On the Luminosity Distance and the Hubble constant (Revised)

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Abstract

By differentiating the standard formula for the luminosity distance with respect to time, we find that the equation is inconsistent with light propagation. Therefore, a new definition of the luminosity distance is provided for an expanding Universe. From supernovae observations, using this definition we find that the Hubble parameter is a constant of physics equal to $H_0 = 63.2\, km\, s^{-1}\, Mpc^{-1}$.

I. INTRODUCTION

This is a revision of a manuscript [1]. The luminosity distance is an important concept in cosmology, as this is the distance measure obtained from supernovae data using the distance modulus. The standard formula of the luminosity distance is $d_L = (1+z)\, d_M = \frac{d_M}{a}$, where d_L is the luminosity distance and d_M the comoving transverse distance [2]. As shown below, this definition implies that the peculiar velocity is a time-varying velocity of light and the expansion term has a negative sign; therefore a new definition is proposed.

II. DIFFERENTIATING THE STANDARD LUMINOSITY DISTANCE EQUATION

From there we will use the notation r_L for the luminosity distance as it represents the radius of a sphere for light propagating from the center point of emission of the light source. The standard formula of the luminosity distance for a flat Universe is as follows:

$$r_L = \frac{\chi}{a}\,, \tag{1}$$

and

$$\chi = c \int_0^{t_0} \frac{dt}{a}\,, \tag{2}$$

where r_L is the luminosity distance, χ the comoving distance, a the scale factor at the time of emission, t the time equal to zero at the origin set at the center of the sphere from which light is emitted, and t_0 the time when light reaches the earth.

By differentiating (1) with respect to t we get:

$$\frac{dr_L}{dt} = \frac{\dot{\chi}}{a} - \frac{\dot{a}}{a^2}\,\chi\,. \tag{3}$$

As $I = \int_{t_1}^{t_2} f(t)\, dt$ leads to $\frac{dI}{dt} = \frac{dt_2}{dt}\, f(t_2) - \frac{dt_1}{dt}\, f(t_1)$, from (2) we get:

$$\dot{\chi} = \frac{c}{a}\,. \tag{4}$$

Using (1) we get:

$$\frac{\dot{a}}{a^2}\,\chi = \frac{\dot{a}}{a}\,r_L\,. \tag{5}$$

Because $H = \frac{1}{a}\frac{da}{dt}$, equation (5) can be rewritten as follows:

$$\frac{\dot{a}}{a^2}\,\chi = H\,r_L\,. \tag{6}$$

Combining (3), (4), and (6) we get:

$$\frac{dr_L}{dt} = \frac{c}{a^2} - H\,r_L\,. \tag{7}$$

The term $H\,r_L$ represents the expansion for the radius of our sphere, and $\frac{c}{a^2}$ is the peculiar velocity. We see that (7) is not consistent with light propagation.

A new equation is proposed for the luminosity distance where the peculiar velocity is always equal to c. Considering a sphere of radius r'_L for the propagation of light emitted from a point at the center, and that the sphere inflates over time due to the expansion of the Universe and the velocity of light, we obtain:

$$\frac{dr'_L}{dt} = c + H\,r'_L\,, \tag{8}$$

with boundary condition $r'_L = 0$ at $t = 0$, r'_L is the luminosity distance, t the time between emission and reception of the light source, and H the Hubble constant at time t.

III. SOLVING OUR NEW EQUATION FOR THE LUMINOSITY DISTANCE

In this section we assume that the Hubble constant does not vary over time and is always equal to H_o.

By integrating (8) between 0 and T, we get:

$$r'_L = \frac{c}{H_0}\left(\exp(H_0\,T) - 1\right)\,. \tag{9}$$

where T is the light travel time between the earth and the emission point of the supernovae.

This equation can be rewritten as follows:

$$T = \frac{1}{H_0}\,\ln\left(1 + \frac{H_0}{c}\,r'_L\right)\,. \tag{10}$$

Because $\frac{da}{dt} = H a$, and given the cosmological redshift equation $(1+z) = \frac{1}{a}$, the expression of the light travel time versus redshift is as follows:

$$T = \int_{1/(1+z)}^{1} \frac{da}{H\,a} = \frac{1}{H_0}\,\ln(1+z)\,. \tag{11}$$

where a is the scale factor.

By combining (10) and (11) we get:

$$r'_L = \frac{c}{H_0}\,z\,. \tag{12}$$

IV. CALCULATION OF THE HUBBLE CONSTANT FROM SUPERNOVAE DATA

Let us compute the Hubble constant from supernovae using the relationship in (12) . In order to compute the luminosity distance we use the redshift-adjusted distance modulus provided in [3] which is as follows:

$$m - M = -5 + 5\log r'_L + 2.5\log(1+z)\,. \quad (13)$$

The distance modulus $\mu = m - M$ is the difference between the apparent magnitude m and the absolute magnitude M.

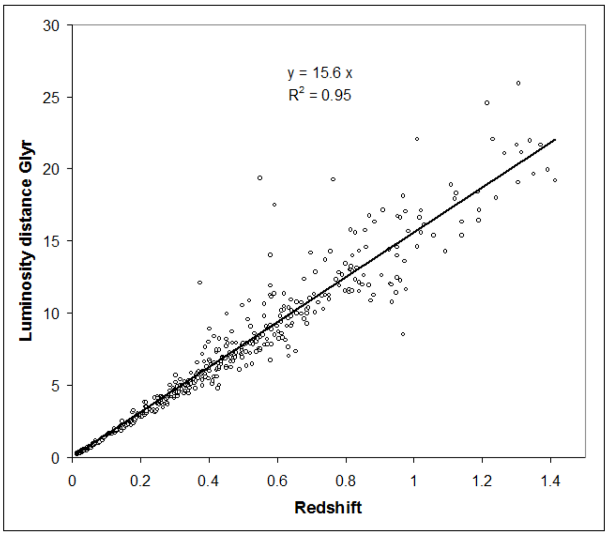

FIG. 1. Luminosity distance in Glyr versus redshift plot for supernovae. Data source: http://supernova.lbl.gov/Union/

In Fig. 1 we have a plot of the luminosity distance versus redshift that was obtained with (13) using supernovae data. This plot is rectilinear with a slope of 15.6 where the luminosity distance is expressed in $Glyr$ (billion light years). The Hubble constant, which is the inverse of the slope from (12), is equal to $H_0 = 63.2\,km\,s^{-1}\,Mpc^{-1}$.

V. CONCLUSION

This study show that the standard formula of the luminosity distance implies that the peculiar velocity is a time-varying velocity of light and the Hubble expansion has a negative sign. Given our choice for the luminosity distance equation, we find that the solution to this equation requires a Hubble parameter that does not change over time in order to fit the supernovae data.

[1] Y. Heymann, Progress in Physics **3**, 5 (2013).

[2] S. Weinberg, “Gravitation and cosmology: Principles and applications of the general theory of relativity,” (John Wiley and Sons, 1972) p. 421.

[3] Y. Heymann, Progress in Physics **1**, 6 (2012).