A mathematical model of reality

Abstract

It is possible to derive a model from well selected first principles that shows many similarities with what we know from observing reality.

The presented model is based on first principles that formulate a recipe for modular construction and this foundation is mathematically known as an orthocomplemented weakly modular lattice. When starting from this foundation it is mathematically inescapable evident that this model confines to a quaternionic space-progression model that proceeds with model-wide progression steps through an ordered sequence of static sub-models that each represent a static status quo of the whole model.

The model is a hybrid construct that consists of a sub model that treats all discrete objects and a continuum based model that embeds the discrete objects.

Despite the fact that the target of the model is to reach a level in which it shows many features and phenomena that we know from observing reality, this model is not claimed to be a model of physics. The reason for this restriction is that many of its aspects cannot be observed. Physicists tend to deny completely or largely deduced models.

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If the model introduces new science, then it has fulfilled its purpose.

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1 Generating the model

1.1 First principles

We use a paginated space progression model that emerges from a skeleton relational structure. This structure can mathematically be characterized as an orthocomplemented weakly modular lattice[1]. Another name for this lattice is orthomodular lattice. Quantum logic has this lattice structure. Classical logic has a slightly different lattice structure. It is an orthocomplemented modular lattice. For our purpose it is better to interpret the elements of the orthomodular lattice as construction elements rather than as propositions. The ~25 axioms that define an orthomodular lattice form the first principles on which the whole model will be built. Only trustworthy, mostly mathematical methods will be used to extend this model until a level is reached in which it shows many features and phenomena that we recognize from observing reality.

1.2 The next level

The set of closed subspaces of an infinite dimensional separable Hilbert space is also an orthomodular lattice. The Hilbert space adds extra functionality to this orthomodular lattice. This extra functionality concerns the superposition principle and the possibility to store data in eigenspaces of normal operators. In the form of Hilbert vectors the Hilbert space features a finer structure than the orthomodular lattice has.

The Hilbert space can only handle members of a division ring for specifying superposition coefficients, for the eigenvalues of its operators and for the values of its inner products. Only three suitable division rings exist: the real numbers, the complex numbers and the quaternions. Quaternions enable the storage of 1+3D data that have an Euclidean geometric structure.

Thus, selecting a skeleton relational structure that is an orthomodular lattice as the foundation of the model already puts significant restrictions to the model. On the other hand, this choice promotes modular construction and in this way it significantly reduces the relational complexity of the final model.

1.3 Adding dynamics

This primitive model does not provide means to control dynamics and it does not support the representation of continuums.

Dynamics can be added by using an ordered sequence of the models that can represent a static status quo. This choice makes the model paginated. The model proceeds with model-wide progression steps. All discrete objects in the model can be considered to be regenerated at every progression step.

With this decision, an extra mechanism must be added that ensures sufficient coherence between subsequent elements of the sequence. The coherence must not be too stiff, otherwise no dynamics occurs. On the other hand it must be sufficient restrictive, otherwise the result is dynamical chaos.

In order to reach sufficient coherence the next sequence member must not differ much from the considered member.

[1] See the appendix

This mechanism shares many aspects with a real time operating system. The RTOS schedules subtasks and it ensures that these subs-tasks occur in sync.

1.4 Adding continuums

Continuums can be supported by adding the Gelfand triple to the Hilbert space. The Gelfand triple can be used to check the coherence. This is done by ***embedding*** the subsequent Hilbert spaces into a common Gelfand triple. As a consequence progression steps along the Hilbert spaces and it flows inside the Gelfand triple. This allows the embedding process to control the dynamic as well as the spatial coherence.

The embedding process already puts many restrictions that at least partly ensure the dynamical coherence.

2 Modular construction

Thus, if the orthomodular lattice is considered as the foundation of the model, then the separable Hilbert space is the next level of extension of the model. The foundation can be considered as part of a recipe for modular construction. What is missing are the binding mechanism and a way to hide part of the relations that exist inside the modules from the outside of the modules. These ingredients are delivered by the superposition principle and by the embedding mechanism.

3 Exploiting the model

At every progression instant, each discrete construct in this model is supposed to expose the skeleton relational structure that is defined as an orthomodular lattice.

At each progression instant, every discrete construct in this model can be represented by a closed subspace of a single separable Hilbert space.

3.1 Embedding the most elementary objects

The embedding process gives every elementary building block an actual location. At the next progression step that location will differ. At each progression step the elementary building block will hop to the next location. The next location is not known in advance. It is determined by a stochastic process.

At each progression instant, every discrete building block in this model owns an exact hopping value and an exact location that together form a *dual*. Both members of the dual share the same real part, which stores the progression value.

Other elementary objects exist that have an exact location at a series of progression steps. Also these objects hop from the current location to the next location. The location and the hop form a *dual.*

The *duals* form the most elementary objects in the model. On themselves they do not have any other characteristic than their quaternionic value. Only as sets these *duals* become extra significance. Extra data are obtained from the statistics of the set or from the symmetry properties of the set. The hops form a path and this path adds its own characteristics.

The set can have one of three forms:

- A coherent swarm
- A closed path
- An open path string

3.2 Swarms

The swarm differs from the two other forms in the fact that it can be characterized by a density distribution. The swarm is a coherent set. Two interpretations are possible:

- The swarm is generated by an ongoing stochastic process. After a while the statistic characteristics of the swarm stabilize.
- The swarm is prepared in advance. Its elements are used one by one. The currently active element is obtained by random selection from the set of not yet used elements. When all elements are used, then the swarm is regenerated.

Here we take the second interpretation. We do this because it is easier to understand. It means that the swarm is generated by a cyclic stochastic process. The swarm contains a huge number of elements. The swarm can be described by a normalized continuous location density distribution[2]. This continuous distribution has a Fourier transform. As a consequence the swarm owns a displacement generator. Thus, at first approximation the swarm moves as one unit.

These measures are part of the task of the mechanism that must ensure sufficient coherence between the elements of the sequence of static sub-models that together form the dynamic model.

The swarm includes a closed path. We suppose that the statistics of the planned swarm are stable. Under the mentioned conditions, the swarm is at rest. It means that the sum of all hops equals zero. In this condition the swarm has a fixed number of elements.

3.3 Moving the swarm

Adding extra duals to the swarm causes a movement of the extended set. Adding particular sets of hops may cause an oscillation of the swarm. This occurs in typical oscillation modes. These extra sets form cycles. They are closed path objects. Adding or retrieving such sets must be done in sync with the swarm regeneration process. The sets that leave the oscillating swarm are open path strings. Such open path strings can also enter the free swarm or an already oscillating swarm. In principle the oscillations keep the swarm on average at the same location.

Adding a more arbitrary set of duals or an open path string that does not fit for establishing an oscillation, will cause a translation of the possibly oscillating swarm. An entering string can be broken into one or more fitting open path strings and a translation set. The translation set increases the kinetic energy of the composite.

3.3.1 Synchronization

However, adding duals to the swarm itself will disturb the regeneration synchronization of higher order constructs. The generation of the swarm represents a cycle period that is used in the construction of composites. The composites are generated by parallel process that all act within the same cycle period.

Thus the addition of series of duals that cause movement of the swarm occurs in a parallel process and the action of the members of the series are spread over the cycle period. This gives these extra duals a different character than the duals that form the swarm. The extra duals act on the full swarm and can be considered as incremental displacement generators in configuration space. They can be seen as superposition coefficients in Fourier (=momentum) space.

In order to explain this a mathematical intermezzo is inserted.

[2] The normalized continuous location density distribution corresponds to the squared modulus of the wave function that characterizes elementary particles in contemporary physics.

4 Mathematical intermezzo

The equations in this intermezzo are based on application in a flat continuum. In practice this only holds under special conditions. In general the embedding continuum is curved. Later we treat the influence of curvature.

4.1 Functions as Hilbert space operators

By using bra-ket notation, operators that reside in the Hilbert space and correspond to continuous functions, can easily be defined starting from an orthogonal base of vectors.

Let $\{q_i\}$ be the set of rational quaternions and $\{|q_i\rangle\}$ be the set of corresponding base vectors.

$|q_i\rangle q_i\langle q_i|$ is the configuration parameter space operator.

Let $f(q)$ be a quaternionic function.

$|q_i\rangle f(q_i)\langle q_i|$ defines a new operator that is based on function $f(q)$.

In the Gelfand triple, the continuous function $f(q)$ can be defined between a continuum eigenspace that acts as target space and the eigenspace of the reference operator $|q\rangle q\langle q|$ $t$hat acts as parameter space.

In the Gelfand triple the dimension of a subspace loses its significance. Thus a function that is derived from the representation of a coherent swarm in Hilbert space has a dimension in Hilbert space, but loses that characteristic in its representation in the Gelfand triple.

4.2 Quaternion geometry and arithmetic

Quaternions and quaternionic functions offer the advantage of a very compact notation of items that belong together.

Quaternions can be considered as the combination of a real scalar and a 3D vector that has real coefficients. This vector forms the imaginary part of the quaternion. Quaternionic number systems are division rings.

Bi-quaternions exist whose parts exist of a complex scalar and a 3D vector that has complex coefficients. Bi-quaternions do not form division rings. This model does not use them.

4.2.1 Notation

We indicate the real part of quaternion a by the suffix a_0.

We indicate the imaginary part of quaternion a by bold face $\boldsymbol{a}$.

$$a = a_0 + \boldsymbol{a} \tag{1}$$

4.2.2 Sum

$$c = c_0 + \boldsymbol{c} = a + b \quad (1)$$

$$c_0 = a_0 + b_0 \quad (2)$$

$$\boldsymbol{c} = \boldsymbol{a} + \boldsymbol{b} \quad (3)$$

4.2.3 Product

$$f = f_0 + \boldsymbol{f} = d\,e \quad (1)$$

$$f_0 = d_0 e_0 - \langle \boldsymbol{d}, \boldsymbol{e} \rangle \quad (2)$$

$$\boldsymbol{f} = d_0 \boldsymbol{e} + e_0 \boldsymbol{d} \pm \boldsymbol{d} \times \boldsymbol{e} \quad (3)$$

The $\pm$ sign indicates the influence of right or left handedness of the number system[3].

$\langle \boldsymbol{d}, \boldsymbol{e} \rangle$ is the inner product of $\boldsymbol{d}$ and $\boldsymbol{e}$.

$\boldsymbol{d} \times \boldsymbol{e}$ is the outer product of $\boldsymbol{d}$ and $\boldsymbol{e}$.

4.2.4 Norm

$$|a| = \sqrt{a_0 a_0 + \langle \boldsymbol{a}, \boldsymbol{a} \rangle} \quad (1)$$

4.2.5 Rotation

Quaternions are often used to represent rotations.

$$c = ab/a \quad (1)$$

rotates the imaginary part of b that is perpendicular to the imaginary part of a[4].

4.3 Quaternionic functions

4.3.1 Norm

Square-integrable functions are normalizable. The norm is defined by:

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[3] Quaternionic number systems exist in 16 symmetry flavors. Within a coherent set all elements belong to the same symmetry flavor.

[4] See Q-FORMULÆ

$$\|\psi\|^2 = \int_V |\psi|^2 \, dV \tag{1}$$

$$= \int_V \{|\psi_0|^2 + \ |\boldsymbol{\psi}|^2 \,\} dV$$

$$= \|\psi_0\|^2 + \|\boldsymbol{\psi}\|^2$$

4.3.2 Differentiation

If g is differentiable then the quaternionic nabla ∇g of g exists.

The quaternionic nabla ∇ is a shorthand for $\nabla_0 + \boldsymbol{\nabla}$

$$\nabla_0 = \frac{\partial}{\partial \tau} \tag{3}$$

$$\boldsymbol{\nabla} = \left\{\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right\} \tag{4}$$

$$h = h_0 + \boldsymbol{h} = \nabla g \tag{4}$$

$$h_0 = \nabla_0 g_0 - \langle \boldsymbol{\nabla}, \boldsymbol{g} \rangle \tag{5}$$

$$\boldsymbol{h} = \nabla_0 \boldsymbol{g} + \boldsymbol{\nabla} g_0 \pm \boldsymbol{\nabla} \times \boldsymbol{g} \tag{6}$$

$$\phi = \nabla\psi \ \Rightarrow \phi^* = (\nabla\psi)^* \tag{7}$$

$$(\nabla\psi)^* = \nabla_0\psi_0 - \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle - \nabla_0 \boldsymbol{\psi} - \boldsymbol{\nabla}\psi_0 \mp \boldsymbol{\nabla} \times \boldsymbol{\psi} \tag{8}$$

$$\nabla^*\psi^* = \ \nabla_0\psi_0 - \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle - \nabla_0 \boldsymbol{\psi} - \boldsymbol{\nabla}\psi_0 \pm \boldsymbol{\nabla} \times \boldsymbol{\psi} \tag{9}$$

Similarity of these equations with Maxwell equations is not accidental. In Maxwell equations several terms in the above equations have been given special names.

4.3.3 Gauge transformation

For a function χ that obeys

$$\nabla^*\nabla\chi = \nabla_0\nabla_0\chi + \langle \boldsymbol{\nabla}, \boldsymbol{\nabla}\chi \rangle = 0 \tag{1}$$

the value of ϕ in

$$\phi = \nabla\psi \tag{2}$$

does not change after the gauge transformation[5]

$$\psi \to \psi + \xi = \psi + \nabla^*\chi \tag{3}$$

$$\nabla\xi = 0$$

Thus in general:

$$\nabla^*\nabla\psi = \nabla_0\nabla_0\psi + \langle \boldsymbol{\nabla}, \boldsymbol{\nabla}\psi \rangle = \rho \neq 0 \tag{4}$$

ρ is a quaternionic function.

Its real part ρ_0 represents an object density distribution.

Its imaginary part $\boldsymbol{\rho}$ represents a current density distribution.

$$\nabla^*\nabla\chi_0 = 0 \tag{5}$$

$$\nabla^*\nabla\boldsymbol{\chi} = 0 \tag{6}$$

Equation (5) forms the basis of the Huygens principle.

It is the reason of the emission of a 3D wave front during the process of embedding of a discrepant object at a given location in a continuum.

[5] The qualification gauge transformation is usually given to a transformation that leaves the Laplacian untouched. Here we use that qualification for transformations that leave the quaternionic differential untouched.

In a similar way equation (6) forms the basis of the emission of a 1D wave front during the embedding of a discrepant displacement in the continuum. The direction of the displacements is coupled to the direction of the emitted 1D wave front.

Small displacements can be treated in a local complex number based sub-model.

In a complex number based model the 3D gauge transformation becomes a 1D gauge transformation.

4.3.4 Displacement generator

The definition of the differential is

$$\Phi = \nabla\psi \tag{1}$$

In Fourier space the nabla becomes a displacement generator.

$$\tilde{\Phi} = \mathcal{M}\tilde{\psi} \tag{2}$$

$\mathcal{M}$ is the ***displacement generator***

A small displacement in configuration space becomes a multiplier in Fourier space.

In a paginated space-progression model the displacements are small and the displacement generators work incremental. The multipliers act as superposition coefficients.

4.3.5 The coupling equation

The coupling equation follows from peculiar properties of the differential equation. We start with two normalized functions ψ and φ and a normalizable function $\Phi = m\ \varphi$.

$$||\psi|| = ||\varphi|| = 1 \tag{1}$$

These normalized functions are supposed to be related by:

$$\Phi = \nabla\psi = m\ \varphi \tag{2}$$

$\Phi = \nabla\psi$ defines the differential equation. (3)

$\nabla\psi = \Phi$ formulates a continuity equation. (4)

$\nabla\psi = m\ \varphi$ formulates the coupling equation. (5)

It couples ψ to φ. m is the coupling factor.

$$\nabla\psi = m_1\ \varphi \tag{6}$$

$$\nabla^*\varphi = m_2\ \zeta \tag{7}$$

$$\nabla^*\nabla\psi = m_1\ \nabla^*\varphi = m_1 m_2 \zeta = \rho \tag{8}$$

Each double differentiable quaternionic function represents a normalized density distribution.

4.3.5.1 In Fourier space

The Fourier transform of the coupling equation is:

$$\mathcal{M}\tilde{\psi} = m\tilde{\varphi} \tag{1}$$

$\mathcal{M}$ is the ***displacement generator***

5 What happens to the continuum?

5.1 Embedding

In the previous chapter only mathematical formulas were listed. Here we give these formulas an interpretation in the model.

The quaternionic differential equation

$$\phi = \nabla\psi \tag{1}$$

can be interpreted as a continuity equation. It describes how a coherent set of discrete objects are embedded in a continuum.

$$\nabla^*\nabla\psi = \nabla_0\nabla_0\psi + \langle\boldsymbol{\nabla},\boldsymbol{\nabla}\psi\rangle = \rho \neq 0 \tag{2}$$

$$\nabla\chi = 0 \tag{3}$$

$$\nabla^*\nabla\chi = \nabla_0\nabla_0\chi + \langle\boldsymbol{\nabla},\boldsymbol{\nabla}\chi\rangle = 0 \tag{4}$$

Here ψ describes the embedding continuum including the added coherent swarm of objects ρ in the form of a normalized continuous quaternionic density distribution. χ describes the embedding continuum without the swarm.

The quaternionic function χ describes how the embedding continuum reacts on the embedding.

Formula (4) is used by the Huygens principle. For each embedding of the location part of a dual a 3D wave front is generated at that location.

For each embedding of the displacement part of a dual a 1D wave front is generated at the landing location of the hop. For this case the formula can better be considered in a local complex number based context. The angular distribution of the 1D wave fronts depends on the angular distribution of the hops.

Only a discrepancy in the symmetry flavors that are coupled leads to the singularity that causes the emission of a wave front that goes together with the embedding of a discrete object into an embedding continuum.

5.2 Superposition

The coupling equation shows that an incremental displacement in configuration space corresponds to a multiplication factor in Fourier space.

$$\nabla\psi = m\,\varphi \tag{1}$$

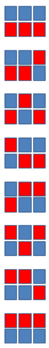
$$\mathcal{M}\tilde{\psi} = m\tilde{\varphi} \tag{2}$$

This multiplication factor can be interpreted as a superposition coefficient.

5.3 Quaternionic function symmetry flavors

Continuous quaternionic functions do not switch to other symmetry flavors.

- If the real part is ignored, then still ***8*** symmetry flavors result
- They are marked by special indices, for example $\boldsymbol{\psi}^{\textcircled{4}}$
- $\boldsymbol{\psi}^{\textcircled{0}}$is the reference symmetry flavor
- They are also marked by colors $N, R, G, B, \bar{B}, \bar{G}, \bar{R}, \bar{N}$
- Half of them is right handed, **R**
- The other half is left handed, **L**



$\boldsymbol{\psi}^{\textcircled{0}} N$ ***R***	The colored rectangles reflect the directions of the axes
$\boldsymbol{\psi}^{\textcircled{1}} R$ ***L***	
$\boldsymbol{\psi}^{\textcircled{2}} G$ ***L***	
$\boldsymbol{\psi}^{\textcircled{3}} B$ ***L***	
$\boldsymbol{\psi}^{\textcircled{4}} \bar{B}$ ***R***	
$\boldsymbol{\psi}^{\textcircled{5}} \bar{G}$ ***R***	
$\boldsymbol{\psi}^{\textcircled{6}} \bar{R}$ ***R***	
$\boldsymbol{\psi}^{\textcircled{7}} W$ ***L***	

Also continuums feature a symmetry flavor. The reference symmetry flavor is the symmetry flavor of the parameter space. The parameter space is a flat continuum.

If the continuous quaternionic function describes the density distribution of a set of discrete objects, then this set can be attributed with the same symmetry flavor.

5.4 Influence of symmetries

The embedding process is controlled by the symmetry flavors of the embedded object and the embedding continuum. Quaternions number systems as well as continuous quaternionic functions exist in 16 symmetry flavors. Even when the real parts are ignored this results in a variety of 8×8=16 different embedding products. Enough to cover all first generation members of the standard model.

For example the Dirac equation for the free electron in quaternionic format runs:

$$\nabla\psi = m_e\, \psi^* \tag{1}$$

And the Dirac equation for the positron runs:

$$\nabla^*\psi^* = m_e\, \psi \tag{2}$$

Thus

$$\nabla^*\nabla\psi = m_e\nabla^* \psi^* = m_e^2\, \psi \tag{3}$$

For electrons ψ represents its own normalized object density distribution.

ψ^* and ψ are symmetry flavors of the same base function.

Other elementary particles couple different symmetry flavors $\{\psi^x, \psi^y\}$ of their shared base function:

$$\nabla\psi^x = m_{xy}\, \psi^y \tag{4}$$

And for the antiparticle:

$$\nabla^*(\psi^x)^* = m_{xy}\, (\psi^y)^* \tag{5}$$

The difference in the symmetry flavors between the members of the pair $\{\psi^x, \psi^y\}$ can be related to the electric charge, color charge and spin of the corresponding elementary particle.

Fermions appear to couple to the reference symmetry flavor $\boldsymbol{\psi}^{⓪}$.

5.5 Coupling properties

Discrepancies between the coupled symmetry flavors determine the properties of the coupling result. For example electric charge depends on the number of dimensions in which symmetry flavors differ. Also the direction in which they differ is important. Further is important whether handedness switches. Color charge also changes with the number of dimensions in which symmetry flavors differ. Spin appears to depend on the fact whether the embedding continuum has the reference flavor.

5.6 Curvature

In practice the emission of 3D wave fronts will cause a local folding and thus a curvature of the embedding continuum. This effect is the basis of the gravitation potential, which represents the averaged effects of these wave fronts.

In a curved environment the quaternionic nabla must be replaced by a differential that is constituted of 16 partial derivatives.

Where the 3D wave fronts decrease their amplitude with distance from the source, will the amplitude of the 1D wave fronts stay constant. As a consequence the 1D wave fronts do not curve the embedding continuum. Depending on the angular distribution of the hops that generated them, the 1D wave fronts also combine and average down to an up to 3D potential. In contemporary physics this potential is known as electromagnetic potential. The messengers keep the amplitude of their 1D wave fronts.

Curvature affects the map of the swarm onto the curved embedding continuum. The overall map $\mathcal{P}$ produces a blurred target. It is described by the convolution of a sharp continuous quaternionic allocation function $\wp$ and a stochastic spatial spread function $\mathcal{S}$.

$$\mathcal{P} = \wp \circ \mathcal{S} \tag{1}$$

The allocation function has a flat parameter space. It describes the history of the path of the concerned particle. At the same time it describes the embedding continuum.

The stochastic spatial spread function $\mathcal{S}$ produces a blurred image ψ of a swarm. The swarm is produced by the combination of a Poisson process and a binomial process. The spread function implements the binomial process. $\mathcal{S}$ has progression as its single real parameter. The Fourier transform $\check{\psi}$ of the density distribution ψ that describes this swarm acts as a mapping quality characteristic.

5.7 Metric

The differential of the sharp allocation function defines a kind of quaternionic metric.

$$ds(q) = ds^{\nu}(q)e_{\nu} = d\wp = \sum_{\mu=0\ldots3} \frac{\partial \wp}{\partial q_{\mu}} dq_{\mu} = c^{\mu}(q) dq_{\mu}$$

q is the quaternionic location.

ds is the metric.

c^{μ} is a quaternionic function.

Pythagoras:

$$c^2 dt^2 = ds\, ds^* = dq_0^2 + dq_1^2 + dq_2^2 + dq_3^2 \tag{2}$$

Minkowski:

$$dq_0^2 = d\tau^2 = c^2\, t^2 - dq_1^2 - dq_2^2 - dq_3^2 \tag{3}$$

In flat space:

$$\Delta s_{flat} = \Delta q_0 + \boldsymbol{i}\, \Delta q_1 + \boldsymbol{j}\, \Delta q_2 + \boldsymbol{k}\, \Delta q_3 \tag{4}$$

In curved space:

$$\Delta s_{\wp} = c^0\, \Delta q_0 + c^1\, \Delta q_1 + \ c^2\, \Delta q_2 + c^3\, \Delta q_3 \tag{5}$$

$d\wp$ is a quaternionic metric

It is a linear combination of 16 partial derivatives

6 Energy

In the model the energy of a composite is directly related to the number of duals that constitute the composite. It is also directly related to the dimension of the dual subspace that represents the composite.

In the open path objects energy is related to the number of hops that constitute the object. This is also equal to the number of 1D wave fronts that constitute the object.

Oscillations that are internal to a composite are represented by closed path objects. The enclosed extra hops add to the energy of the composite.

7 Potentials

In this model potentials form the averages over a small period of progression and a region of space of the wave fronts that are emitted during embedding of particles.

7.1 The location potential

The wave fronts that are emitted during the embedding of the location members of the duals are isotropic 3D wave fronts. Their spreading is controlled by the 3D version of the Huygens principle. This means that their amplitude decreases with the distance from the source as $1/r$.

Here we consider a simplified situation. With an isotropic density distribution $\rho_0(r)$ in the swarm the scalar potential $\varphi_0(R)$ can be estimated as:

$$\varphi_0(R) = \int_0^R \rho_0(r) dr \tag{1}$$

R is the distance to the center of the swarm.

If the density distribution approaches a 3D Gaussian distribution, then this integral equals[6]:

$$\varphi_0(R) = \mathrm{Erf}(R)/R \tag{2}$$

We suppose that this distribution is a good estimate for the structure of the swarm of a free electron. It is remarkable that this potential has no singularity at $R = 0$. At the same time, already at a short distance of the center the function very closely approaches $1/R$.

7.1.1 Moving swarm

If the swarm moves with uniform speed $\boldsymbol{v}$ than this conforms to a vector potential $\boldsymbol{\varphi}(R)$:

$$\boldsymbol{\varphi}(R) = \int_0^R \boldsymbol{v}\, \rho_0(r) dr = \boldsymbol{v}\ \varphi_0(R) \tag{1}$$

Now we use

$$\mathfrak{E} = \nabla\varphi = \nabla_0\varphi_0 - \langle \boldsymbol{\nabla}, \boldsymbol{\varphi} \rangle + \nabla_0 \boldsymbol{\varphi} - \boldsymbol{\nabla}\varphi_0 \mp \boldsymbol{\nabla} \times \boldsymbol{\varphi}$$

An acceleration of the swarm goes together with an extra vector field $\boldsymbol{\mathfrak{E}}$:

$$\boldsymbol{\mathfrak{E}} \approx \nabla_0\, \boldsymbol{\varphi}(R) = \dot{\boldsymbol{v}}\, \varphi_0\,(R) \tag{2}$$

[6] http://en.wikipedia.org/wiki/Poisson's_equation#Potential_of_a_Gaussian_charge_density

Other terms are small.

7.2 The hop potential

The wave fronts that are emitted during the embedding of the displacement members of the duals are planar 1D wave fronts. Their spreading is controlled by the 1D version of the Huygens principle. This means that their amplitude does not decrease with distance from the source. As a consequence these wave fronts do not curve the embedding continuum.

For an isotropic swarm the angular distribution of the wave fronts is isotropic. The corresponding location distribution may again approach a Gaussian distribution. The sum of all hops is supposed to equal zero. The spread of the influences behave similar to the spread of the influences of the location related wave fronts. Thus here the same formulas holds as for the location related wave fronts.

8 Inertia

In the model, universe is ***coarsely uniformly*** covered with swarms. These swarms represent a location potential $\varphi_0(R)$ that at larger distances deceases as $1/r$. Together the most distant swarms together deliver the largest contribution. Locally these potentials combine in a scalar potential ϕ_0 :

$$\phi_0 = \int \varphi_0(R)\, dV \tag{1}$$

A moving swarm will go together with a vector potential $\boldsymbol{\phi}$.

$$\boldsymbol{\phi} = \int \boldsymbol{v}\, \varphi_0(R)\, dV = \boldsymbol{v}\, \phi_0 \tag{2}$$

Now we use

$$\mathfrak{G} = \nabla\phi = \nabla_0\phi_0 - \langle \boldsymbol{\nabla}, \boldsymbol{\phi} \rangle + \nabla_0\boldsymbol{\phi} - \boldsymbol{\nabla}\phi_0 \mp \boldsymbol{\nabla} \times \boldsymbol{\phi}$$

An accelerating swarm will encounter a vector field $\boldsymbol{\mathfrak{G}}$:

$$\boldsymbol{\mathfrak{G}} \approx \nabla_0\boldsymbol{\phi} = \dot{\boldsymbol{v}}\, \phi_0 \tag{3}$$

Other terms are small.

This field counteracts the acceleration.

9 Cosmos

9.1 Integral continuity equations

The integral equations that describe cosmology are:

$$\int_V \nabla \rho \, dV = \int_V s \, dV \tag{1}$$

$$\int_V \nabla_0 \rho_0 \, dV = \int_V \langle \boldsymbol{\nabla}, \boldsymbol{\rho} \rangle \, dV + \int_V s_0 \, dV \tag{2}$$

$$\int_V \nabla_0 \, \boldsymbol{\rho} \, dV = - \int_V \boldsymbol{\nabla} \rho_0 \, dV - \int_V \boldsymbol{\nabla} \times \boldsymbol{\rho} \, dV + \int_V \boldsymbol{s} \, dV \tag{3}$$

$$\frac{d}{d\tau} \int_V \rho \, dV + \oint_S \widehat{\boldsymbol{n}} \rho \, dS = \int_V s \, dV \tag{4}$$

Here $\widehat{\boldsymbol{n}}$ is the normal vector pointing outward the surrounding surface S, $\boldsymbol{v}(\tau, \boldsymbol{q})$ is the velocity at which the charge density $\rho_0(\tau, \boldsymbol{q})$ enters volume V and s_0 is the source density inside V. In the above formula ρ stands for

$$\rho = \rho_0 + \boldsymbol{\rho} = \rho_0 + \frac{\rho_0 \boldsymbol{v}}{c} \tag{4}$$

It is the flux (flow per unit of area and per unit of progression) of ρ_0 . τ stands for progression.

9.2 Space cavities

A static space cavity is characterized by:

$$\frac{d}{d\tau} \int_V \rho \, dV = 0 \tag{1}$$

All properties of this object depend on the surrounding surface.

These objects are known as black holes.

9.3 Inversion surfaces

An inversion surface S is characterized by:

$$\oint_S \hat{\boldsymbol{n}}\rho \, dS = 0 \tag{1}$$

It is supposed that duals are stopped, but that potentials and their constituting wave fronts can still pass this inversion surface.

The inversion surfaces divide universe into compartments.

10 Messengers

10.1 Observed behavior

Photons are very special objects that are emitted by oscillating composites when they step down from a higher oscillation mode to a lower oscillation mode. Absorption of photons by a composite occurs when the composite steps up from a lower oscillation mode to a higher oscillation mode.

Further photons play a role in the creation and the annihilation of pairs of elementary building blocks. The pair consists of a merge of an elementary particle and its antiparticle.

A very particular difference occurs between young and old photons. At recent detection, old photons appear to be red-shifted.

10.2 Representation in the model

In the model messengers are represented by open chains of duals. Duals are constituted from a location and a hop. At embedding the location produces a 3D wave front, whose amplitude quickly diminishes and the hop produces a 1D wave front that keeps its amplitude. The 3D wave front produces a slight curvature of the embedding continuum. The messengers are the equivalents of photons. The emission and the absorption of messengers are controlled by processes that work in parallel to the generation of swarms. Locally these processes act in sync. These processing periods depend on the number of involved progression steps. It is not probable that the number of involved progression steps will change with progression. However, the duration of a single progression step may change with progression.

The 3D wave fronts in the messengers cause a slight space curvature that is spread along the chain that represents the messenger. Thus in the model messengers have mass. On the other hand the 1D wave fronts represent energy.

With other words, the emission of an old messenger lasted as long as in those conditions the absorption of a messenger lasted, but the emission and absorption of young messengers takes a shorter duration. This means that when an old messenger is recently absorbed, then only part of the number of wave fronts are detected. With other words the detected old messenger appears to be red-shifted.

11 Interpretation of red shift

In the model the speed of information transfer is taken as a model constant. This means that extension of the progression step goes together with space extension.

In this paper, red shift of old messengers is explained as extension of the progression step with smaller progression values. This is in direct contrast to the interpretation that contemporary physics gives to red-shift of old photons.

This model relates the energy of messengers to the number of contained wave fronts. The model considers the duration of the emission, passage and absorption of messengers as a variable that decreases with progression. The difference between the emission duration and the absorption duration causes the observed red-shift.

Contemporary physics ignores the duration of these processes. It relates the energy of the photon to the frequency of the photon.

12 Object graininess

In the model, locally the duration of emission, passage and absorption of messengers is suspected to be equal or it depends on the graininess of the emitter. Measuring the duration and the frequency of the messenger will reveal the number of wave fronts that is contained in the messenger.

This is also the case for messengers that are released at pair annihilation. In this case the number of contained wave fronts will give information about the number of duals that were contained in the members of the annihilated pair. With other words it will reveal the dimension of the dual subspace that represented the annihilated object.

In elementary particles, the chain of hops is folded into a coherent swarm of locations. As a consequence in the swarm the 3D wave fronts superpose into a significant space curvature.

13 The dimension of the subspaces

In the Gelfand triple the dimension of subspaces that correspond to subspaces in Hilbert space is not defined. However, the coupling equation can give some indication of a measure that can replace the dimension. In the Hilbert space the dimension relates to the number of eigenvalues of the operator whose eigenvectors span the subspace. Now consider the eigenvalues that store the values of the hops. Compare:

$$\Phi = \nabla\psi = m\,\varphi$$

$$\|\Phi\|^2 = \int_V |\Phi|^2\, dV = \int_V |\nabla\psi|^2\, dV = m^2 \|\varphi\|^2 = m^2$$

Now let us apply this to the swarm by replacing the integral by a summation of squared hop sizes $|h_i|$.

$$m^2 = \sum_i^N |h_i|^2 = N\overline{|h_i|^2}$$

$h_i$is the quaternionic value of the *i*-th hop. It includes a fixed size progression step. The resulting part is an imaginary quaternion. N is the number of elements in the location swarm.

Thus m is related to the dimension of the subspace in the Hilbert space.

13.1 Binding energy

If the dimension of the subspace that represents a composite is smaller than the sum of the dimensions of its constituents, then the difference is spent on binding energy. The hops that left have gone in the form of messengers or these hops are used to support the oscillations of the constituents that occur inside of the composite. At the same time the constituents have lost part of their identity. They differ from the free versions of the constituents.

14 Spurious duals

The most elementary discrete objects in the model are the location-hop duals. Embedding of a location causes a 3D wave front. Embedding of a hop causes a 1D wave front that relates its direction to the direction of the hop. Embedding a dual generates both wave fronts.

Spurious duals cause the generation of spurious wave fronts. The amplitude of the 3D wave front diminishes quickly, but these wave fronts curve the embedding continuum a bit. Thus this effect may be causing dark “matter”. The 1D wave fronts do not curve the embedding continuum, but they may represent “dark energy”.

14.1 Why swarms ?

The fact that spurious duals appear in large numbers, raises the question why swarms, which are coherent collections of large numbers of duals, also exist. What keeps these duals together. Some kind of binding principle must exist. What is even more peculiar is the fact that these swarms have fixed statistical properties, while the set elements have the same symmetry flavor.

The binding can be caused by the common gravity pitch. The embedding of each separate dual causes only a small gravitation pinch. The gravitation pinch diminishes with distance r as $1/r$. The swarm contains a huge number of duals and these duals together produce a gravitation pitch that is shaped as a much broader $Erf(r)/r$ dependence on distance r. This relatively flat potential covers a major part of the swarm. At much larger distances this function also diminishes as $1/r$.

In large numbers spurious duals can still bring a noticeable gravitation contribution that is characterized by a very large spread.

15 Appendix: History of discoveries

The concept of "Universe" follows with mathematical inescapable evidence from first principles that constitute a recipe for modular construction. These first principles define an orthomodular lattice. This is the structure of quantum logic.

Quantum logic was introduced by Garret Birkhoff and John von Neumann in their 1938 paper. G. Birkhoff and J. von Neumann, *The Logic of Quantum Mechanics*, Annals of Mathematics, Vol. 37, pp. 823–843

The Hilbert space was discovered in the first decades of the 20-th century by David Hilbert and others. http://en.wikipedia.org/wiki/Hilbert_space.

Paul Dirac introduced the bra-ket notation, which popularized the usage of Hilbert spaces.

In 1843 quaternions were discovered by Rowan Hamilton.
http://en.wikipedia.org/wiki/History_of_quaternions

In the sixties Constantin Piron and Maria Pia Solèr proved that the number systems that a separable Hilbert space can use must be division rings. “Division algebras and quantum theory” by John Baez. http://arxiv.org/abs/1101.5690

In the sixties Israel Gelfand and Georgyi Shilov introduced a way to model continuums via an extension of the separable Hilbert space into a so called Gelfand triple. The Gelfand triple often gets the name rigged Hilbert space, which is confusing, because this construct is not a Hilbert space.

These discoveries are used as foundations by the e-book “The Hilbert Book Model Game”. http://vixra.org/abs/1405.0340 .