbons-Hawking one. We conclude in Sec. V.

II. Relations of Energy Density of scalar field and Its Entropic Density with Hawking-Unruh Temperature near Event Horizon

In this section, we review [4]; obtain that the energy density of the scalar field is proportional to the square of the Hawking-Unruh temperature near the event horizon of SBH, and the entropic density is directly proportional to the Hawking-Unruh one.

A. Energy density being proportional to square of Hawking-Unruh temperature

In [4], the Schwarzschild coordinates is used, and the energy density ρ_{sf} of the scalar field of SBH is (we work with ħ = c = G = k = 1 units)

$$\rho_{sf} = 6\pi^2 M^2 T_{H-U}^4 \,/\, 45\varepsilon(\varepsilon+\delta) \tag{1}$$

Where δ = nε is the film thickness, ε is the distance from the film to the horizon of SBH, and n = 1, 2, 3….

Substituting the Hawking-Unruh temperature T_{H-U} = 1 / 8πM into (1), we obtain

$$\rho_{sf} = \pi T_{H-U}^2 / 60\varepsilon^2(\mathrm{n}+1) \tag{2}$$

Therefore the energy density of the scalar field is proportional to the square of Hawking-Unruh temperature near the horizon. Recovering ħ, c, G and k, we obtain

$$\rho_{sf} = \pi \mathrm{c}^2\mathrm{k}^2 T_{H-U}^2 / 60\varepsilon^2(\mathrm{n}+1)\hbar^2\mathrm{G} \tag{3}$$

So it belongs to quantum gravity.

B. Entropic density being directly proportional to Hawking-Unruh temperature

The entropic density s_{sf} of the scalar field near the horizon is [4]

$$s_{sf} = 8\pi^2 M^2 / 45\beta^3\varepsilon(\varepsilon+\delta) \tag{4}$$

where β is [4]

$$\beta = 8\pi M \tag{5}$$

Substituting (5) and T_{H-U} = 1 / 8πM into (4), we obtain

$$s_{sf} = T_{H-U} / 360\varepsilon^2(\mathrm{n}+1) \tag{6}$$

Therefore the entropic density is directly proportional to the Hawking-Unruh temperature.

III. Relations of Gravitational Energy Density and Its Entropic Density with effective temperature far from Event Horizon

In this section, we review [6] briefly; find that the gravitational energy density is proportional to the square of the effective temperature far from the event horizon inside SBH, and its entropic density is directly proportional to the effective one.

A. Relations for Coulomb-like gravitational fields

First let us review [6] briefly. The gravitational fields can be classified two types: Coulomb-like gravitational fields and wave-like ones. In general they are mixed. For the Coulomb-like gravitational fields

$$8\pi\rho_{grav} = 2\alpha\sqrt{2W/3} \ \text{ and } \ p_{grav} = 0 \tag{7}$$

where ρ_{grav} is the gravitational energy density, α is a constant, $W = T_{abcd}u^a u^b u^c u^d$, T_{abcd} is the Weyl tensor, u^a, u^b, u^c, u^d are the timelike unit vectors, and p_{grav} is the isotropic pressure. The Schwarzschild geometry can be written in Gullstrand–Painlevé coordinates as

$$\mathrm{d}s^2 = -[1-(2m / r)]\mathrm{d}t^2 - 2\sqrt{2m/r}\,\mathrm{d}r\mathrm{d}t + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2 \tag{8}$$

where m is the constant mass parameter. The gravitational energy density and temperature is given at each point in the region $r < 2m$ by

$$\rho_{grav} = 2\alpha m / 8\pi r^3 \tag{9}$$

$$T_{grav} = m / 2\pi r^2\sqrt{|1-(2m/\mathrm{r})|} \tag{10}$$

where T_{grav} is the effective temperature. Taking (10) to (9), we find

$$\rho_{grav} = \alpha\pi[2-(r / m)]T_{grav}^2 \tag{11}$$

It is the equation concerning the gravitational energy density and the effective temperature in the region r < 2m. Note that the isotropic pressure is zero. When $r << 2m$, we derive

$$\rho_{grav} = 2\alpha\pi T_{grav}^2 \tag{12}$$

So the gravitational energy density is proportional to the square of the effective temperature far from the horizon inside SBH. It includes two regions far from horizon inside SBH: Singularity and vacuum. In (11) when $r \rightarrow 0$, we gain (12) also, that is the gravitational energy density being proportional to the square of the effective temperature in the singularity and near the one.

B. Relations for the wave-like gravitational fields

In [6], for the wave-like gravitational fields

$$8\pi\rho_{grav} = \beta\sqrt{4W} \ \text{ and } \ p_{grav} = \rho_{grav} / 3 \tag{13}$$

For the SBH, the gravitational energy density is given at each point in region r < 2m by

$$\rho_{grav} = \sqrt{6}\beta m / r^3 \tag{14}$$

Taking (10) to (14), we find

$$\rho_{grav} = \sqrt{6}\beta\pi[2-(r / m)]T_{grav}^2 \tag{15}$$

When $r << 2m$ and $r \rightarrow 0$, we obtain

$$\rho_{grav} = 2\sqrt{6}\beta\pi T_{grav}^2 \tag{16}$$

It is very similar to (12). Therefore the gravitational energy density is proportional to the square of the effective temperature far from the horizon inside SBH, whether the gravitational fields are Coulomb-like or wave-like.

C. Relations of entropic density

For the Coulomb-like gravitational fields, its entropic density s_{grav} is [6]

$$\delta s_{grav} = \delta(\rho_{grav} v) / T_{grav} \tag{17}$$

where $v = z^a\eta_{abcd}\mathrm{d}x^b\mathrm{d}x^c\mathrm{d}x^d$ and we can set an arbitrary constant to zero. Substituting (11) into (17) and integral, we get

$$s_{grav} = \alpha\pi[2-(r / m)]T_{grav} \tag{18}$$

This is the equation concerning the entropic density and the effective temperature in the region r < $2m$. When $r << 2m$ and $r \rightarrow 0$, we derive

$$s_{grav} = 2\alpha\pi T_{grav} \tag{19}$$

So the entropic density is directly proportional to the effective temperature far from the horizon inside SBH.

For the wave-like gravitational fields, substituting (15) into (17) and integral, we obtain

$$s_{grav} = \sqrt{6}\beta\pi[2-(r / m)]T_{grav} \tag{20}$$

When $r << 2m$ and $r \rightarrow 0$, we derive

$$s_{grav} = 2\sqrt{6}\beta\pi T_{grav} \tag{21}$$

Therefore the entropic density is directly proportional to the effective temperature far from the horizon inside SBH, whether the gravitational fields are Coulomb-like or wave-like.

IV. Relations of HDE Density and Its Entropic Density with Gibbons-Hawking Temperature

In this section, we review [7, 8, 9], obtain that the HDE density is proportional to square of the Gibbons-Hawking temperature, and its entropic density is directly proportional to the Gibbons-Hawking one.

A. HDE density being proportional to square of Gibbons-Hawking temperature

In [7, 8, 9], the equation of HDE model can be rewritten as

$$\rho_{de} = 3c_L^2\mathrm{M}_{\mathrm{pl}}^2L^{-2} \tag{22}$$

where ρ_{de} is the HDE density, $c_L \geq 0$ is a dimensionless model parameter, $\mathrm{M}_{\mathrm{Pl}} \equiv 1 / \sqrt{8\pi\mathrm{G}}$ is the reduced Planck mass and L is the cosmic cutoff.

Substituting the Gibbons-Hawking temperature $T_{G-H} = 1 / 2\pi L$ into (22), we gain

$$\rho_{de} = 12\pi^2c_L^2\mathrm{M}_{\mathrm{pl}}^2T_{G-H}^2 = 3\pi c_L^2\mathrm{M}_{\mathrm{p}}^2T_{G-H}^2 / 2 \tag{23}$$

where $\mathrm{M}_{\mathrm{P}} \equiv 1 / \sqrt{\mathrm{G}}$ is the Planck mass. So the holographic dark energy density is proportionate to square of the Gibbons-Hawking temperature.

B. HDE entropic density being directly proportional to Gibbons-Hawking temperature

In [9], the HDE entropy S_{de} is

$$S_{de} = \pi\mathrm{M}_{\mathrm{pl}}^2L^2 \tag{24}$$

The HDE entropic density s_{de} is

$$s_{de} = \pi\mathrm{M}_{\mathrm{pl}}^2L^2 / L^3 = \pi\mathrm{M}_{\mathrm{pl}}^2 / L = 2\pi^2\mathrm{M}_{\mathrm{pl}}^2T_{G-H} \tag{25}$$

Therefore the HDE entropic density is directly proportional to the Gibbons-Hawking temperature.

V. Conclusion

In this paper, we have obtained the energy density of the scalar field [4] being proportional to the square of the Hawking-Unruh temperature near the horizon, and that the entropic density is directly proportional to the Hawking-Unruh one; found the equation concerning the gravitational energy density and the effective temperature in the region $r < 2m$ [6] inside SBH; derived that the gravitational energy density is proportional to the square of the effective temperature far from the horizon in SBH interior, whether the gravitational fields of SBH are Coulomb-like or wave-like; obtained that their entropic density are directly proportional to the effective one. These equations are true in the singularity and near one of SBH also. Also we have got that the HDE density [7] is proportional to square of the Gibbons-Hawking temperature, and its entropic density is directly proportional to the Gibbons- Hawking one.

Eq. (2), (12), (16), and (23) are similar. Eq. (2) belongs to the Hawking radiation; (12) and (16) belong to the quantum gravity. It is well-known that the Hawking radiation is produced by the quantum gravity, so Eq. (2) has the relation with (12) and (16). Eq. (12), (16) and (23) are similar also. Eq. (23) belongs to the dark energy which can produce the repulsion; therefore it has the relation with (12) and (16). Their entropic density is also. We suggest that these relations are interesting and significant for SBH and HDE.

References

[1] J. Stefan, Wien. Ber., **79** (1879), 391

[2] L. E. Boltzmann, Wied. Ann. Phys., **22** (1884), 291

[3] Meng Q M, Jiang J J, Liu J L, *et al*, Physical Journal, 2009, **58** (1): 78-82

[4] DENG De-Li, Journal of Guizhou University (Natural Sciences), 27, **4** (2010), 1000-5269(2010)04-0022-03

[5] A. I. Fisenko, V. Lemberg, arXiv: 1403.6868

[6] T. Clifton, G. F. R. Ellis and R. Tavakol, Class. Quantum Grav.**30** (2013) 125009 (15pp)

[7] M. Li, Phys.Lett. B, **603** (2004) 1; arXiv: 0403127 [hep-th]

[8] K.Ke, M. Li, Phys. Lett. B **606** (2005) 173-176

[9] Q-G. Huang and M. Li, arXiv: 0404229 [astro-ph]

[10] S.W. Hawking, Nature, **248**, (1974) 30; Commun. Math. Phys. **43** (1975)199-220

[11] W.G. Unruh, Phys.Rev.D**10**, (1974) 3194

[12] G.W. Gibbons and S.W. Hawking, Phys. Rev. D **15**, 2738 (1977)

[13] J.W. Lee, J. Lee, and H.C. Kim, J. Cosmol. Astropart. Phys. **0708** (2007) 005

[14]LI Miao, LI Xiao-Dong, WANG Shuang, and WANG Yi, Commun. Theor. Phys. **56** (2011) 525–604

[15] S. Mohanty and A. Nautiyal, arXiv: 1404.2222 [hep-ph]