

Homology Classes of Generalised Triangulations Made up of a Small Number of Simplexes

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Abstract

By means of a computer, all the possible homogeneous compact Generalised Triangulations made up of a small number of 3-simplexes (from 1 to 3) have been classified in homology classes. The analysis shows that, with a small number of simplexes, it is already possible to build quite a large number of separate topological spaces.

Key Words: topology, generalised triangulation, homology.

1 Introduction

Given a number T of 3-simplexes, having $n = 4 \cdot T$ faces, it is possible to build several compact topological spaces by identifying the n faces in couples.

There are:

$$(n - 1) \cdot (n - 3) \cdot \dots \cdot 1 \quad (1)$$

different combinations possible for identifying faces and, for each couple of faces, there are 6 different orientations for the identification.

In total, all the possible compact Generalised Triangulations that is possible to build up with a number T of 3-simplexes (i.e. $4 \cdot T$ faces) are:

$$(n - 1) \cdot (n - 3) \cdot \dots \cdot 1 \cdot 6^{\frac{n}{2}} \quad (2)$$

where the above equation takes already into account the most obvious symmetries.

For example, with 1 tetrahedron is possible to build 108 different compact Generalised Triangulations, with 2 tetrahedra is possible to build 136080 compact Generalised Triangulations and so on. Note that some of the combinations lead to non feasible triangulations meaning that, given the instruction for identifying faces, it is simply not possible to build a space that makes sense.

By means of a computer we have evaluated the homology groups of all the possible Generalised Triangulations made up of T simplexes (with T going from 1 to 3) and we have classified them in classes of triangulations having the same homology groups.

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The homology groups have been evaluated from the boundary maps ∂_i expressed as a matrix D_i of integers. In particular the rank of the homology group H_i has been evaluated from the ranks of D_{i+1}, D_i , and the chain space dimensions while the torsion part of the group H_i has been red out from the boundary map D_{i+1} expressed in Smith Normal Form. The results are reported below.

Type of Triang.	Num. of Triang.	Note
Non feasible triangulations	69	
Path connected triangulations	39	in 8 homology classes
Non path connected triangulations	0	
Total	108	

Table 1 : Triangulations made of 1 simplex

Type of Triang.	Num. of Triang.	Note
Non feasible triangulations	98991	
Path connected triangulations	35568	in 50 homology classes
Non path connected triangulations	1521	in 30 homology classes
Total	136080	

Table 2 : Triangulations made of 2 simplexes

Type of Triang.	Num. of Triang.	Note
Non feasible triangulations	366924249	
Path connected triangulations	113844096	in 234 homology classes
Non path connected triangulations	4220775	in 340 homology classes
Total	484989120	

Table 3 : Triangulations made of 3 simplexes

2 Spaces made of 1 simplex - sum up table

The following table sums up the homology classes of all path connected compact homogeneous Generalised Triangulations made up of only 1 tetrahedra.

n	H_0	H_1	H_2	H_3	χ	Num. of Triang.
1	\mathbb{Z}	0	0	\mathbb{Z}	0	20
2	\mathbb{Z}	\mathbb{Z}_4	0	\mathbb{Z}	0	5
3	\mathbb{Z}	\mathbb{Z}_5	0	\mathbb{Z}	0	5
4	\mathbb{Z}	0	\mathbb{Z}	\mathbb{Z}	1	3
5	\mathbb{Z}	0	0	0	1	3
6	\mathbb{Z}	0	\mathbb{Z}	0	2	1
7	\mathbb{Z}	\mathbb{Z}_4	0	0	1	1
8	\mathbb{Z}	\mathbb{Z}_5	0	0	1	1

Table 4 : Homology classes (spaces made of 1 simplex)

3 Spaces made of 2 simplexes - sum up table

The following table sums up the homology classes of of all path connected compact homogeneous Generalised Triangulations made up of 2 tetrahedra.

n	H_0	H_1	H_2	H_3	χ	Num. of Triang.
1	\mathbb{Z}	0	\mathbb{Z}	\mathbb{Z}^2	0	11359
2	\mathbb{Z}	0	0	\mathbb{Z}^2	-1	6473
3	\mathbb{Z}	0	0	\mathbb{Z}	0	5621
4	\mathbb{Z}	0	\mathbb{Z}	\mathbb{Z}	1	3226
5	\mathbb{Z}	\mathbb{Z}_3	0	\mathbb{Z}	0	1765
6	\mathbb{Z}	0	0	0	1	1595
7	\mathbb{Z}	0	\mathbb{Z}	0	2	903
8	\mathbb{Z}	\mathbb{Z}_2	0	\mathbb{Z}	0	732
9	\mathbb{Z}	\mathbb{Z}_3	0	0	1	578
10	\mathbb{Z}	0	\mathbb{Z}^2	\mathbb{Z}^2	1	440
11	\mathbb{Z}	\mathbb{Z}	0	\mathbb{Z}	-1	396
12	\mathbb{Z}	\mathbb{Z}_7	0	\mathbb{Z}	0	364
13	\mathbb{Z}	\mathbb{Z}_2	0	0	1	358
14	\mathbb{Z}	\mathbb{Z}_5	0	\mathbb{Z}	0	212
15	\mathbb{Z}	\mathbb{Z}_8	0	\mathbb{Z}	0	180
16	\mathbb{Z}	\mathbb{Z}_7	0	0	1	165
17	\mathbb{Z}	0	\mathbb{Z}^2	0	3	154
18	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}	0	2	142
19	\mathbb{Z}	0	\mathbb{Z}_2	0	1	139
20	\mathbb{Z}	\mathbb{Z}	0	0	0	130
21	\mathbb{Z}	\mathbb{Z}_5	0	\mathbb{Z}^2	-1	114
22	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}	0	2	94
23	\mathbb{Z}	\mathbb{Z}_8	0	0	1	87
24	\mathbb{Z}	\mathbb{Z}_5	0	0	1	83
25	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}	0	2	40

Table 5 : Homology classes (spaces made of 2 simplexes) n:1-25

n	H_0	H_1	H_2	H_3	χ	Num. of Triang.
26	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}	0	1	32
27	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}	\mathbb{Z}	1	25
28	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	\mathbb{Z}	0	22
29	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	0	1	21
30	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}	0	2	20
31	\mathbb{Z}	\mathbb{Z}_8	\mathbb{Z}	0	2	14
32	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}	0	12
33	\mathbb{Z}	0	\mathbb{Z}^3	0	4	11
34	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^2	0	3	11
35	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}	\mathbb{Z}	1	8
36	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}	\mathbb{Z}	1	5
37	\mathbb{Z}	0	\mathbb{Z}^2	\mathbb{Z}	2	4
38	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_2	0	1	4
39	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}	0	2	4
40	\mathbb{Z}	\mathbb{Z}_8	\mathbb{Z}	\mathbb{Z}	1	4
41	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^2	0	2	4
42	\mathbb{Z}	0	\mathbb{Z}^4	0	5	3
43	\mathbb{Z}	\mathbb{Z}_8	\mathbb{Z}^2	0	3	3
44	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^2	0	3	2
45	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}	\mathbb{Z}	1	2
46	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}^2	0	3	2
47	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}_2	0	0	2
48	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}^2	0	3	1
49	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^3	0	4	1
50	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}^2	0	3	1

Table 5 : Homology classes (spaces made of 2 simplexes) n:26-50

4 Spaces made of 3 simplexes - sum up table

The following table sums up the homology classes of of all path connected compact homogeneous Generalised Triangulations made up of 3 tetrahedra.

n	H_0	H_1	H_2	H_3	χ	Num. of Triang.
1	\mathbb{Z}	0	\mathbb{Z}	\mathbb{Z}^3	-1	27794856
2	\mathbb{Z}	0	0	\mathbb{Z}^2	-1	21297916
3	\mathbb{Z}	0	\mathbb{Z}	\mathbb{Z}^2	0	14583893
4	\mathbb{Z}	0	0	\mathbb{Z}^3	-2	13190400
5	\mathbb{Z}	0	0	\mathbb{Z}	0	8256514
6	\mathbb{Z}	0	\mathbb{Z}	\mathbb{Z}	1	7289062
7	\mathbb{Z}	0	\mathbb{Z}	0	2	3337122
8	\mathbb{Z}	0	0	0	1	1952888
9	\mathbb{Z}	0	\mathbb{Z}^2	0	3	1664641
10	\mathbb{Z}	0	\mathbb{Z}^2	\mathbb{Z}	2	1573652
11	\mathbb{Z}	0	\mathbb{Z}^2	\mathbb{Z}^3	0	1464514
12	\mathbb{Z}	0	\mathbb{Z}^2	\mathbb{Z}^2	1	1350393
13	\mathbb{Z}	\mathbb{Z}_2	0	\mathbb{Z}	0	1039055
14	\mathbb{Z}	0	\mathbb{Z}_2	0	1	777592
15	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}	\mathbb{Z}	1	648812
16	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}	\mathbb{Z}^2	0	612947
17	\mathbb{Z}	\mathbb{Z}_2	0	0	1	591869
18	\mathbb{Z}	\mathbb{Z}_3	0	\mathbb{Z}	0	492526
19	\mathbb{Z}	0	$\mathbb{Z} \oplus \mathbb{Z}_2$	0	2	462966
20	\mathbb{Z}	\mathbb{Z}_3	0	\mathbb{Z}^2	-1	453676
21	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}	0	2	445639
22	\mathbb{Z}	0	\mathbb{Z}^3	0	4	417638
23	\mathbb{Z}	\mathbb{Z}_2	0	\mathbb{Z}^2	-1	369919
24	\mathbb{Z}	\mathbb{Z}_4	0	\mathbb{Z}^2	-1	338930
25	\mathbb{Z}	\mathbb{Z}_5	0	\mathbb{Z}^2	-1	295738
26	\mathbb{Z}	\mathbb{Z}_5	0	\mathbb{Z}	0	236229
27	\mathbb{Z}	\mathbb{Z}_4	0	\mathbb{Z}	0	226886
28	\mathbb{Z}	\mathbb{Z}_3	0	0	1	226760
29	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}	0	163893
30	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}	0	2	144341
31	\mathbb{Z}	\mathbb{Z}_4	0	\mathbb{Z}^3	-2	141960
32	\mathbb{Z}	\mathbb{Z}_5	0	\mathbb{Z}^3	-2	141960
33	\mathbb{Z}	0	\mathbb{Z}^3	\mathbb{Z}	3	125687
34	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^2	0	3	115262
35	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}	\mathbb{Z}	1	86689
36	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_2	0	1	86288
37	\mathbb{Z}	\mathbb{Z}_5	0	0	1	81232
38	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^2	\mathbb{Z}	2	74244
39	\mathbb{Z}	\mathbb{Z}_4	0	0	1	69779
40	\mathbb{Z}	0	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}	1	68910

Table 6 : Homology classes (spaces made of 3 simplexes) n:1-40

n	H_0	H_1	H_2	H_3	χ	Num. of Triang.
41	\mathbb{Z}	0	\mathbb{Z}^4	0	5	68511
42	\mathbb{Z}	\mathbb{Z}	0	\mathbb{Z}	-1	64013
43	\mathbb{Z}	\mathbb{Z}	0	\mathbb{Z}^2	-2	43511
44	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^2	0	3	39050
45	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}	\mathbb{Z}	1	39043
46	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}	\mathbb{Z}	1	38081
47	\mathbb{Z}	\mathbb{Z}_7	0	\mathbb{Z}	0	37748
48	\mathbb{Z}	\mathbb{Z}	0	0	0	37237
49	\mathbb{Z}	0	\mathbb{Z}_3	0	1	35734
50	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}	\mathbb{Z}^2	0	34861
51	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}_2	0	1	30964
52	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}	0	2	29680
53	\mathbb{Z}	\mathbb{Z}_7	0	0	1	28507
54	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^2	\mathbb{Z}^2	1	27856
55	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}	0	2	26523
56	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}	0	1	24640
57	\mathbb{Z}	\mathbb{Z}_6	0	\mathbb{Z}	0	23764
58	\mathbb{Z}	\mathbb{Z}_6	0	0	1	21663
59	\mathbb{Z}	\mathbb{Z}_9	0	\mathbb{Z}	0	19200
60	\mathbb{Z}	\mathbb{Z}_{11}	0	\mathbb{Z}	0	19180
61	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^3	0	4	18905
62	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}	0	2	17551
63	\mathbb{Z}	\mathbb{Z}_8	0	\mathbb{Z}	0	16746
64	\mathbb{Z}	\mathbb{Z}_8	0	0	1	16650
65	\mathbb{Z}	0	\mathbb{Z}^3	\mathbb{Z}^2	2	15037
66	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}	0	1	13680
67	\mathbb{Z}	\mathbb{Z}_{11}	0	0	1	13321
68	\mathbb{Z}	\mathbb{Z}_9	0	0	1	13235
69	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}	0	11151
70	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}	0	2	10450
71	\mathbb{Z}	\mathbb{Z}_{10}	0	\mathbb{Z}	0	9654
72	\mathbb{Z}	\mathbb{Z}_{12}	0	\mathbb{Z}	0	9536
73	\mathbb{Z}	\mathbb{Z}_{13}	0	\mathbb{Z}	0	9528
74	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}_2	0	1	9410
75	\mathbb{Z}	0	\mathbb{Z}^4	\mathbb{Z}	4	8774
76	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}_2	0	0	8684
77	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}_2	0	0	8441
78	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	0	1	8176
79	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	0	0	0	7947
80	\mathbb{Z}	0	\mathbb{Z}^5	0	6	7444

Table 6 : Homology classes (spaces made of 3 simplexes) n:41-80

n	H_0	H_1	H_2	H_3	χ	Num. of Triang.
81	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}	0	2	7416
82	\mathbb{Z}	\mathbb{Z}_3	0	\mathbb{Z}^3	-2	7360
83	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^2	0	2	7113
84	\mathbb{Z}	\mathbb{Z}_{13}	0	0	1	6729
85	\mathbb{Z}	\mathbb{Z}_{12}	0	0	1	6683
86	\mathbb{Z}	\mathbb{Z}_{10}	0	0	1	6593
87	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^3	0	4	6509
88	\mathbb{Z}	\mathbb{Z}_8	\mathbb{Z}	0	2	6474
89	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^2	-1	6414
90	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}	\mathbb{Z}^3	-1	6256
91	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}^2	0	2	6067
92	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	\mathbb{Z}	0	5606
93	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}	\mathbb{Z}^2	0	5550
94	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}	\mathbb{Z}^3	-1	5364
95	\mathbb{Z}	\mathbb{Z}_9	\mathbb{Z}	0	2	5252
96	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	5225
97	\mathbb{Z}	\mathbb{Z}_2	$\mathbb{Z} \oplus \mathbb{Z}_2$	0	2	5220
98	\mathbb{Z}	\mathbb{Z}_{11}	\mathbb{Z}	0	2	5140
99	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}^2	0	3	5058
100	\mathbb{Z}	\mathbb{Z}_2	0	\mathbb{Z}^3	-2	5052
101	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}^2	0	3	5029
102	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}^2	0	3	4826
103	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}	\mathbb{Z}^2	0	4673
104	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^3	\mathbb{Z}	3	4504
105	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^2	\mathbb{Z}	2	4097
106	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}_2	\mathbb{Z}	0	4048
107	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}	\mathbb{Z}	1	3489
108	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_3	0	1	3046
109	\mathbb{Z}	\mathbb{Z}_{10}	\mathbb{Z}	0	2	2655
110	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}^2	0	3	2624
111	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}_2	0	1	2597
112	\mathbb{Z}	\mathbb{Z}_{12}	\mathbb{Z}	0	2	2524
113	\mathbb{Z}	\mathbb{Z}_{13}	\mathbb{Z}	0	2	2494
114	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}^2	\mathbb{Z}	2	2289
115	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}^2	\mathbb{Z}	2	2252
116	\mathbb{Z}	\mathbb{Z}_7	0	\mathbb{Z}^2	-1	2210
117	\mathbb{Z}	0	$\mathbb{Z}^2 \oplus \mathbb{Z}_2$	0	3	2026
118	\mathbb{Z}	\mathbb{Z}_9	\mathbb{Z}	\mathbb{Z}	1	2023
119	\mathbb{Z}	\mathbb{Z}_{11}	\mathbb{Z}	\mathbb{Z}	1	2021
120	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}^2	0	3	1905

Table 6 : Homology classes (spaces made of 3 simplexes) n:81-120

n	H_0	H_1	H_2	H_3	χ	Num. of Triang.
121	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	0	\mathbb{Z}	-1	1744
122	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^4	0	5	1654
123	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}	\mathbb{Z}	0	1616
124	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}	\mathbb{Z}	1	1607
125	\mathbb{Z}	0	\mathbb{Z}_4	0	1	1524
126	\mathbb{Z}	\mathbb{Z}_8	\mathbb{Z}^2	0	3	1458
127	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}_3	\mathbb{Z}	0	1440
128	\mathbb{Z}	0	\mathbb{Z}_3	\mathbb{Z}	0	1332
129	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_3	\mathbb{Z}	0	1332
130	\mathbb{Z}	0	\mathbb{Z}^3	\mathbb{Z}^3	1	1312
131	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^3	0	3	1301
132	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}^3	0	3	1236
133	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^2	\mathbb{Z}	1	1186
134	\mathbb{Z}	\mathbb{Z}_{12}	\mathbb{Z}	\mathbb{Z}	1	1129
135	\mathbb{Z}	\mathbb{Z}_{11}	\mathbb{Z}^2	0	3	1091
136	\mathbb{Z}	\mathbb{Z}_{13}	\mathbb{Z}	\mathbb{Z}	1	1083
137	\mathbb{Z}	\mathbb{Z}_9	\mathbb{Z}^2	0	3	1034
138	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^3	-2	1028
139	\mathbb{Z}	\mathbb{Z}_{10}	\mathbb{Z}	\mathbb{Z}	1	919
140	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}_3	0	1	864
141	\mathbb{Z}	0	$\mathbb{Z}^2 \oplus \mathbb{Z}_2$	\mathbb{Z}	2	858
142	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^2	\mathbb{Z}^2	1	820
143	\mathbb{Z}	\mathbb{Z}_2	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}	1	804
144	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}^3	0	4	714
145	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^4	0	5	621
146	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}^3	0	4	589
147	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}^3	0	4	584
148	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}	-1	576
149	\mathbb{Z}	\mathbb{Z}_{10}	\mathbb{Z}^2	0	3	568
150	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^2	\mathbb{Z}^3	0	544
151	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}^2	\mathbb{Z}	2	543
152	\mathbb{Z}	\mathbb{Z}_{13}	\mathbb{Z}^2	0	3	532
153	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^3	\mathbb{Z}^2	2	512
154	\mathbb{Z}	\mathbb{Z}_{12}	\mathbb{Z}^2	0	3	502
155	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}^3	0	4	502
156	\mathbb{Z}	\mathbb{Z}_6	0	\mathbb{Z}^2	-1	498
157	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}_2	0	1	488
158	\mathbb{Z}	0	\mathbb{Z}^6	0	7	479
159	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}	\mathbb{Z}	1	479
160	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}_2	\mathbb{Z}	0	448

Table 6 : Homology classes (spaces made of 3 simplexes) n:121-160

n	H_0	H_1	H_2	H_3	χ	Num. of Triang.
161	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}_2	\mathbb{Z}	0	440
162	\mathbb{Z}	0	\mathbb{Z}^5	\mathbb{Z}	5	393
163	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}	\mathbb{Z}^2	0	325
164	\mathbb{Z}	\mathbb{Z}_9	\mathbb{Z}	\mathbb{Z}^2	0	310
165	\mathbb{Z}	\mathbb{Z}_{11}	\mathbb{Z}	\mathbb{Z}^2	0	306
166	\mathbb{Z}	\mathbb{Z}_{11}	\mathbb{Z}^2	\mathbb{Z}	2	296
167	\mathbb{Z}	\mathbb{Z}_9	\mathbb{Z}^2	\mathbb{Z}	2	292
168	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}^2	\mathbb{Z}	1	244
169	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	0	\mathbb{Z}^2	-2	236
170	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}_2	0	1	235
171	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^3	\mathbb{Z}	3	216
172	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}^4	0	4	212
173	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^2	\mathbb{Z}^2	0	176
174	\mathbb{Z}	\mathbb{Z}_{10}	\mathbb{Z}^2	\mathbb{Z}	2	165
175	\mathbb{Z}	\mathbb{Z}_3	$\mathbb{Z} \oplus \mathbb{Z}_2$	0	2	164
176	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}^3	0	4	162
177	\mathbb{Z}	\mathbb{Z}_{13}	\mathbb{Z}	\mathbb{Z}^2	0	158
178	\mathbb{Z}	\mathbb{Z}_{10}	\mathbb{Z}	\mathbb{Z}^2	0	152
179	\mathbb{Z}	\mathbb{Z}_{12}	\mathbb{Z}	\mathbb{Z}^2	0	150
180	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^4	0	4	144
181	\mathbb{Z}	\mathbb{Z}_{13}	\mathbb{Z}^2	\mathbb{Z}	2	134
182	\mathbb{Z}	\mathbb{Z}_8	\mathbb{Z}^3	0	4	132
183	\mathbb{Z}	\mathbb{Z}_{12}	\mathbb{Z}^2	\mathbb{Z}	2	116
184	\mathbb{Z}	\mathbb{Z}_9	\mathbb{Z}^3	0	4	114
185	\mathbb{Z}	\mathbb{Z}_{11}	\mathbb{Z}^3	0	4	102
186	\mathbb{Z}	\mathbb{Z}_{12}	\mathbb{Z}^3	0	4	90
187	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^5	0	6	90
188	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}^2	\mathbb{Z}	2	80
189	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}_2	0	1	78
190	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}^2	\mathbb{Z}	2	77
191	\mathbb{Z}	\mathbb{Z}_3	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}	1	72
192	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}^4	0	5	70
193	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^3	\mathbb{Z}	2	60
194	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^3	\mathbb{Z}^2	1	60
195	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^4	\mathbb{Z}	4	59
196	\mathbb{Z}	\mathbb{Z}_{13}	\mathbb{Z}^3	0	4	54
197	\mathbb{Z}	0	\mathbb{Z}^4	\mathbb{Z}^2	3	48
198	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}^3	\mathbb{Z}	2	37
199	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^2	\mathbb{Z}^3	-1	36
200	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}^3	\mathbb{Z}	3	34

Table 6 : Homology classes (spaces made of 3 simplexes) n:161-200

n	H_0	H_1	H_2	H_3	χ	Num. of Triang.
201	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}^4	0	5	34
202	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}	\mathbb{Z}^2	0	31
203	\mathbb{Z}	\mathbb{Z}_{10}	\mathbb{Z}^3	0	4	30
204	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}^4	0	5	30
205	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^3	\mathbb{Z}^2	2	24
206	\mathbb{Z}	\mathbb{Z}_2	$\mathbb{Z}^2 \oplus \mathbb{Z}_2$	0	3	22
207	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}^4	0	5	21
208	\mathbb{Z}	\mathbb{Z}_2	$\mathbb{Z}^2 \oplus \mathbb{Z}_2$	\mathbb{Z}	2	20
209	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}^3	\mathbb{Z}	3	19
210	\mathbb{Z}	0	\mathbb{Z}^7	0	8	18
211	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^5	0	5	14
212	\mathbb{Z}	\mathbb{Z}_{13}	\mathbb{Z}^3	\mathbb{Z}	3	12
213	\mathbb{Z}	\mathbb{Z}_{13}	\mathbb{Z}^4	0	5	12
214	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^4	\mathbb{Z}^2	3	12
215	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}^3	\mathbb{Z}	3	12
216	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^4	\mathbb{Z}	4	12
217	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}^5	0	6	12
218	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}^4	0	5	12
219	\mathbb{Z}	\mathbb{Z}_8	\mathbb{Z}^4	0	5	12
220	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}^6	0	6	12
221	\mathbb{Z}	\mathbb{Z}_{11}	\mathbb{Z}^3	\mathbb{Z}	3	9
222	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}^3	\mathbb{Z}	3	8
223	\mathbb{Z}	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}	0	8
224	\mathbb{Z}	\mathbb{Z}_{11}	\mathbb{Z}^4	0	5	6
225	\mathbb{Z}	\mathbb{Z}_{12}	\mathbb{Z}^3	\mathbb{Z}	3	6
226	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^5	0	6	6
227	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}^5	0	6	6
228	\mathbb{Z}	\mathbb{Z}_9	\mathbb{Z}^3	\mathbb{Z}	3	6
229	\mathbb{Z}	\mathbb{Z}_9	\mathbb{Z}^4	0	5	6
230	\mathbb{Z}	0	$\mathbb{Z}^3 \oplus \mathbb{Z}_2$	0	4	5
231	\mathbb{Z}	\mathbb{Z}	$\mathbb{Z}^2 \oplus \mathbb{Z}_2$	\mathbb{Z}	1	4
232	\mathbb{Z}	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	0	1	4
233	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}^5	0	6	2
234	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}^6	0	7	2

Table 6 : Homology classes (spaces made of 3 simplexes) n:201-234