The Theory of Massive Photons
(Draft)

Rodolfo A. Frino – September 2014 (v1)
Electronics Engineer – Degree from the National University of Mar del Plata - Argentina
rodolfo_frino@yahoo.com.ar

Abstract

This paper is concerned with the theory on the rest mass of the photon. The equations for the energy of the photon are developed. The theory predicts an equation for the "massive" photoelectric effect similar to the Einstein's original photoelectric effect. This new theory predicts that the value of the black hole's cutoff radius is a function of the frequency of the photons generated inside the black hole. This means that the black hole's event horizon is not a spherical surface but a three-dimensional shell. Because this formulation is not a gravity theory but a theory on massive photons, it cannot predict exactly the same Schwarzchild radius as general relativity. Despite of not being a gravity theory, this theory predicts that, should the equivalent mass of the photon equal its rest mass, the black hole cutoff radius would be equal to the Schwarzchild radius. I am convinced that the full potential of this formulation will be shown when it is used in conjunction with the Proca equations, the standard model and quantum gravity theories.

***Keywords**: Black hole, Schwarzschild radius, rest mass, relativistic mass, equivalent mass, event horizon, Proca equations, quantum gravity theories.*

1. Introduction

In this paper I introduced a theory that account for the rest mass of the photon. The idea of massive photons is not new. Proca [1] developed a set of equations for electromagnetism that take into account the rest mass of the photon. These equations, known as the Proca equations, are the Maxwell equations for massive photons. More recently Pani [2, 3] has applied these equations to super massive black holes to place new limits on the mass of the photon. My calculations of the rest mass of the photon are below these new limits, and therefore they are in accordance with the observed stability of super massive black holes.

In an article published by Scientific American, Goldhaber et al [1] quoted:

"The particle of electromagnetic radiation often assumed to be massless, but the laws of physics do not require that assumption. If the photon has a mass, however, it must be exceedingly small." (p 86-96)

In his book "Cosmic impressions", Walter Thirring [4] quotes:

"It is been always said that photons have zero rest mass, but maybe we should say that its mass has a value which is lower than the limits we are able to measure. If the mass [of the photon] were exactly zero, then its Compton wavelength would be infinite, but because the radius of the universe is finite, it is time to begin to work meticulously."

The equations for the energy of the photon are developed from simple modifications carried out to the special theory of relativity.

2. The Equations for Massive Photons

Because the equivalent mass of a photon depends on its frequency and not on its velocity the Einstein's relativistic mass law is not applicable to massive photons. Therefore, assuming that photons have a non-zero rest mass, they have to be treated differently.

Let us consider the Einstein's total relativistic energy equation

$$E_{rel}^2 = (pc)^2 + (m_0 c^2)^2 \tag{1-1}$$

I have used E_{rel} to denote the relativistic energy of a body to avoid confusion with the concept of equivalent energy used in this paper.

According to Einstein the rest mass, m_0 , of a photon is zero, therefore the total relativistic energy of a photon will be

$$E_{rel} = pc \tag{1-2}$$

the relativistic kinetic energy is, by definition

$$K_{rel} = E_{rel} - m_0 c^2 \tag{1-3}$$

But because $m_0 = 0$ we get

$$K_{rel} = E_{rel} = pc \tag{1-4}$$

Therefore we reach the following conclusion for massless photons: the relativistic total energy and the kinetic energy are identical.

However, if photons have non-zero rest mass, their kinetic energy must be different to their equivalent energy. I called the energy of a photon equivalent rather than relativistic. Since the mass of the photon does not depend on its velocity, the concept of relativistic energy does not make any sense in this context. Thus, relativistic energy, in this case, must be replaced by the concept of equivalent energy.

I will start the derivation of the equivalent kinetic energy of the photon by re-writing equation (1-1) as follows

$$E_{rel}^2 - (m_0 c^2)^2 = (pc)^2 \tag{1-5}$$

$$(E_{rel} + m_0 c^2)(E_{rel} - m_0 c^2) = (pc)^2 \tag{1-6}$$

$$(E_{rel} - m_0 c^2) = \frac{(pc)^2}{E_{rel} + m_0 c^2} \tag{1-7}$$

This is the point where equations give birth to a new theory.

There should be two different sets of equations: The first set should be for subluminar massive particles (not photons) such as electrons, muons, tau particles, protons, neutrons, pi mesons, etc. This set already exists and it was discovered by Albert Einstein. The Einstein's relativistic energy equation (1-1) is an example.

A second set of equations should befor luminar massive particles such as photons and gravitons. This is the set of equations I shall develop in this formulation.

According to De Broglie a particle's wavelength and its momentum are related by

$$\lambda = \frac{h}{mv} \tag{1-8}$$

Thus a massless particle should have an infinite wavelength given by

$$\lambda_\infty = \lim_{m \to 0} \lambda = \lim_{m \to 0} \frac{h}{mv} = \infty \tag{1-9}$$

However this is impossible because our Universe (not the Meta-universe) was born 13,822 billion years ago and since then it expanded at finite rates. Therefore there is not enough room in our Universe to accommodate a particle with infinite wavelength even if we take into account the expansion of space itself.

Now, the time has come to complete Einstein's work by developing the second set of equations – the equations for massive photons. (It is worthy to remark that there is only one type of photon but I will refer to either massive or massless photons to differentiate the two theories).

In order to do this I shall do two things. Firstly I shall introduce two postulates

Postulate 1
Both the value of the equivalent mass, m , of the photon and its rest mass, m_0 *, are not relativistic masses. This means that they do not obey the relativistic mass law (eq. 1-11)*

Postulate 2
The value of the rest mass of a photon, m_0 *, cannot be modified by the action of gravity.*

Secondly, I shall modify the Einstein's equations as follows

a) **First Modification**

The first modification will be to define the *kinetic energy for massive photons* as follows

Kinetic energy for massive photons

$$K = \frac{(pc)^2}{E + m_0 c^2} \qquad (1\text{-}10\ a)$$

where E is the equivalent energy of the photon

$$K = E_{\text{total}} - E_0 \qquad (1\text{-}10\ b)$$

where E_{total} is the total energy of the photon

Note that equation (1-10 a) coincides with special relativity for massive particles. This can be easily shown by separating equation (1-7) into two equations as follows

$$K_{rel} = \frac{(pc)^2}{E_{rel} + m_0 c^2} \qquad (1\text{-}10\ c)$$

$$K_{rel} = E_{rel} - m_0 c^2 \qquad (1\text{-}10\ d)$$

One of the differences between Einstein's theory and this theory is that in special relativity both equations of the kinetic energy are functions of the relativistic energy E_{rel} as shown by equations (1-10 c) and (1-10 d). In this formulation, however, equation (1-10 a) is a function of the equivalent energy while equation (1-10 b) is a function of the total energy. This makes a huge difference, the difference we just needed.

b) **Second Modification**

The second modification deals with the *equivalent mass* and *equivalent energy*.

Since a photon travels at the speed of light, its equivalent mass m is not a function of its velocity. The Einstein's photoelectric effect has shown that the energy of a photon (in other words the equivalent energy of the photon) is a function of its frequency f. This means that the relativistic mass formula, which is a function of the velocity of the particle, is not valid for massive photons (see Postulate 1).

Massive photons do not obey the relativistic mass formula

$$m \neq \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (1\text{-}11)$$

Relativistic energy and relativistic mass have no meaning for massive photons

Instead, I shall define the *equivalent energy* and the *equivalent mass* (for a massive photon) and I will relate them through the Einstein's famous equation

Equivalent energy and equivalent mass

$$E = m c^2 \tag{1-12}$$

In this theory equivalent energy, E, and equivalent mass, m, for massive photons are related by the same equation that relates relativistic energy, E_rel, and relativistic mass ,m_rel, in the special theory of relativity.

Equivalent energy and relativistic energy are both energies. The difference between the two is that the former applies to electromagnetic radiation and depends only on its frequency (does not depend on any velocity) while the latter depends on the velocity of the body.

c) **Third Modification**

The third modification (see **note**) I will make will be the introduction of the *total energy for massive photons*. Thus, I shall define this energy as follows

Total energy for massive photons

$$E_{\text{total}} = K + E_0 \tag{1-13}$$

(**Note**: In order to explain the equivalent kinetic energy of this formulation I have already introduced the concept of total energy in the **First Modification**. However, I think that adding this point separately adds clarity).

The rest energy, E_0 , will be, as per special relativity defined as follows

$$E_0 = m_0 c^2 \tag{1-14}$$

In summary, the relativistic theory of massive photons is based on the following set of equations

i- Total Energy: $$E_{\text{total}} = K + E \tag{1-15}$$

ii- Kinetic energy: $$K = \frac{(pc)^2}{E + m_0 c^2} \tag{1-16}$$

iii- Rest energy: $$E_0 = m_0 c^2 \tag{1-17}$$

iv- Equivalent energy: $$E = m c^2 \tag{1-18}$$

v- Equivalent momentum: $$p = mc \tag{1-19}$$

vi- Equivalent mass: $$m = \frac{hf}{c^2} \tag{1-20}$$

From the above five equations we can derived the rest of the formulas for massive photons. The results are shown in Table 1.

The first column of the table is used to identify the equation. The second column contains the equations for massive photons. The third column contains the corresponding equations for massless photons according to special relativity (when there is one).

Variable	Eq. no.	Equations for massive photons	Equations for massless photons
Rest mass m_0	(T1)	$\approx 1.345\times 10^{-69}\,Kg$	0
Mass	(T2)	Equivalent mass m	Relativistic mass m_{rel}
Mass $m=f(f)$	(T3)	Equivalent mass $\frac{hf}{c^2}$	Relativistic mass $\frac{hf}{c^2}$
Mass $m=f(E)$	(T4)	Equivalent mass $\frac{E}{c^2}$	Relativistic mass $\frac{E_{rel}}{c^2}$
Mass $m=f(K)$	(T5)	Equivalent mass $\frac{1}{2\mathrm{c}^2}\left(K+\sqrt{K^2+4\mathrm{Km}_0 c^2}\right)$	Relativistic mass $\frac{E_{rel}}{c^2}$
Momentum $p=f(m)$	(T6)	Equivalent momentum mc	Relativistic momentum $m_{rel}\,c$
Momentum $p=f(f)$	(T7)	Equivalent momentum $\frac{hf}{c}$	Relativistic momentum $\frac{hf}{c}$
Momentum $p=f(E)$	(T8)	Equivalent momentum $\frac{E}{c}$	Relativistic momentum $\frac{E_{rel}}{c}$

Momentum $p = f(K)$	(T9)	Equivalent momentum $\frac{1}{2c}\left(K + \sqrt{K^2 + 4Km_0c^2}\right)$	Relativistic momentum $\frac{E_{rel}}{c}$
Rest Energy E_0	(T10)	Equivalent rest energy $m_0 c^2$	Relativistic rest energy 0
Energy $E = f(m)$	(T11)	Equivalent energy $m c^2$	Relativistic energy $m_{rel} c^2$
Energy $E = f(f)$	(T12)	Equivalent energy hf	Relativistic energy hf
Energy $E = f(p)$	(T13)	Equivalent energy pc	Relativistic energy $p_{rel} c$
Energy $E = f(K)$	(T14)	Equivalent energy $\frac{1}{2}\left(K + \sqrt{K^2 + 4Km_0c^2}\right)$	Relativistic energy E_{rel}
Total Energy E_{total}	(T15)	Total energy (Equivalent energy + rest energy) $K + E_0$	Relativistic total energy $m_{rel} c^2$
Kinetic Energy $K = f(m)$	(T16)	Equivalent kinetic Energy $\frac{m^2}{(m + m_0)} c^2$	Relativistic kinetic energy $m_{rel} c^2$
Kinetic Energy $K = f(f)$	(T17)	Equivalent kinetic Energy $\frac{(hf)^2}{hf + m_0 c^2}$	Relativistic kinetic energy hf
Kinetic Energy $K = f(p)$	(T18)	Equivalent Kinetic Energy $\frac{(pc)^2}{pc + m_0 c^2}$	Relativistic kinetic energy $K_{rel} = E_{rel}$
Kinetic Energy $K = f(E)$	(T19)	Equivalent kinetic Energy $\frac{(E)^2}{E + m_0 c^2}$	Relativistic kinetic energy $K_{rel} = E_{rel}$
Kinetic Energy $K = f(p, E)$	(T20)	Equivalent kinetic Energy $\frac{(pc)^2}{E + m_0 c^2}$	Relativistic kinetic energy $K_{rel} = E_{rel}$

Minimum Energy $E_{min} = K_{min}$	(T21)	0	0

Table 1: *This table shows the equations for the mass, momentum and energy of both massive and massless photons.*

Using these equations I will make four predictions. These predictions are the subject of the next section.

3. Predictions

This sections contains the prediction of this formulation:

3.1 The Schwarzschild radius.
3.2 The Photoelectric effect.
3.3 Massive photons and the harmonic oscillator
3.4 Electron-positron annihilation

3.1 The Schwarzschild Radius

Let us consider the problem of finding the Schwarzschild radius for a star.

According to the principle of conservation of energy the kinetic energy plus the gravitational energy must be constant. If this constant is zero, then we can write

$$K + U = 0 \tag{3.1-1}$$

The gravitational potential energy is

$$U = -\frac{GMm}{r} \tag{3.1-2}$$

The non-emission condition is

$$\frac{(pc)^2}{E + m_0 c^2} \leq \frac{GMm}{r} \tag{3.1-3}$$

Now we use the Einstein's equation, that in this case relates the equivalent energy to the equivalent mass:

$$E = mc^2 \tag{3.1-4}$$

$$r \leq \frac{GMm\,(m + m_0)}{p^2} \tag{3.1-5}$$

and considering the momentum of the photon

$$p = mc \tag{3.1-6}$$

we arrive to the following equation

$$r \le \left(1 + \frac{m_0}{m}\right)\frac{GM}{c^2} \tag{3.1-7}$$

When the radius of the black hole satisfies the above inequation, a photon created inside the black hole will not be able to escape to empty space because its kinetic energy will be less than its gravitational energy.

Thus this radius is a *cutoff radius* or *non-emission radius*. Therefore we can write:

$$r_{cutoff} \le \left(1 + \frac{m_0}{m}\right)\frac{GM}{c^2} \tag{3.1-8 a}$$

Equation (3.1-8 a) tells us that the cutoff radius depends on both the equivalent mass of the photon and on its rest mass.

Equation (3.1-8 a) can be re-written in terms of the frequency and in terms of the wavelength of the photon. These equations are

$$r_{cutoff} \le \left(1 + \frac{f_0}{f}\right)\frac{GM}{c^2} \tag{3.1-8 b}$$

$$r_{cutoff} \le \left(1 + \frac{\lambda}{\lambda_0}\right)\frac{GM}{c^2} \tag{3.1-8 c}$$

I shall define a new concept that we shall need soon

Partial cutoff horizon

The partial cutoff horizon is the radius of a black hole for which at least one wavelength of electromagnetic radiation can escape to infinity.

Example 1*: according to equation* (3.1-8 c)*, a radius equal to* $\frac{2\,GM}{c^2}$ *is a partial cutoff horizon because photons whose wavelenghts are* $\lambda \neq \lambda_0$ *can escape to infinity.*

Example 2*: Also according to equation* (3.1-8 c)*, a radius equal to* $\frac{GM}{c^2}$ *is NOT a partial cutoff horizon because there are no photons that can escape to infinity.*

Total cutoff horizon (Schwarzchild radius)

The total cutoff horizon (minimum cutoff radius) is the radius of a black hole from which no electromagnetic radiation can escape to infinity.

Example*: According to equation* (3.1-8 c)*, a radius equal to* $\frac{GM}{c^2}$ *is a total cutoff horizon or Schwarzchild radius because there are no photons that can escape to infinity.*

This result is consistent with the fact that haven't used any theory of gravitation in conjunction with this model. We cannot expect this theory to produce the exact value of the Schwarzchild radius because we did not take into account the curvature of space-time.

Fuzzy shell or fuzzy event horizon

The fuzzy shell (or fuzzy cutoff shell) is a spherical shell of space (in general a three dimensional region of space of any shape, normally spherical or ellipsoidal) defined by:

$$\frac{GM}{c^2} < R_{\text{fuzzy}} \leq \frac{2\,GM}{c^2}$$

where at least one wavelength can escape to infinity

Now I will analyze two different cases

Case a)

I will assume that because of the intense gravitational fields inside the black hole, a photon generated by the black hole will have an equivalent mass equal to its rest mass. This means that any photon generated inside the black hole will have $m = m_0$ (or in other words $f = f_0$, and $\lambda = \lambda_0$).

Thus the cutoff radius will be

$$r_{\text{cutoff}} = \frac{2\,GM}{c^2} \tag{3.1-9}$$

But this is twice the Schwarzschild radius of the black hole

$$r_{cutoff} = R_S \tag{3.1-10}$$

In summary, if the radius of a star becomes equal or smaller than $2\,GM/c^2$ the star will transform into a black hole and it will not be able to emit any electromagnetic radiation except through the Hawking mechanism.

This result is in agreement with general relativity.

Case b)

I will assume that despite the intense gravitational fields inside the black hole, the energy of the photons created inside this cosmic body will vary between a minimum energy given by $E_{min}(\lambda=\lambda_0)=\frac{hc}{\lambda_0}$ and a maximum energy given by $E_{max}(\lambda=L_P)=\frac{hc}{L_P}$

Thus the corresponding minimum and a maximum cutoff radii will be

b1) The Minimum Cutoff Radius

The minimum radius corresponds to the minimum wavelength (maximum energy)

$$r_{\text{cutoff\_min}} \le \left(1+\frac{L_P}{\lambda_0}\right)\frac{GM}{c^2} \tag{3.1-11}$$

thus neglecting the term in L_P we have approximately

$$r_{\text{cutoff\_min}} \le \frac{GM}{c^2} \tag{3.1-12}$$

b2) The Maximum Cutoff Radius

The maximum radius corresponds to the maximum wavelength (minimum energy)

$$r_{\text{cutoff\_max}} \le \left(1+\frac{\lambda_0}{\lambda_0}\right)\frac{GM}{c^2} \tag{3.1-13}$$

$$r_{\text{cutoff\_max}} \le \frac{2\,GM}{c^2} \tag{3.1-14}$$

Therefore a black hole has a **fuzzy shell** (see definition above), depending on the wavelength of the photons generated inside this cosmic body.

In this case the Schwarzschild radius will be the minimum cutoff radius (also known as total cutoff radius)

$$R_S \le \frac{GM}{c^2} \tag{3.1-15}$$

If the radius of a star becomes equal or smaller than $R_S=GM/c^2$ (condition for maximum energy photons) the star will transform into a black hole and it will not be able to emit any electromagnetic radiation except through the Hawking mechanism.

Therefore if nature behaves this way (**case b2**), the description of non-rotating black holes given by general relativity (through the Schwarzschild radius) is an approximation.

In this regards Wikipedia [5] quotes: *"The description of event horizons given by general relativity is thought to be incomplete. When the conditions under which event horizons occur are modeled using a more comprehensive picture of the way the universe works, that includes both relativity and quantum mechanics, event horizons are expected to have properties that are different from those predicted using general relativity alone."*

This result I obtained is more conservative than the new Hawking's claim [6]

"The absence of event horizons means that there are no black holes — in the sense of regimes from which light can't escape to infinity".

I believe that for black holes whose radii are greater than $\frac{GM}{c^2}$ and smaller than $\frac{2GM}{c^2}$ there is a fuzzy horizon (or a fuzzy shell) - a 3-D horizon from which at least some light can escape to infinity (but not all) depending on the frequency of the photons.

Below the minimum radius of $\frac{GM}{c^2}$ no light originating inside the black hole can escape to infinity. In the light of a more sophisticated quantum gravitational theory, this formulation could predict a shifted fuzzy horizon extending from $\frac{2GM}{c^2}$ to $\frac{4GM}{c^2}$, for example. If this were the case it would be in full agreement with general relativity.

3.2 The Photoelectric Effect

The photoelectric effect was explained by A. Einstein and the equation is

$$\frac{1}{2} m_e v_{max}^2 = hf - W \tag{3.2-1}$$

where

m_e = electron rest mass
v_{max} = maximum emission velocity of photoelectrons for a monochromatic frequency f
h = Planck's constant
f = frequency of monochromatic light
W = work function. Work done to eject one of the electrons from the surface of the material.

The first side of equation (3.2-1) is the maximum kinetic energy of the emitted electrons. Einstein assumed that these electrons were the least firmly bound to an atom of the material. This energy is also known as the maximum emission energy.

Taking into account the rest mass of the photon the new equation for the photoelectric effect is

$$\frac{1}{2} m_e v_{max}^2 = \frac{(hf)^2}{hf + m_0 c^2} - W \tag{3.2-2}$$

where $m_0 = m_{\gamma 0}$ is the rest mass of the photon.

As expected, equation (3.2-2) reduces to equation (3.2-1) when the photon rest mass is zero.

3.3 Massive Photons and the Harmonic Oscillator

The kinetic energy of a photon is

$$K = \frac{(pc)^2}{E + m_0 c^2} \tag{3.3-1}$$

If the equivalent mass of the photon equals its rest mass,

$$m = m_0 \tag{3.3-2}$$

then the kinetic energy will be

$$K_0 = \frac{(m_0 c^2)^2}{m_0 c^2 + m_0 c^2} = \frac{m_0^2 c^2}{m_0 + m_0} = \frac{m_0 c^2}{2} \tag{3.3-3}$$

but

$$m_0 c^2 = h f_0 \tag{3.3-4}$$

Thus equation (3.3-2) transforms into

$$K_0 = \frac{h f_0}{2} \tag{3.3-5}$$

Let us compare this energy – equation (3.3-4) with the energy of the quantum harmonic oscillator

$$E_{oscillator}(n) = \left(n + \frac{1}{2}\right) h f_0 \tag{3.3-6}$$

n = 0, 1, 2, 3, ...

If n=0 then the minimum energy of the oscillator is

$$E_{oscillator}(0) = \frac{h f_0}{2} \tag{3.3-7}$$

Comparing equations (3.3-5) and (3.3-7) we see that they are exactly the same equation. Then at very low frequencies, massive photons behave like a harmonic oscillator. Thus the condition (3.3-2) defines a zero point energy condition for photons.

3.4 Electron-Positron Annihilation

Let us consider the standard theory of electron-positron annihilation.

If an electron and a low energy positron are in contact they annihilate producing two or three gamma rays photons.

When two photons are produced the reaction is

$$e^- + e^+ \rightarrow \gamma + \gamma$$

If the two particles are almost at rest before the annihilation the total momentum will be almost zero, therefore assuming the total momentum is zero, due to the conservation of momentum two gamma rays photons must be created and these two photons must move in opposite directions.

Because of the conservation of energy we can write

$$2\, m_{e0}\, c^2 = 2\, hf \tag{3.4-1}$$

where m_{e0} is the mass of the either the electron or the positron and f is the frequency of each massless photon. This frequency is given by

$$f = \frac{m_{e0}\, c^2}{h} \tag{3.4-2}$$

The frequency and the wavelength of a photon are related by the expression

$$f = \frac{c}{\lambda} \tag{3.4-3}$$

eliminating the frequency from equations (3.4-2) and (3.4-3) we get

$$\lambda = \frac{h}{m_{e0}\, c} \tag{3.4-4}$$

Now let us consider the equivalent situation but this time we assume that photons have a rest mass.

The conservation of energy and equation (T17) allow us to write

$$2\, m_{e0}\, c^2 = 2 \frac{(hf)^2}{hf + m_0\, c^2} \tag{3.4-5}$$

This leads to the following quadratic equation

$$(hf)^2 - m_{e0} c^2 hf - m_{e0} m_0 c^4 = 0 \tag{3.4-6}$$

this equation can be easily solved to get

$$hf = \frac{1}{2} m_{e0} c^2 \pm \frac{1}{2} \sqrt{m_{e0}(m_{e0} + 4 m_0)}\, c^2 \tag{3.4-7}$$

Taking the positive solution and using equation (3.4-4) we get the wavelength of each massive photon

$$\lambda = \frac{2h}{\left[m_{e0} + \sqrt{m_{e0}(m_{e0} + 4 m_0)} \right] c} \tag{3.4-8}$$

As expected, equation (3.4-8) reduces to equation (3.4-4) when the photon rest mass $, m_0 ,$ is zero.

Thus this theory predicts that the wavelength of the gamma ray photons is actually lower than the prediction from special relativity (massless photons).

4. Calculation of the Photon Rest Mass

In another paper entitled *"Cosmology and the First Model Meta-law"* [7] I published the equation for the age of the universe I discovered in 2012. This equation is

$$T = \frac{h^2}{2\pi^2 c G m_e m_p^2} \qquad \text{(Formula for the age of the universe)} \tag{4-1}$$

According to the temporal form of the Heisenberg uncertainty principle the product of the age of the universe times the rest energy of the photon must be approximately equal to the reduced Planck's constant divide by two. Mathematically this means that

$$T\, m_{\gamma_0} c^2 \simeq \frac{\hbar}{2} \qquad \text{(Heisenberg uncertainty principle)} \tag{4-2}$$

Let us eliminate the age of the universe from the above equations: (3-1) and (3-2) to obtain the rest mass of the photon

$$\left(\frac{h^2}{2\pi^2 c G m_e m_p^2} \right) m_{\gamma_0} c^2 = \frac{\hbar}{2} \tag{4-3}$$

Thus, the formula for the rest mass of the photon is

$$m_{\gamma_0} = \frac{\pi G m_e m_p^2}{2hc} \qquad \text{(Formula for the rest mass of the photon)} \tag{4-4}$$

which yields the following value

$$m_{\gamma_0} \simeq 1.34494 \times 10^{-69}\, Kg$$

This result is in accordance to the latest models of super massive black holes theories based on a new model that incorporates the Proca equations. In the web article *"Mass of the photon has been narrowed"* Pani [2] quotes:

"Ultralight photons with nonzero mass would produce a 'black hole bomb': a strong instability that would extract energy from the black hole very quickly, ..., The very existence of such particles is constrained by the observation of spinning black holes. With this technique, we have succeeded in constraining the mass of the photon to unprecedented levels: the mass must be one hundred billion of billions times smaller than the present constraint on the neutrino mass, which is about two electron-volts."

Pani [3] found the following bounds

$$m_{\gamma_0} \simeq 10 \times^{-18} eV \times 1.782658104 \times 10^{-36} Kg/eV = 1.78 \times 10^{-54} Kg$$
$$m_{\gamma_0} \simeq 10 \times^{-21} eV \times 1.782658104 \times 10^{-36} Kg/eV = 1.78 \times 10^{-57} Kg$$

The rest mass of the photon calculated using equation (4-4) $m_{\gamma_0} \simeq 1.34494 \times 10^{-69}\, Kg$ is well below the values calculated by Pani.

In Table 1 of the article entitled "Photon and Graviton Mass Limits", Goldhaber et al [8], list a the mass limits for both photons and gravitons. These limits are in excellent agreement with my calculations.

5. Conclusions

In quantum electrodynamics the electromagnetic field is a gauge field. The corresponding gauge particle is the photon. Because standard gauge symmetries requires photons to be massless particles, theoretical physicists also have the same demand. They will not easily abandon a model that is already obsolete. This is like adoring a dinosaur that is already extinct.

These results presented in this paper strongly suggest that the equations for massive photons of this formulation are correct. The implications of this formulation should be evaluated in the light of a new standard model and quantum gravity theories.

Readers interested in further reading about the implications of massive photons can refer to the article entitled "The mass of the photon" by Tu, Luo and Gilles [9].

Appendix 1

Limits

In this appendix we take the limits of the expressions (equivalent energy, kinetic energy and momentum) shown in column 2 of table 1 when the photon rest mass tends to zero. The results of these limits coincide with the corresponding values for massless photons. shown in column 3.

Equivalent energy

$$\lim_{m_0 \to 0} E = \lim_{m_0 \to 0} \frac{1}{2}\left(K + \sqrt{K^2 + 4\mathrm{Km}_0 c^2} \right) = K$$

Kinetic energy

$$\lim_{m_0 \to 0} K = \lim_{m_0 \to 0} \frac{(E)^2}{E + m_0 c^2} = E$$

$$\lim_{m_0 \to 0} K = \lim_{m_0 \to 0} \left(\frac{m^2}{m + m_0} \right) c^2 = m c^2$$

$$\lim_{m_0 \to 0} K = \lim_{m_0 \to 0} \frac{(hf)^2}{hf + m_0 c^2} = hf$$

$$\lim_{m_0 \to 0} K = \lim_{m_0 \to 0} \frac{(pc)^2}{pc + m_0 c^2} = pc$$

Mass

$$\lim_{m_0 \to 0} m = \lim_{m_0 \to 0} \frac{1}{2\mathrm{c}^2}\left(K + \sqrt{K^2 + 4\mathrm{Km}_0 c^2} \right) = \frac{K}{c^2} = \frac{E}{c^2}$$

Momentum

$$\lim_{m_0 \to 0} p = \lim_{m_0 \to 0} \frac{1}{2\mathrm{c}}\left(K + \sqrt{K^2 + 4\mathrm{Km}_0 c^2} \right) = \frac{K}{c} = \frac{E}{c}$$

Appendix 2

Comparison of Energy Formulas Between Massive Photons and Massive Particles (Special Relativity)

It is interesting to compare the formulas for the energy of massive photons described by this formulation with the relativistic energy for massive particles (for example fermions and hadrons) described by the special theory of relativity (which also applies to massless photons). This is done in Table A2.

Equations for massive photons	Einstein's Equations for massive particles
Equivalent energy $E=\frac{1}{2}\left(K+\sqrt{K^2+4\mathrm{Km}_0c^2}\right)$ (A2 - a1)	Relativistic energy $E_{rel}=K_{rel}+m_0c^2$ (A2 - b1)
Equivalent kinetic energy $K=\frac{(pc)^2}{E+m_0c^2}$ (A2 - a2) The equation for the equivalent kinetic energy *K* looks like its counterpart, K_{rel} , however the energy *E* here is an equivalent energy while in Einstein's model is a relativistic energy, E_{rel} . A similar story can be told about the momentum.	Relativistic kinetic energy $K_{rel}=\frac{(pc)^2}{E_{rel}+m_0c^2}$ (A2 - b2)
The equivalent energy in terms of *pc* can be found solving the third degree equation: $E^3+(pc)c^2E^2-(pc)^2E-(pc)^2m_0c^2=0$ The solution to this equation is *E = pc* Therefore the equation for the equivalent energy in terms of *pc* is $E=pc$ The equivalent energy of the photon does not depend on its rest mass	Relativistic energy in terms of *pc* and the rest energy Normally the above equations (A2 – b1) and (A2 – b2) are combined into the famous Einstein's relativistic energy equation $E_{rel}=\sqrt{(pc)^2+(m_0c^2)^2}$ The relativistic energy of fermions and hadrons depends on its rest mass
Total energy $E_{total}=E+m_0c^2$ or $E_{total}=pc+m_0c^2$ Therefore the total energy of this formulation is the closest we can get to the equivalent energy of Einstein's massless model.	Relativistic energy (same equation as above) $E_{rel}=\sqrt{(pc)^2+(m_0c^2)^2}$

Table A2: This table compares the energies for massive photons (this theory) with the energies for massive particles (relativistic model).

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