A mathematical model of reality

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Abstract

It is possible to derive a model from a set of well selected first principles. After a series of extensions of this foundation the model shows many similarities with what we know from observing reality.

The first principles formulate a skeleton relational structure that is mathematically known as an orthocomplemented weakly modular lattice. It can be considered as part of a recipe for modular construction. When starting from this foundation it is mathematically inescapable evident that this model confines to a quaternionic space-progression model that proceeds with model-wide progression steps through an ordered sequence of sub-models that each represent a static status quo of the whole model.

This base model is a hybrid construct that consists of a sub model that treats all discrete objects and a continuum based model that embeds the discrete objects. The discrete part of the model keeps its data in an infinite dimensional separable Hilbert space. The continuum part stores its data in a corresponding non-separable Hilbert space.

An extra mechanism that controls the coherence and the scheduling of dynamics applies the recurrent embedding of the discrete objects into appropriate continuums.

Despite the fact that the target of the model is planned to reach a level in which it shows many features and phenomena that we know from observing reality, this model is not claimed to be a model of physics. The reason for this restriction is that many of its aspects cannot be observed and will never become observable. Physicists tend to deny completely or largely deduced models.

If the model introduces new science, then it has fulfilled its purpose.

Contents

1 Introduction

The original target of the mathematical model is far reaching. That target is to reach a level in which the model shows many features and phenomena that we also know from observing reality. The current result of that approach is rather modest.

Due to the fact that the model starts at a foundation that not even shows fundamental concepts such as progression and a spatial geometry, it covers merely what is generally considered as the lowest levels of the model of reality.

The model covers elementary particles, photons and the origins of physical fields, but it does not cover hadrons and higher order composites. Still it lays the base on which such composites can be comprehended.

The current model uncovers the white spots that were left by physicists that only want to consider observable and thus experimentally testable facts.

The current state of the model already proves that alternatives exist for the approaches that are implemented in the contemporary models of physics.

The model shows that the physicists that developed the contemporary models of physics made some choices that were not very lucky choices and that made physics more complicated than necessary.

2 Generating the model

2.1 First principles

The ~25 axioms that define an orthocomplemented weakly modular lattice[1] form the first principles on which the whole model will be built. Another name for this lattice is orthomodular lattice. Quantum logic has this lattice structure. Classical logic has a slightly different lattice structure. It is an orthocomplemented modular lattice. For our purpose it is better to interpret the elements of the orthomodular lattice as construction elements rather than as propositions. Only trustworthy, mostly mathematical methods will be used to extend this model until a level is reached in which it shows many features and phenomena that we recognize from observing reality.

Space and progression emerge from the selected foundation. The first principles only deliver a static foundation. Via a trick we convert this static model in an ordered sequence that implements stepwise dynamics. As a consequence, the dynamic model uses a paginated space progression model that emerges from a skeleton relational structure. Paginated means that the model steps with model wide progression steps.

2.2 The next level

The set of closed subspaces of an infinite dimensional separable Hilbert space also forms an orthomodular lattice. The Hilbert space adds extra functionality to this orthomodular lattice. This extra functionality concerns the superposition principle and the possibility to store data in eigenspaces of normal operators. In the form of Hilbert vectors the Hilbert space features a finer structure than the orthomodular lattice has.

[1] See the appendix. This lattice is described in detail in the e-book.

The Hilbert space can only handle members of a division ring for specifying superposition coefficients, for the eigenvalues of its operators and for the values of its inner products. Only three suitable division rings exist: the real numbers, the complex numbers and the quaternions. Quaternions enable the storage of 1+3D data that have an Euclidean geometric structure.

Thus, selecting a skeleton relational structure that is an orthomodular lattice as the foundation of the model already puts significant restrictions to the model. On the other hand, as will be shown, this choice promotes modular construction. In this way it eases system configuration and the choice significantly reduces the relational complexity of the final model.

2.3 Adding dynamics

The primitive model that is reached after this first extension step does not provide any means to control dynamics and it does not support the representation of continuums.

Dynamics can be added by using an ordered sequence of the models that can represent a static status quo. This choice makes the model paginated. As a consequence the model proceeds with model-wide progression steps. In this way, all discrete objects in the model can be considered to be regenerated. That does not need to be done in the smallest model wide steps. It can be done in progression cycles that depend on the type of the discrete object.

With this decision, an extra mechanism must be added that ensures sufficient coherence between subsequent elements of the sequence. In order to reach sufficient coherence the next sequence member must not differ much from the considered member. With other words, the coherence must not be too stiff, otherwise no dynamics occurs. On the other hand it must be sufficient restrictive, otherwise the result is dynamical chaos.

Due to potential difference in generation time, conflicts during the generation of composites can occur. Thus, the mechanism that ensures dynamic al coherence must also schedule the composite configuration subtasks. As a consequence, this mechanism shares many aspects with a real time operating system. This RTOS schedules subtasks and it ensures that these subs-tasks occur in sync.

2.4 Adding continuums

Every infinite dimensional separable Hilbert space owns a Gelfand triple[2]. Continuums can be supported by adding the Gelfand triple to the Hilbert space. The Gelfand triple can be used to check the coherence. This is done by ***embedding*** the subsequent Hilbert spaces into a common Gelfand triple. As a consequence progression steps along the Hilbert spaces and it flows inside the common Gelfand triple. This allows the embedding process to control both the dynamic as well as the spatial coherence.

The embedding process already puts many restrictions that at least partly ensure the dynamical coherence.

The embedding of discrete objects in their surrounding continuums appears to be a very delicate process. It is not yet fully supported by dedicated mathematical theory.

[2] The name Gelfand triple indicates a non-separable (rigged) Hilbert space.

3 Modular construction

Thus, if the orthomodular lattice is considered as the foundation of the model, then the separable Hilbert space is the next level of extension of the model. The foundation can be considered as part of a recipe for modular construction. Closed subspaces of an infinite dimensional separable quaternionic Hilbert space represent the modules. What is missing are the binding mechanism and a way to hide part of the relations that exist inside the modules from the outside of the modules. These ingredients are delivered by the superposition principle and by the embedding mechanism.

The modular construction recipe is certainly the most influential rule that exists in the generation of reality. Even without intelligent design it achieved the construction of intelligent species.

4 Exploiting the model

4.1 Conformance to the foundation

We can now reformulate the foundation as follows:

At every used progression instant, each discrete construct in this model is supposed to expose the skeleton relational structure that is defined as an orthomodular lattice.

At each used progression instant, every discrete construct in this model can be represented by a closed subspace of a single infinite dimensional separable quaternionic Hilbert space.

This does not mean that every closed subspace represents a discrete object. It also does not mean that a unique “ top-subspace” exists that contains all closed subspaces, which represent discrete objects of the model. On the other hand this means that a collection of system subspaces exists that are the disjunction of closed subspaces, which represent discrete objects and that themselves are not contained in another closed subspace that represents a discrete object. Each of these composites represents a modular system.

4.2 Embedding of discrete objects

The closed subspace that represents a discrete object can be spanned by eigenvectors of a location operator, but this does not mean that these eigenvalues must form a coherent set. However, if they form a coherent set, then the collection of eigenvalues might characterize the discrete object. This set of locations is characterized by its size, its statistical characteristics and by its discrete symmetry set.

Not all Hilbert space vectors that span a Hilbert subspace will be re-embedded into the Gelfand triple at every progression step. For atomic Hilbert subspaces[3] at the utmost one eigenvector of the location operator is embedded at each progression step. Spanning the subspace with these eigenvectors takes a series of discrete embedding occurrences. During a regeneration cycle[4] of the corresponding discrete object that eigenvector is used only once. The embedding lasts no more than that progression step. The next embedding of the atomic object concerns a different eigenvector and a different eigenvalue.

Thus, per discrete embedding the embedding itself only lasts a very small instance. It is quickly replaced by another embedding occurrence. What stays are the consequences of these very short embedding occurrences. These consequences will only appear in the embedding continuum.

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[3] An atomic Hilbert subspace represents an atom of the orthomodular lattice.

[4] See next section

However, the eigenvalue of the embedded eigenvector stays stored in the Hilbert space. During the regeneration cycle these eigenvalues form a swarm or some other geometrical configuration.

The fact that the embedding itself only takes a single progression step, does not mean that at every progression step the discrete object is re-embedded. Zero or more progression steps may separate the embedding occurrences. The embedding occurrences may, but must not be evenly distributed over the complete regeneration cycle.

Due to the transfer of the embedding locations into the continuum the statistical characteristics of the set of original eigenvalues lose their sense and are replaced by integral characteristics of the representing continuum. However, the discrete symmetry characteristics are preserved. In fact the discrete symmetries of the embedded object can become mixed with the symmetry set of the embedding continuum and this mixture appears to be a requirement for becoming a discernable discrete object.

4.3 Modularization, dimension and duration

Filling a subspace with eigenvectors of a suitable location operator takes a number of embedding occurrences. That number equals the dimension of the subspace. Such subspaces represent construction modules. If subspaces must be combined in higher level modules, then the generation periods must be synchronized. With other words, the components of a modular system all are synchronized on some basic generation cycle and all these cycles are synchronized.

If the subspaces that participate in the generation of a composite differ in dimension then there must exist a common cycle period in which all participating subspaces are filled. The filling of a subspace with appropriate eigenvectors is a stepwise action. The eigenvalues are generated one by one and at every progression instant only one eigenvalue is the current value that characterizes the owner of the subspace.

Thus, not only categories of subspaces exist that belong to different subspace dimensions. Also several cycle periods might exist[5]. Atomic discrete objects exist in classes that all have the same generation cycle. Only class types that own generation cycles that are compatible in period as well in synchronization can join in a composite and only compatible composites can join in a modular system.

Duration is measured in basic progression steps. However, the fill duration need not equal the dimension of the subspace. It is also not required that the embedding occurrences are evenly distributed over the duration.

Thus, duration takes a significant and at the same time complicated role in this model. During this period the dynamics of the considered point-like object are mapped into a spatial structure that takes the form of a location swarm or some other geometric configuration. The set of locations also corresponds to a set of hops that connect the subsequent locations. Thus, the hops form a micro-path. These hops also correspond to eigenvalues and the corresponding eigenvectors span a different subspace. The two subspaces form a dual. In a similar way the locations and the hops form duals. The hop operator forms a dual with the location operator.

[5] In contemporary physics elementary particles types exist that have different rest masses and elementary particle types exist in different generations. Later it will be shown that rest mass relates to subspace dimension

Thus the discrete part of the model is regenerated in several different ways. Each of these ways corresponds to a modular system wide clock and quite probably these clocks are in sync with the highest frequency clock, but mutually they may differ in their frequency.

4.4 Speed of information transfer

The spatial map of the dynamic behavior of the discrete object is restricted by the current value of the speed of information transfer. Thus a change of the size of the basic progression step translates via this speed into a proportional isotropic change of the spatial map.

4.5 Categories of subspaces

It is clear that not all closed subspaces figure as discrete objects in the model. Only subspaces that are spanned by eigenvectors of the selected location operator will do that. The same holds for the dual subspaces. Inside a modular system, the durations and the synchronization of the fill of subspaces must fit.

The geometrical structures that are contained in the eigenvalues and represent the discrete objects will reflect these restrictions. In fact they represent the fine grain behavior of the discrete object rather than its spatial structure.

4.6 Embedding the most elementary objects

For simplicity we suspect that all model wide clocks and modular system wide clocks operate in sync with the highest frequency clock. The clock with the highest frequency controls progression. Not all steps of this clock need to be used by some action.

Embedding may occur at the pace of this highest frequency clock, but not all ticks must correspond to an actual embedding. Only in this way the generation of objects belonging to subspaces with different dimensions can be synchronized, such that they can be combined into composites.

This means that the discrete objects can be considered to be prepared in advance of their usage in the modular construction process[6].

The embedding of a discrete object lasts very short and is quickly replaced by a subsequent embedding at a slightly different location. Only the consequences of the embedding last and will appear in the embedding continuum. However, the fact that the embedding took place is registered in the separable Hilbert space. Its location is represented by an eigenvector of a location operator and by the corresponding eigenvalue.

The embedding process gives every elementary building block an actual location in the embedding continuum. At a later progression step that location will differ. At each actively used progression step the elementary building block will hop to the next location. For most discrete objects the next location is not known in advance. In that case it is supposed to be determined by a stochastic process.

At each used progression interval, every discrete building block in this model owns an exact hopping value and an exact location that together form a *dual*. Both members of the dual share the same real part, which stores the progression value of the landing instant.

[6] This is just a view. It is possible that the embedding is an ongoing process.

The embedded locations may form a swarm, but also other elementary objects exist that have an exact location at a series of progression steps. Also these objects hop from the current location to the next location. Such objects form another kind of geometrical construct.

4.7 Sets

4.7.1 Forms

The set can have one of four forms:

- A coherent swarm
- A closed path string
- An open path string
- Spurious duals

The swarms contain a folded hopping path string.

4.7.2 Duals

In all cases, the location and the hop form a *dual.* A hop connects two locations. The *duals* form the most elementary objects in the model. On themselves they do not have any other characteristic than their quaternionic value. Only as sets these *duals* become extra significance. Extra data are obtained from the statistics of the set or from the symmetry properties of the set. The hops form a path and this path adds its own characteristics.

Duals can also appear as spurious objects.

4.7.3 Swarms

The swarm differs from the other forms in the fact that it can be characterized by a density distribution. The swarm is a coherent set. It represents the spatial map of the fine grain behavior of the discrete object during a given cycle period.

Two interpretations are possible:

- The swarm is generated by an ongoing stochastic process. After a while the statistic characteristics of the swarm stabilize.
- The swarm is prepared in advance. Its elements are used one by one. The currently active element is obtained by random selection from the set of not yet used elements. When all elements are used, then the swarm is regenerated.

Here we take the second interpretation. We do this because it is easier to understand and to handle. Both interpretations mean that the swarm is generated by a cyclic stochastic process. The swarm contains a huge number of elements.

These measures are part of the task of the mechanism that must ensure sufficient coherence between the elements of the sequence of static sub-models that together form the dynamic model.

The swarm includes a closed path. We suppose that the statistics of the planned swarm are stable. Under the mentioned conditions, the swarm is at rest. It means that the sum of all hops equals zero. In this condition the swarm has a fixed number of elements.

The swarm translates the fine grain dynamics of its owner into a spatial structure. In this way dynamical coherence translates into spatial coherence.

With respect to the swarm three levels of coherence exist.

1. The first level is always present in the representation of the discrete object. In this level all elements of a swarm must belong to a quaternionic number system that has a given symmetry flavor. Due to the four dimensions of quaternions, quaternionic number systems exist in sixteen versions (symmetry flavors) that only differ in their discrete symmetry set.
    - This condition is used by nature in order to create the diversity of elementary particle
2. The second level concerns the extra restriction that the swarm of discrete locations can be described by a normalized continuous location density distribution[7].
    - This condition leads to the existence of the wave function
3. The third level concerns the additional restriction that this continuous location density function has a Fourier transform. If this condition is fulfilled, then the swarms owns a displacement generator and this means that at first approximation the swarm moves as one unit.
    - This condition is used by nature in order to let elementary particles behave like wave-like objects. They are not waves. They are not even wave packages, but they can form detection patterns that look wave-like.

4.8 Moving the swarm

Adding extra duals to the swarm causes a movement of the extended set. This may result in the fact that the sum of all hops is no longer closed. As a consequence the swarm moves.

Adding particular sets of hops may cause an oscillation of the swarm. This occurs in typical oscillation modes. These extra sets form cycles. They are closed path objects. Adding or retrieving such sets must be done in sync with the swarm regeneration process. The sets that leave the oscillating swarm are open path strings. Such open path strings can also enter the free swarm or an already oscillating swarm. In principle the oscillations keep the swarm on average at the same location.

Adding a more arbitrary set of duals or an open path string that does not fit for establishing an oscillation, will cause a translation of the possibly oscillating swarm. An entering string can be broken into one or more fitting open path strings and a translation set. The translation set increases the kinetic energy of the composite.

4.8.1 Synchronization

However, adding duals to the swarm itself will disturb the regeneration synchronization of higher order constructs. The generation of the swarm represents a cycle period that is used in the construction of composites. The composites are generated by parallel process that all act within the same cycle period. So all constituents of the composite must be extended in the same sense.

Thus the addition of series of duals that cause movement of the swarm occurs in well synchronized parallel processes and the possibly the action of the members of the series are spread over the cycle period. This gives these extra duals a different character than the duals that form the swarm that was at rest. The extra duals act on the full swarm and can be considered as incremental displacement generators in configuration space. They can be seen as superposition coefficients in Fourier (=momentum) space.

In order to explain this a mathematical intermezzo is inserted.

[7] The normalized continuous location density distribution corresponds to the squared modulus of the wave function that characterizes elementary particles in contemporary physics.

5 Mathematical intermezzo

The equations in this intermezzo are based on application in a flat continuum. In practice this only holds under special conditions. In general the embedding continuum is curved. Later we treat the influence of curvature.

5.1 Functions as Hilbert space operators

Paul Dirac introduced the bra-ket notation that eases the formulation of Hilbert space habits. By using bra-ket notation, operators that reside in the Hilbert space and correspond to continuous functions, can easily be defined starting from an orthogonal base of vectors.

Let $\{q_i\}$ be the set of rational quaternions and $\{|q_i\rangle\}$ be the set of corresponding base vectors.

$|q_i\rangle q_i \langle q_i|$ is the configuration parameter space operator.

Let $f(q)$ be a quaternionic function.

$|q_i\rangle f(q_i) \langle q_i|$ defines a new operator that is based on function $f(q)$.

In the Gelfand triple, the continuous function $f(q)$ can be defined between a continuum eigenspace that acts as target space and the eigenspace of the reference operator $|q\rangle q\langle q|$ $t$hat acts as parameter space. $|q\rangle f(q)\langle q|$ defines a curved continuum.

In the Gelfand triple the dimension of a subspace loses its significance. Thus a function that is derived from the representation of a coherent swarm in Hilbert space has a dimension in Hilbert space, but loses that characteristic in its representation in the Gelfand triple.

The continuums that appear as eigenspaces in the Gelfand triple can be considered as quaternionic functions that also have a representation in the corresponding infinite dimensional separable Hilbert space.

5.2 Quaternion geometry and arithmetic

Quaternions and quaternionic functions offer the advantage of a very compact notation of items that belong together.

Quaternions can be considered as the combination of a real scalar and a 3D vector that has real coefficients. This vector forms the imaginary part of the quaternion. Quaternionic number systems are division rings. Other division rings are real numbers and complex numbers. Octonions do not form a division ring.

Bi-quaternions exist whose parts exist of a complex scalar and a 3D vector that has complex coefficients. Bi-quaternions do not form division rings. This model does not use them.

5.2.1 Notation

We indicate the real part of quaternion a by the suffix a_0.

We indicate the imaginary part of quaternion a by bold face $\boldsymbol{a}$.

$$a = a_0 + \boldsymbol{a} \quad (1)$$

5.2.2 Conjugation

$$a^* = a_0 - \boldsymbol{a} \quad (1)$$

5.2.3 Sum

(1)

$$c = c_0 + \boldsymbol{c} = a + b$$

$$c_0 = a_0 + b_0 \quad (2)$$

$$\boldsymbol{c} = \boldsymbol{a} + \boldsymbol{b} \quad (3)$$

5.2.4 Product

$$f = f_0 + \boldsymbol{f} = d\, e \quad (1)$$

$$f_0 = d_0 e_0 - \langle \boldsymbol{d}, \boldsymbol{e} \rangle \quad (2)$$

$$\boldsymbol{f} = d_0 \boldsymbol{e} + e_0 \boldsymbol{d} \pm \boldsymbol{d} \times \boldsymbol{e} \quad (3)$$

The $\pm$ sign indicates the influence of right or left handedness of the number system[8].

$\langle \boldsymbol{d}, \boldsymbol{e} \rangle$ is the inner product of $\boldsymbol{d}$ and $\boldsymbol{e}$.

$\boldsymbol{d} \times \boldsymbol{e}$ is the outer product of $\boldsymbol{d}$ and $\boldsymbol{e}$.

5.2.5 Norm

$$|a| = \sqrt{a_0 a_0 + \langle \boldsymbol{a}, \boldsymbol{a} \rangle} \quad (1)$$

5.2.6 Rotation

Quaternions are often used to represent rotations.

$$c = ab/a \quad (1)$$

[8] Due to the four dimensional structure of quaternions, quaternionic number systems exist in 16 symmetry flavors. Within a coherent set all elements belong to the same symmetry flavor.

rotates the imaginary part of b that is perpendicular to the imaginary part of a[9].

[9] See Q-FORMULÆ

5.3 Quaternionic functions

5.3.1 Norm

Square-integrable functions are normalizable. The norm is defined by:

$$\|\psi\|^2 = \int_V |\psi|^2 \, dV \tag{1}$$

$$= \int_V \{|\psi_0|^2 + |\boldsymbol{\psi}|^2\} dV$$

$$= \|\psi_0\|^2 + \|\boldsymbol{\psi}\|^2$$

5.3.2 Differentiation

If g is differentiable then the quaternionic nabla ∇g of g exists.

The quaternionic nabla ∇ is a shorthand for $\nabla_0 + \boldsymbol{\nabla}$

$$\nabla_0 = \frac{\partial}{\partial \tau} \tag{1}$$

$$\boldsymbol{\nabla} = \left\{\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right\} \tag{2}$$

$$h = h_0 + \boldsymbol{h} = \nabla g \tag{3}$$

$$h_0 = \nabla_0 g_0 - \langle \boldsymbol{\nabla}, \boldsymbol{g} \rangle \tag{4}$$

$$\boldsymbol{h} = \nabla_0 \boldsymbol{g} + \boldsymbol{\nabla} g_0 \pm \boldsymbol{\nabla} \times \boldsymbol{g} \tag{5}$$

$$\phi = \nabla\psi \Rightarrow \phi^* = (\nabla\psi)^* \tag{6}$$

$$(\nabla\psi)^* = \nabla_0\psi_0 - \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle - \nabla_0 \boldsymbol{\psi} - \boldsymbol{\nabla}\psi_0 \mp \boldsymbol{\nabla} \times \boldsymbol{\psi} \tag{7}$$

$$\nabla^*\psi^* = \nabla_0\psi_0 - \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle - \nabla_0 \boldsymbol{\psi} - \boldsymbol{\nabla}\psi_0 \pm \boldsymbol{\nabla} \times \boldsymbol{\psi} \tag{8}$$

Similarity of these equations with Maxwell equations is not accidental. In Maxwell equations several terms in the above equations have been given special names and special symbols. Similar equations occur in other branches of physics. Apart from these differential equations also integral equations exist.

5.3.3 Gauge transformation

For a function χ that obeys the ***quaternionic wave equation***[10]

$$\nabla^{*}\nabla\chi = \nabla_0\nabla_0\chi + \langle\boldsymbol{\nabla},\boldsymbol{\nabla}\chi\rangle = 0 \qquad (1)$$

the value of ϕ in

$$\phi = \nabla\psi \qquad (2)$$

does not change after the gauge transformation[11]

$$\psi \rightarrow \psi + \xi = \psi + \nabla^{*}\chi \qquad (3)$$

$$\nabla\xi = 0 \qquad (4)$$

$$\chi = \chi_0 + \boldsymbol{\chi} \qquad (5)$$

Thus in general:

$$\nabla^{*}\nabla\psi = \nabla_0\nabla_0\psi + \langle\boldsymbol{\nabla},\boldsymbol{\nabla}\psi\rangle = \rho \neq 0 \qquad (6$$

ρ is a quaternionic function.

Its real part ρ_0 represents an object density distribution.

Its imaginary part $\boldsymbol{\rho} = \boldsymbol{v}\,\rho_0$ represents a current density distribution.

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[10] Be aware, this is the quaternionic wave equation. This is not the common form of the wave equation, which is complex number based.

[11] The qualification gauge transformation is usually given to a transformation that leaves the Laplacian untouched. Here we use that qualification for transformations that leave the quaternionic differential untouched.

Equation (1) forms the basis of the generalized (quaternionic) Huygens principle[12].

$$\nabla^*\nabla\chi_0 = 0 \tag{7}$$

Equation (7) has 3D isotropic wave fronts as its solution. χ_0 is a scalar function. By changing to polar coordinates it can be deduced that a general solution is given by:

$$\chi_0(r,\tau) = \frac{f_0(\boldsymbol{i}r - c\tau)}{r} \tag{8}$$

Where $c = \pm 1$ and $\boldsymbol{i}$ represents a base vector in radial direction. In fact the parameter $\boldsymbol{i}r - c\tau$ of f_0 can be considered as a complex number valued function.

$$\nabla^*\nabla\boldsymbol{\chi} = 0 \tag{9}$$

Here $\boldsymbol{\chi}$ is a vector function.

Equation (9) has one dimensional wave fronts as solutions:

$$\boldsymbol{\chi}(z,\tau) = \boldsymbol{f}(\boldsymbol{i}z - c\tau) \tag{10}$$

Again the parameter $\boldsymbol{i}z - c\tau$ of $\boldsymbol{f}$ can be interpreted as a complex number based function.

The imaginary $\boldsymbol{i}$ represents the base vector in the x, y plane. Its orientation θ may be a function of z.

That orientation determines the polarization of the wave front.

$$\begin{aligned}
&\frac{\partial}{\partial\tau}\boldsymbol{f} = c\boldsymbol{f}' \\
&\frac{\partial^2\boldsymbol{f}}{\partial\tau^2} = c\frac{\partial}{\partial\tau}\boldsymbol{f}' = c^2\boldsymbol{f}'' \\
&\frac{\partial\boldsymbol{f}}{\partial z} = \boldsymbol{i}\boldsymbol{f}' \\
&\frac{\partial^2\boldsymbol{f}}{\partial z^2} = \boldsymbol{i}\frac{\partial}{\partial z}\boldsymbol{f}' = -\boldsymbol{f}''
\end{aligned} \tag{11}$$

[12] The papers on Huygens principle use the complex number based wave equation, which differs from the quaternionic wave equation.

$$\frac{\partial^2 \boldsymbol{f}}{\partial \tau^2} + \frac{\partial^2 \boldsymbol{f}}{\partial z^2} = (c^2 - 1)\boldsymbol{f}''$$

If $c = \pm 1$, then $\boldsymbol{f}$ is a solution of the quaternionic wave equation.

5.3.4 Displacement generator

The definition of the differential is

$$\Phi = \nabla\psi \tag{1}$$

In Fourier space the nabla becomes a displacement generator.

$$\widetilde{\Phi} = \mathcal{M}\tilde{\psi} \tag{2}$$

$\mathcal{M}$ is the ***displacement generator***

A small displacement in configuration space becomes a multiplier in Fourier space.

In a paginated space-progression model the displacements are small and the displacement generators work incremental. The multipliers act as superposition coefficients.

5.3.5 The coupling equation

The coupling equation follows from peculiar properties of the differential equation. We start with two normalized functions ψ and φ and a normalizable function $\Phi = m\,\varphi$.

$$||\psi|| = ||\varphi|| = 1 \tag{1}$$

These normalized functions are supposed to be related by:

$$\Phi = \nabla\psi = m\,\varphi \tag{2}$$

$\Phi = \nabla\psi$ defines the differential equation. (3)

$\nabla\psi = \Phi$ formulates a continuity equation. (4)

$\nabla\psi = m\,\varphi$ formulates the coupling equation. (5)

It couples ψ to φ. m is the coupling factor.

$$\nabla\psi = m_1\, \varphi \tag{6}$$

$$\nabla^*\varphi = m_2\, \zeta \tag{7}$$

$$\nabla^*\nabla\psi = m_1\, \nabla^*\varphi = m_1 m_2 \zeta = \rho \tag{8}$$

Each double differentiable quaternionic function corresponds to a normalized density distribution.

5.3.5.1 In Fourier space

The Fourier transform of the coupling equation is:

$$\mathcal{M}\tilde{\psi} = m\tilde{\varphi} \tag{1}$$

$\mathcal{M}$ is the ***displacement generator***

6 What happens to the continuum?

6.1 Description of embedding

An almost continuous quaternionic function describes the continuum in which a dual is embedded. The place where the dual is embedded is a target value of this function and is related to the parameter value of this position. The embedding of duals is almost instantaneously and is immediately released. However, the result of the embedding survives. The embedding of a dual concerns two quaternionic data. The first relates to the jump from the previous location to the new location. The second relates to the new location. Both data are stored in eigenvalues and the corresponding eigenvectors of matching operators. As a result of the embedding wave fronts are transmitted from the new location. These wave fronts keep running and thereby form the traces of the embedding event in the continuum. Embedding of a location causes the emission of a 3D wave front. Embedding of a hop causes the emission of a 1D wave front in the direction of the hop.

6.2 Embedding

The embedding process of a single dual in a continuum can be split in three phases and two occurrences.

- The first phase treats the situation before the embedding takes place.
- At the start of the second phase the location is embedded.
- The second phase describes the situation during which the location is embedded.
- At the start of the third phase the embedding of the location is released and the hop is embedded.
- The third phase describes the situation after the release of the embedding of the location.

The third phase is the first phase of the next embedding process.

In the previous chapter only mathematical formulas were listed. Here we give these formulas an interpretation in the model.

The quaternionic differential equation

$$\phi = \nabla\psi \tag{1}$$

can be interpreted as a continuity equation. It describes how a coherent set of discrete objects are embedded in a continuum.

$$\nabla^*\nabla\psi = \nabla_0\nabla_0\psi + \langle\boldsymbol{\nabla},\boldsymbol{\nabla}\psi\rangle = \rho \neq 0 \tag{2}$$

$$\nabla^*\nabla\chi = \nabla_0\nabla_0\chi + \langle\boldsymbol{\nabla},\boldsymbol{\nabla}\chi\rangle = 0 \tag{3}$$

Here ψ describes the embedding continuum including the added coherent swarm of objects ρ in the form of a continuous quaternionic density distribution. χ may describe the embedding continuum without the swarm.

The embedding of a discrete object only lasts very shortly and is quickly replaced by another embedding occurrence.

The quaternionic function χ may describe how the embedding continuum reacts on the embedding when the embedding itself is resolved again.

$$\nabla^*\nabla\chi_0 = 0 \tag{4}$$

χ_0 is a scalar function . Equation (4) forms the basis of the Huygens principle.

For each temporary embedding of the location part of a dual a 3D wave front is generated at that location that keeps moving away from that location.

$$\nabla^*\nabla\boldsymbol{\chi} = 0 \tag{5}$$

In a similar way equation (5) forms the basis of the emission of a 1D wave front during the embedding of a discrepant displacement in the continuum. The direction of the displacements is coupled to the direction of the emitted 1D wave front. Here $\boldsymbol{\chi}$ is a vector function.

For each temporary embedding of the displacement part of a dual a 1D wave front is generated at the landing location of the hop. The wave front keeps moving away from that location.

For this case the formula can also be considered in a local complex number based context. In the 3D environment, the angular distribution of the 1D wave fronts depends on the angular distribution of the hops.

Only a discrepancy in the symmetry flavors that are coupled leads to the singularity that causes the emission of a wave front that goes together with the embedding of a discrete object into an embedding continuum.

The local situation between two subsequent embedding occurrences fits in existing field theory. What occurs during embedding is not yet covered by available mathematics.

6.3 Superposition

The coupling equation shows that an incremental displacement in configuration space corresponds to a multiplication factor in Fourier space.

$$\nabla\psi = m\,\varphi \tag{1}$$

$$\mathcal{M}\tilde{\psi} = m\tilde{\varphi} \tag{2}$$

This multiplication factor can be interpreted as a superposition coefficient.

6.4 Symmetry flavors

- Members of coherent sets $\{a_i\}$ of quaternions all feature the same symmetry flavor.
- Continuous quaternionic functions $\psi(q)$ do not switch to other symmetry flavors.
- If the real part is ignored, then still ***8*** symmetry flavors result
- Symmetry flavors are marked by special indices, for example $\boldsymbol{a}^{\textcircled{4}}$
- They are also marked by colors $N, R, G, B, \bar{B}, \bar{G}, \bar{R}, \bar{N}$
- Half of them is right handed, **R**
- The other half is left handed, **L**
- $\boldsymbol{\psi}^{\textcircled{0}}$ is the reference symmetry flavor of function ψ
- The colored rectangles reflect the directions of the axes

Symmetry flavors of members of coherent sets:		Symmetry flavors of continuous functions:	
	$\boldsymbol{a}^{\textcircled{0}} N\ \boldsymbol{R}$		$\boldsymbol{\psi}^{\textcircled{0}} N\ \boldsymbol{R}$
	$\boldsymbol{a}^{\textcircled{1}}\ R\ \boldsymbol{L}$		$\boldsymbol{\psi}^{\textcircled{1}}\ R\ \boldsymbol{L}$
	$\boldsymbol{a}^{\textcircled{2}}\ G\ \boldsymbol{L}$		$\boldsymbol{\psi}^{\textcircled{2}}\ G\ \boldsymbol{L}$
	$\boldsymbol{a}^{\textcircled{3}}\ B\ \boldsymbol{L}$		$\boldsymbol{\psi}^{\textcircled{3}}\ B\ \boldsymbol{L}$
	$\boldsymbol{a}^{\textcircled{4}}\ \bar{B}\ \boldsymbol{R}$		$\boldsymbol{\psi}^{\textcircled{4}}\ \bar{B}\ \boldsymbol{R}$
	$\boldsymbol{a}^{\textcircled{5}}\ \bar{G}\ \boldsymbol{R}$		$\boldsymbol{\psi}^{\textcircled{5}}\ \bar{G}\ \boldsymbol{R}$
	$\boldsymbol{a}^{\textcircled{6}}\ \bar{R}\ \boldsymbol{R}$		$\boldsymbol{\psi}^{\textcircled{6}}\ \bar{R}\ \boldsymbol{R}$
	$\boldsymbol{a}^{\textcircled{7}} W\ \boldsymbol{L}$		$\boldsymbol{\psi}^{\textcircled{7}} W\ \boldsymbol{L}$

Also continuums feature a symmetry flavor. The reference symmetry flavor of a continuous function is the symmetry flavor of the parameter space. The parameter space is a flat continuum. It is a coherent set

If the continuous quaternionic function describes the density distribution of a set $\{a_i\}$ of discrete objects a_i, then this set can be attributed with the same symmetry flavor.

6.5 Influence of symmetries

The embedding process is controlled by the symmetry flavors of the embedded object and the embedding continuum. Quaternions number systems as well as continuous quaternionic functions exist in 16 symmetry flavors. Even when the real parts are ignored this results in a variety of 8×8=64 different embedding products. Enough to cover all first generation members of the standard model.

For example the Dirac equation for the free electron in quaternionic format runs:

$$\nabla\psi = m_e\,\psi^* \qquad (1)$$

And the Dirac equation for the positron runs:

$$\nabla^*\psi^* = m_e\,\psi \qquad (2)$$

Thus

$$\nabla^{*}\nabla\psi = m_e \nabla^{*}\,\psi^{*} = m_e^2\,\psi \qquad (3)$$

For electrons ψ represents its own normalized object density distribution.

ψ^{*} and ψ are symmetry flavors of the same base function.

Other elementary particles couple different symmetry flavors $\{\psi^x, \psi^y\}$ of their shared base function:

$$\nabla\psi^x = m_{xy}\,\psi^y \qquad (4)$$

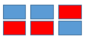
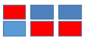
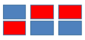
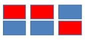
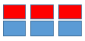
And for the antiparticle:

$$\nabla^{*}(\psi^x)^{*} = m_{xy}\,(\psi^y)^{*} \qquad (5)$$

The difference in the symmetry flavors between the members of the pair $\{\psi^x, \psi^y\}$ can be related to the electric charge, color charge and spin of the corresponding elementary particle.

Fermions appear to couple to the reference symmetry flavor $\boldsymbol{\psi}^{\textcircled{0}}$.

- Continuous quaternionic functions do not switch to other symmetry flavors.
- If the real part is ignored, then still ***8*** symmetry flavors result
- Symmetry flavors are marked by special indices, for example $\boldsymbol{\psi}^{\textcircled{4}}$
- They are also marked by colors $N, R, G, B, \bar{B}, \bar{G}, \bar{R}, \bar{N}$
- Half of them is right handed, **R**
- The other half is left handed, **L**
- $\boldsymbol{\psi}^{\textcircled{0}}$is the reference symmetry flavor
- The colored rectangles reflect the directions of the axes

Symmetry flavors ψ^x		Result of coupling ψ^x to $\psi^{\textcircled{0}}$		
	$\boldsymbol{\psi}^{\textcircled{0}} N$ ***R***		$\boldsymbol{\psi}^{\textcircled{0}}$ *neutrino*	***0***
	$\boldsymbol{\psi}^{\textcircled{1}} R$ ***L***		$\boldsymbol{\psi}^{\textcircled{1}}$ *R upquark*	*⅔*
	$\boldsymbol{\psi}^{\textcircled{2}} G$ ***L***		$\boldsymbol{\psi}^{\textcircled{2}}$ *G upquark*	*⅔*
	$\boldsymbol{\psi}^{\textcircled{3}} B$ ***L***		$\boldsymbol{\psi}^{\textcircled{3}}$ *B upquark*	*⅔*
	$\boldsymbol{\psi}^{\textcircled{4}} \bar{B}$ ***R***		$\boldsymbol{\psi}^{\textcircled{4}}$ $\bar{B}$ *downquark*	*-⅓*
	$\boldsymbol{\psi}^{\textcircled{5}} \bar{G}$ ***R***		$\boldsymbol{\psi}^{\textcircled{5}}$ $\bar{G}$ *downquark*	*-⅓*
	$\boldsymbol{\psi}^{\textcircled{6}} \bar{R}$ ***R***		$\boldsymbol{\psi}^{\textcircled{6}}$ $\bar{R}$ *downquark*	*-⅓*
	$\boldsymbol{\psi}^{\textcircled{7}} W$ ***L***		$\boldsymbol{\psi}^{\textcircled{7}}$ *electron*	***-1***

Fermions have half integer spin. Their "color" structure becomes noticeable. Quarks have "partial" electric charge. Up-quarks have electric charge $+$ ⅔e. Down-quarks have electric charge - ⅓ e.

Bosons couple to other sign flavors. Bosons have integer spin.

For bosons the spin axis may be coupled to the polar axis. The polar angle runs from 0 through 2π. For fermions the spin axis may be coupled to the azimuth axis. The azimuth angle runs from 0 through π.

Massive bosons are observable as W_+, W_- and Z particles. Their "color" structure cannot be observed. Until now, quark-like bosons are not observed. This may be due to color confinement.

6.6 Coupling properties

Discrepancies between the coupled symmetry flavors determine the properties of the coupling result. For example electric charge depends on the number of dimensions in which symmetry flavors differ. Also the direction in which they differ is important. Further is important whether handedness switches. Color charge also changes with the number of dimensions in which symmetry flavors differ. Spin appears to depend on the fact whether the embedding continuum has the reference flavor.

Like spin, electric charge and color charge do not depend on the number of duals that form the swarm.

Electric charge is related to the electrostatic potential. In that respect the location of the electric charge seems to coincide with the estimated location of the complete swarm.

6.7 The Palestra

In order to explain the existence of massive bosons, a bundle of embedding continuums must exist. The members of this bundle must differ in their symmetry flavor. One of them has the reference flavor. The bundle is called Palestra. It represents our curved living space.

6.8 Curvature

In contrast to spin, electric charge and color charge, the influence of the swarm on local curvature does depend on the number of duals that is involved in the swarm. The location of the sources of the gravitation potential appear to be spread over the whole swarm. However, the center location coincides with the location of the source of the electric potential.

In practice the emission of 3D wave fronts will cause a local folding and thus a curvature of the embedding continuum. This effect is the basis of the gravitation potential, which represents the averaged effects of these wave fronts.

In a curved environment the quaternionic nabla must be replaced by a differential that is constituted of 16 partial derivatives.

Where the 3D wave fronts decrease their amplitude with distance from the source, will the amplitude of the 1D wave fronts stay constant. As a consequence the 1D wave fronts do not curve the embedding continuum. Depending on the angular distribution of the hops that generated them, the influences of 1D wave fronts combine and average for electrons to a to 3D potential. In contemporary physics this potential is known as electromagnetic potential. The messengers keep the amplitude of their 1D wave fronts.

Curvature affects the map of the swarm onto the curved embedding continuum. The overall map $\mathcal{P}$ produces a blurred target. It is described by the convolution of a sharp continuous quaternionic allocation function $\wp$ and a stochastic spatial spread function $\mathcal{S}$.

$$\mathcal{P} = \wp \circ \mathcal{S} \tag{1}$$

The allocation function has a flat parameter space. It describes the history of the path of the concerned particle. At the same time it describes the form of the curved embedding continuum.

The stochastic spatial spread function $\mathcal{S}$ produces a blurred image ψ of the owner of a swarm. The swarm is produced by the combination of a Poisson process and a binomial process. The spread function implements the binomial process. $\mathcal{S}$ has progression as its single real parameter. The Fourier transform $\tilde{\psi}$ of the density distribution ψ that describes this swarm acts as a mapping quality characteristic.

6.9 Metric

The differential of the sharp allocation function defines a kind of quaternionic metric.

$$ds(q) = ds^{\nu}(q)e_{\nu} = d\wp = \sum_{\mu=0\ldots3} \frac{\partial \wp}{\partial q_{\mu}} dq_{\mu} = c^{\mu}(q)dq_{\mu}$$

q is the quaternionic location.

ds is the metric.

c^{μ} is a quaternionic function.

Pythagoras:

$$c^2 dt^2 = ds\, ds^* = dq_0^2 + dq_1^2 + dq_2^2 + dq_3^2 \tag{2}$$

Minkowski:

$$dq_0^2 = d\tau^2 = c^2\, t^2 - dq_1^2 - dq_2^2 - dq_3^2 \tag{3}$$

In flat space:

$$\Delta s_{flat} = \Delta q_0 + \boldsymbol{i}\, \Delta q_1 + \boldsymbol{j}\, \Delta q_2 + \boldsymbol{k}\, \Delta q_3 \tag{4}$$

In curved space:

$$\Delta s_{\wp} = c^0\,\Delta q_0 + c^1\,\Delta q_1 + \;c^2\,\Delta q_2 + c^3\,\Delta q_3 \tag{5}$$

$d\wp$ is a quaternionic metric

It is a linear combination of 16 partial derivatives

7 Energy

In the model the energy of a composite is directly related to the number of duals that constitute the composite. It is also directly related to the dimension of the dual subspace that represents the composite.

In the open path objects energy is related to the number of hops that constitute the object. This is also equal to the number of 1D wave fronts that constitute the object.

Oscillations that are internal to a composite are represented by closed path objects. The enclosed extra hops add to the energy of the composite.

8 Potentials

In this model potentials form the averages over a small period of progression and over a region of space of the wave fronts that are emitted during the embedding of particles.

8.1 The location potential

The wave fronts that are emitted during the embedding of the location members of the duals are isotropic 3D wave fronts. Their spreading is controlled by the 3D version of the Huygens principle. This means that their amplitude decreases with the distance from the source as $1/r$.

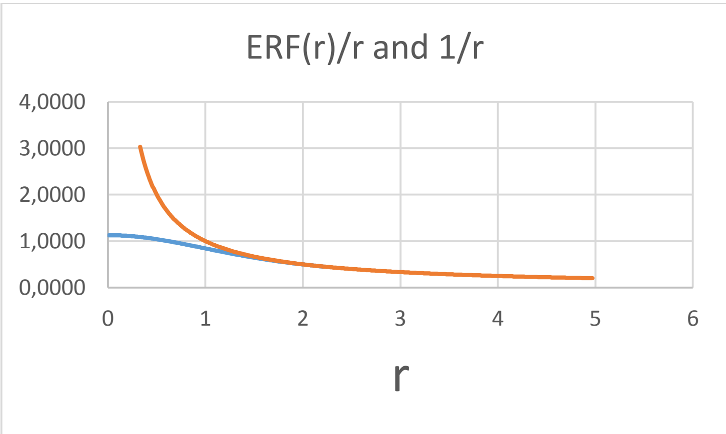
Here we consider a simplified situation. With an isotropic density distribution $\rho_0(r)$ in the swarm the scalar potential $\varphi_0(R)$ can be estimated as:

$$\varphi_0(R) = \int_0^R \rho_0(r)dr \qquad (1)$$

R is the distance to the center of the swarm.

If the density distribution approaches a 3D Gaussian distribution, then this integral equals[13]:

$$\varphi_0(R) = \mathrm{Erf}(R)/R \qquad (2)$$



We suppose that this distribution is a good estimate for the structure of the swarm of a free electron. It is remarkable that this potential has no singularity at $R = 0$. At the same time, already at a short distance of the center the function very closely approaches $1/R$.

[13] http://en.wikipedia.org/wiki/Poisson's_equation#Potential_of_a_Gaussian_charge_density

8.1.1 Moving swarm

The swarm can be described by a continuous density distribution. This function has a Fourier transform. Thus, the swarm owns a displacement generator. This means that at first instance the swarm can be considered to move as one unit.

If the swarm moves with uniform speed $\boldsymbol{v}$ than this conforms to a vector potential $\boldsymbol{\varphi}(R)$:

$$\boldsymbol{\varphi}(R) = \int_0^R \boldsymbol{v}\, \rho_0(r) dr = \boldsymbol{v}\ \varphi_0(R) \tag{1}$$

Now we use

$$\mathfrak{E} = \nabla\varphi = \nabla_0\varphi_0 - \langle\boldsymbol{\nabla}, \boldsymbol{\varphi}\rangle + \nabla_0\boldsymbol{\varphi} - \boldsymbol{\nabla}\varphi_0 \mp \boldsymbol{\nabla} \times \boldsymbol{\varphi}$$

An acceleration of the swarm goes together with an extra vector field $\boldsymbol{\mathfrak{E}}$:

$$\boldsymbol{\mathfrak{E}} \approx \nabla_0\, \boldsymbol{\varphi}(R) = \dot{\boldsymbol{v}}\, \varphi_0\, (R) \tag{2}$$

Other terms are small.

8.1.2 Inertia

In the model, universe is ***coarsely uniformly*** covered with swarms. These swarms represent a location potential $\varphi_0(R)$ that at larger distances deceases as $1/r$. Together the most distant swarms together deliver the largest contribution. Locally these potentials combine in a scalar potential ϕ_0 :

$$\phi_0 = \int \varphi_0(R)\, dV \tag{1}$$

A moving swarm will go together with a vector potential $\boldsymbol{\phi}$.

$$\boldsymbol{\phi} = \int \boldsymbol{v}\, \varphi_0(R)\, dV = \boldsymbol{v}\, \phi_0 \tag{2}$$

Now we use

$$\mathfrak{G} = \nabla\phi = \nabla_0\phi_0 - \langle\boldsymbol{\nabla}, \boldsymbol{\phi}\rangle + \nabla_0\boldsymbol{\phi} - \boldsymbol{\nabla}\phi_0 \mp \boldsymbol{\nabla} \times \boldsymbol{\phi}$$

An accelerating swarm will encounter a vector field $\mathfrak{G}$:

$$\boldsymbol{\mathfrak{G}} \approx \nabla_0 \boldsymbol{\phi} = \dot{\boldsymbol{v}}\, \phi_0 \tag{3}$$

Other terms are small.

This field counteracts the acceleration.

8.2 The hop potential

The wave fronts that are emitted during the embedding of the displacement members of the duals are planar 1D wave fronts. Their spreading is controlled by the 1D version of the Huygens principle. This means that their amplitude does not decrease with distance from the source. As a consequence these wave fronts do not curve the embedding continuum.

For an isotropic swarm the angular distribution of the wave fronts is isotropic. The corresponding location distribution may again approach a Gaussian distribution. The sum of all hops is supposed to equal zero. The spread of the influences behave similar to the spread of the influences of the location related wave fronts. Thus here the same formulas holds as for the location related wave fronts.

8.3 Electrostatic potentials versus gravitation potentials

In this model, gravitation potentials are easily understandable. They represent the smoothed and averaged influences of the 3D wave fronts that are emitted when locations are embedded. The source of the gravitational potential is the sum of the influences of the individual duals that constitute the map of the dynamic behavior of the corresponding discrete object, which reflects that behavior in a spatial location swarm.

Comprehension of electrostatic potentials takes a different route. The source of the electrostatic potential only relates to the difference of the symmetry flavors between the set of embedded hops and the symmetry flavor of the embedding continuum. The corresponding charge does not relate to the number of involved duals. On the other hand, the charge appears to be located at the weighted center location of the swarm. That weighted center location is determined by the normalized continuous density distribution that describes the swarm. This function is a probability density distribution. The charge is located at its weighted average value. The function has a Fourier transform. This means that the behavior of the owner at this scale is described by a displacement generator. At first approximation the density distribution moves as one unit.

The charge acts as a charge of a quaternionic field. That field obeys quaternionic field equations. The symmetry differences not only determine the size and the sign of the charge. They also determine in what dimensions the potential acts.

In fact, due to the hectic fine grain movement of the owner, the location is rather vague. For example if all three dimensions take part, then the shape of the Green's function of the potential will resemble the shape of the Green's function of the gravitation potential.

If less dimensions are involved, then the Green's function of the electrostatic potential will differ correspondingly.

9 Cosmos

9.1 Integral continuity equations

The integral equations that describe cosmology are:

$$\int_V \nabla \rho \, dV = \int_V s \, dV \tag{1}$$

(2)

$$\int_V \nabla_0 \rho_0 \, dV = \int_V \langle \boldsymbol{\nabla}, \boldsymbol{\rho} \rangle \, dV + \int_V s_0 \, dV$$

$$\int_V \nabla_0 \, \boldsymbol{\rho} \, dV = -\int_V \boldsymbol{\nabla} \rho_0 \, dV - \int_V \boldsymbol{\nabla} \times \boldsymbol{\rho} \, dV + \int_V \boldsymbol{s} \, dV \tag{3}$$

$$\frac{d}{d\tau} \int_V \rho \, dV + \oint_S \widehat{\boldsymbol{n}} \rho \, dS = \int_V s \, dV \tag{4}$$

Here $\widehat{\boldsymbol{n}}$ is the normal vector pointing outward the surrounding surface S, $\boldsymbol{v}(\tau, \boldsymbol{q})$ is the velocity at which the charge density $\rho_0(\tau, \boldsymbol{q})$ enters volume V and s_0 is the source density inside V. If ρ_0 is stable, then in the above formula ρ stands for

$$\rho = \rho_0 + \boldsymbol{\rho} = \rho_0 + \frac{\rho_0 \boldsymbol{v}}{c} \tag{4}$$

It is the flux (flow per unit of area and per unit of progression) of ρ_0 . τ stands for progression.

9.2 Space cavities

A static space cavity is characterized by:

$$\frac{d}{d\tau} \int_V \rho \, dV = 0 \tag{1}$$

All properties of this object depend on the surrounding surface.

These objects may represent black holes.

9.3 Inversion surfaces

An inversion surface S is characterized by:

$$\oint_S \widehat{\boldsymbol{n}} \rho \, dS = 0 \tag{1}$$

It is supposed that duals are stopped, but that potentials and their constituting wave fronts can still pass this inversion surface.

The inversion surfaces divide universe into compartments.

10 Messengers

10.1 Observed behavior

Photons are very special objects that are emitted by oscillating composites when they step down from a higher oscillation mode to a lower oscillation mode. Absorption of photons by a composite occurs when the composite steps up from a lower oscillation mode to a higher oscillation mode.

Further photons play a role in the creation and the annihilation of pairs of elementary building blocks. The pair consists of a merge of an elementary particle and its antiparticle.

Photons possess polarization. Photons exist in circular polarized versions and in linear polarized versions.

Photons can travel billions of years and can then still trigger a suitable detector.

A very particular difference occurs between young and old photons. At recent detection, old photons appear to be red-shifted.

10.2 Representation in the model

In the model messengers are represented by open chains of duals. Duals are constituted from a location and a hop. At embedding the location produces a 3D wave front, whose amplitude quickly diminishes and the hop produces a 1D wave front that keeps its amplitude. The 3D wave front produces a slight curvature of the embedding continuum. The messengers are the equivalents of photons. The emission and the absorption of messengers are controlled by processes that work in parallel to the generation of swarms. Locally these processes act in sync. These processing periods depend on the number of involved progression steps. It is not probable that the number of involved progression steps will change with progression. However, the duration of a single progression step may change with progression.

The 3D wave fronts in the messengers cause a slight space curvature that is spread along the chain that represents the messenger. Thus in the model messengers have a linearly spread mass. On the other hand the 1D wave fronts represent energy.

If space extends with increasing progression, then the distance between the wave fronts will grow during the travel of the messenger. With constant speed of information transfer, the absorption of a string of wave fronts will take longer. If the absorption shutter period is locally fixed, then old messengers will not be completely detected and appear to be red-shifted.

10.3 Polarization

In the interpretation of a messenger as a chain of 1D wave fronts, polarization means that in the plane perpendicular to the direction of the travel of the messenger the subsequent wave fronts WILL have a different angular orientation for the direction in which their amplitude extends. Linear polarization means that the plane of the lateral extent is stable. In that plane the direction of the extent is alternating.

11 Cosmological photons

It is a known fact that photons can travel for billions of years and after that can still be detected by a suitable detector. It is also known that in that case the frequency of the detected photon appears to be red-shifted.

Contemporary physics ignores the duration of the emission and absorption processes. It relates the energy of the photon to the frequency of the photon and it uses the Doppler effect and space expansion as a function of progression as explanations for red-shift.

The mathematical model relates the energy of the messengers to the number of the wave fronts that are contained in the messengers. This model takes the speed of information transfer as a model constant. This means that if all messengers feature the same duration, that the relation between energy and frequency also holds for the messengers. In that case red-shift will means that part of the wave fronts did not fit in the available duration. Again space expansion can be used as explanation. During the travel the distance between the wave fronts has increased in space as well as in progression. Thus in the available duration less wave fronts will be counted. However, this does not mean that the arriving photon contained less wave fronts. The other wave fronts are ignored. They might be detected as another photon or the corresponding energy might be converted to kinetic energy.

12 Object graininess

In the model, locally the duration of emission, passage and absorption of messengers is suspected to be equal or it depends on the graininess of the emitter. Measuring the duration and the frequency of the messenger will reveal the number of wave fronts that is contained in the messenger.

This is also the case for messengers that are released at pair annihilation. In this case the number of contained wave fronts will give information about the number of duals that were contained in the members of the annihilated pair. With other words it will reveal the dimension of the dual subspace that represented the annihilated object.

In elementary particles, the chain of hops is folded into a coherent swarm of locations. As a consequence in the swarm the 3D wave fronts superpose into a significant space curvature.

If several durations play a role as is suggested by the existence of generations of elementary particles, then also generations of photons must exist. Thus muon type elementary particles must annihilate into muon type photons. The same should happen for tau types.

Since quarks differ in rest mass from electrons, also here durations may differ, but in that case electrons cannot assembly with electrons into composites.

13 The dimension of the subspaces

In the Gelfand triple the dimension of subspaces that correspond to subspaces in Hilbert space is not defined. However, the coupling equation can give some indication of a measure that can replace the dimension. In the Hilbert space the dimension relates to the number of eigenvalues of the operator whose eigenvectors span the subspace. Now consider the eigenvalues that store the values of the hops. Compare:

$$\Phi = \nabla\psi = m\,\varphi$$

$$\|\Phi\|^2 = \int_V |\Phi|^2\, dV = \int_V |\nabla\psi|^2\, dV = m^2 \|\varphi\|^2 = m^2$$

Now let us apply this to the swarm by replacing the integral by a summation of squared hop sizes $|h_i|$.

$$m^2 = \sum_{i}^{N} |h_i|^2 = N\overline{|h_i|^2}$$

$h_i$is the quaternionic value of the i-th hop. It includes a fixed size progression hop. The resulting part is an imaginary quaternion. N is the number of elements in the location swarm.

Thus m is related to the dimension of the subspace in the Hilbert space.

13.1 Binding energy

If the dimension of the subspace that represents a composite is smaller than the sum of the dimensions of its constituents, then the difference is spent on binding energy. The hops that left have gone in the form of messengers or these hops are used to support the oscillations of the constituents that occur inside of the composite. At the same time the constituents have lost part of their identity. They differ from the free versions of the constituents.

14 Spurious duals

The most elementary discrete objects in the model are the location-hop duals. Embedding of a location causes a 3D wave front. Embedding of a hop causes a 1D wave front that relates its direction to the direction of the hop. Embedding a dual generates both wave fronts.

Spurious duals cause the generation of spurious wave fronts. The amplitude of the 3D wave front diminishes quickly, but these wave fronts curve the embedding continuum a bit. Thus this effect may be causing dark “matter”. The 1D wave fronts do not curve the embedding continuum, but they may represent “dark energy”.

14.1 Why swarms ?

The fact that spurious duals appear in large numbers, raises the question why swarms, which are coherent collections of large numbers of duals, also exist. What keeps these duals together. Some kind of binding principle must exist. What is even more peculiar is the fact that these swarms have fixed statistical properties, while the set elements have the same symmetry flavor.

The binding can be caused by the common gravity pitch. The embedding of each separate dual causes only a small gravitation pinch. The gravitation pinch diminishes with distance r as $1/r$. The swarm contains a huge number of duals and these duals together produce a gravitation pitch that is shaped as a much broader $Erf(r)/r$ dependence on distance r. This relatively flat potential covers a major part of the swarm. At much larger distances this function also diminishes as $1/r$.

In large numbers spurious duals can still bring a noticeable gravitation contribution that is characterized by a very large spread.

15 Difference with Maxwell-like equations

The difference between the Maxwell-Minkowski based approach and the Hamilton-Euclidean based approach will become clear when the difference between the coordinate time t and the proper time τ is investigated. This becomes difficult when space is curved, but for infinitesimal steps space can be considered flat. In that situation holds:

Coordinate time step vector = proper time step vector + spatial step vector

Or in Pythagoras format:

$$(\Delta t)^2 = (\Delta\tau)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$$

This influence is easily recognizable in the corresponding wave equations:

In Maxell-Minkowski format the wave equation uses coordinate time t. It runs as:

$$\partial^2\psi/\partial t^2 - \partial^2\psi/\partial x^2 - \partial^2\psi/\partial y^2 - \partial^2\psi/\partial z^2 = 0$$

Papers on Huygens principle work with this formula or it uses the version with polar coordinates.

For 3D the general solution runs:

$\psi = f(r - ct)/r$, where $c = \pm 1$; f is real

For 1D the general solution runs:

$\psi = f(x - ct)$, where $c = \pm 1$; f is real

For the Hamilton-Euclidean version, which uses proper time τ, we use the quaternionic nabla ∇:

$$\nabla = \left\{\frac{\partial}{\partial\tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right\} = \nabla_0 + \boldsymbol{\nabla};$$

$$\nabla^* = \nabla_0 - \boldsymbol{\nabla}$$

$$\nabla\psi = \nabla_0\,\psi_0 - (\boldsymbol{\nabla}, \boldsymbol{\psi}) + \nabla_0\,\boldsymbol{\psi} + \boldsymbol{\nabla}\,\psi_0 \pm \boldsymbol{\nabla} \times \boldsymbol{\psi}$$

The $\pm$ sign reflects the choice between right handed and left handed quaternions.

In this way the Hamilton-Euclidean format of the wave equation runs:

$$\nabla^*\nabla\psi = \nabla_0\nabla_0\psi + (\boldsymbol{\nabla}, \boldsymbol{\nabla})\psi = 0$$

$$\partial^2\psi/\partial\tau^2 + \partial^2\psi/\partial x^2 + \partial^2\psi/\partial y^2 + \partial^2\psi/\partial z^2 = 0$$

Where $\psi = \psi_0 + \boldsymbol{\psi}$

For the general solution holds: $f = f_0 + \boldsymbol{f}$

For the real part ψ_0 of ψ:

$\psi_0 = f_0\,(\boldsymbol{i}\,r - c\,\tau)/r$, where $c = \pm 1$ and $\boldsymbol{i}$ is an imaginary base vector in radial direction

For the imaginary part $\boldsymbol{\psi}$ of ψ:

$\boldsymbol{\psi} = \boldsymbol{f}(\boldsymbol{i}\,z - c\,\tau)$, where $c = \pm 1$ and $\boldsymbol{i} = \boldsymbol{i}(z)$ is an imaginary base vector in the x, y plane

The orientation $\theta(z)$ of $\boldsymbol{i}(z)$ in the x, y plane determines the polarization of the 1D wave front.

16 Discussion

Accepting of the first principles that constitute the foundation of the model leads undeniably to a basic model that is based on a combination of an infinite dimensional separable quaternionic Hilbert space that owns a corresponding Gelfand triple. Both the Hilbert space and the Gelfand triple are just dumb structured storage media that do not contain any means to control the coherence of dynamics.

Still the discrete part of the model, which is stored in the separable Hilbert space, can be considered to be regenerated at the pace of a model wide progression step and generations of discrete object types are regenerated according to a series of specific model wide progression cycles.

At that instance a mechanism is introduced that controls dynamical and spatial coherence and schedules parallel tasks. This mechanism is restricted by the fact that it must recurrently embed the discrete part of the model into the continuum part of the model. Here a significant piece of mathematics still fails. It seems that discrete objects appear where quaternions are embedded in continuums that belong to a different symmetry flavor. When they appear they produce local singularities in the embedding continuum. The singularities represent sources of wave fronts.

The fact that elementary particles are represented by location swarms is very particular. The swarm is a structural representation of the behavior of the elementary particle, which is a point-like object. In this way this structure can represent the statistical and symmetric properties of that behavior. In addition the swarm can be described by a continuous location density distribution, which has a Fourier transform. These smooth descriptors hide much of the fine grain behavior of the owner of this continuous location density distribution. It also hides the phenomena that happen during the embedding of the locations and the hops in the surrounding continuum. During this embedding process the sources of the potentials that attach to the owner of the swarm originate.

The most fine grain substances of the model are duals of locations and hops that are stored in two quaternions, which share their real part. All discrete objects in the model are represented by these duals. At every progression hop their behavior can be represented by duals of closed subspaces of an infinite dimensional separable Hilbert space. One member of the dual contains the locations. The other contains the hops.

The model suggests that the embedding process causes the emission of wave fronts. The embedding of locations is supposed to cause the emission of 3D wave fronts and these are supposed to curve the embedding continuum. The embedding of hops is supposed to cause the emission of 1D wave fronts. These emissions only occur when there exist a symmetry flavor discrepancy between the embedded quaternions and the embedding quaternionic functions. Probably the 1D wave fronts are only emitted in directions in which this discrepancy occurs. This might then explain the partial electric charge of quarks.

17 Next task

The next task is the precise formulation of the gap that still exist in the mathematical explanation of the embedding process. This embedding process is the factual generator of the discrete building blocks. The germs of these building blocks are duals that consist of a location and a hop. In the embedding process the location and the hop behave differently. The embedding process reacts actively when a discrepancy in symmetry flavor exists between the embedded item and the embedding continuum. Its activity is controlled by the discrepancy. However, embedding of non-discrepant symmetry flavors also produces noticeable effects.

Embedding of a location quaternion into a discrepant quaternionic continuum is supposed to cause a 3D wave front that is emitted at the point of embedment. The 3D wave front is supposed to slightly fold and thus curve the embedding continuum. The amplitude of the 3D wave front and thus its influence diminishes with distance r as 1/r.

Embedding of a hop quaternion into a discrepant quaternionic continuum is supposed to cause a 1D wave front that is emitted at the point of embedment, which is the landing location. The 1D wave fronts keep their amplitude and do not curve the embedding continuum. The 1D wave fronts will feature the same angular distribution as the hops do.

The wave fronts are the results of local singularities in the embedding continuum. A proper mathematical theory that treats these local singularities still fails.

Objects exist for which singularities occur while no symmetry flavor differences exist between their constituting duals and the embedding continuum. These objects have no electric charge. For example photons still contain 1D wave fronts. Z-particles and neutrinos still produce 3D wave fronts. Photons and neutrinos behave differently from more common elementary particles.

18 Appendix: History of discoveries

The concept of "Universe" follows with mathematical inescapable evidence from first principles that constitute a recipe for modular construction. These first principles define an orthomodular lattice. This is the structure of quantum logic.

Quantum logic was introduced by Garret Birkhoff and John von Neumann in their 1938 paper. G. Birkhoff and J. von Neumann, *The Logic of Quantum Mechanics*, Annals of Mathematics, Vol. 37, pp. 823–843

The lattices of quantum logic and classical logic are treated in detail in: http://vixra.org/abs/1411.0175 .

The Hilbert space was discovered in the first decades of the 20-th century by David Hilbert and others. http://en.wikipedia.org/wiki/Hilbert_space.

Paul Dirac introduced the bra-ket notation, which popularized the usage of Hilbert spaces. Dirac also introduced its delta function, which is a generalized function. Generalized functions offered continuums before the Gelfand triple arrived.

Quaternionic Hilbert spaces are treated in: http://vixra.org/abs/1411.0178 .

In 1843 quaternions were discovered by Rowan Hamilton. http://en.wikipedia.org/wiki/History_of_quaternions

In the sixties Constantin Piron and Maria Pia Solèr proved that the number systems that a separable Hilbert space can use must be division rings. “Division algebras and quantum theory” by John Baez. http://arxiv.org/abs/1101.5690

In the sixties Israel Gelfand and Georgyi Shilov introduced a way to model continuums via an extension of the separable Hilbert space into a so called Gelfand triple. The Gelfand triple often gets the name rigged Hilbert space, which is confusing, because this construct is not a Hilbert space. http://www.encyclopediaofmath.org/index.php?title=Rigged_Hilbert_space

These discoveries are used as foundations by the e-book “The Hilbert Book Model Game”. http://vixra.org/abs/1405.0340 .