

**ONE HUNDRED AND EIGHTY  
CONJECTURES ON PRIMES**

**(COLLECTED PAPERS)**

2014

## INTRODUCTION

In two of my previous published books, “Two hundred conjectures and one hundred and fifty open problems on Fermat pseudoprimes”, respectively “Conjectures on primes and Fermat pseudoprimes, many based on Smarandache function”, I already expressed my passion for integer numbers, especially for primes and Fermat pseudoprimes, fascinating numbers that seem to be a little bit more willing to let themselves ordered and understood than the prime numbers.

This book brings together forty-eight papers on prime numbers, many of them supporting the author’s belief, expressed above, namely that new ordered patterns can be discovered in the “undisciplined” set of prime numbers, observing the ordered patterns in the set of Fermat pseudoprimes, especially in the set of Carmichael numbers, the absolute Fermat pseudoprimes, and in the set of Poulet (sometimes also called Sarrus) numbers, the relative Fermat pseudoprimes to base two. Other papers, which are not based on the observation of Fermat pseudoprimes, are based on the observation of Mersenne numbers, Fermat numbers, Smarandache generalized Fermat numbers, and other well known or less known classes of integers which are very much related with the study of primes.

Part One of this book of collected papers contains one hundred and eighty conjectures on primes and Part Two of this book brings together the articles regarding primes, submitted by the author to the preprint scientific database Vixra, representing the context of the conjectures listed in Part One.

## SUMMARY

### **Part one. One hundred and eighty conjectures on primes**

### **Part two. Forty-eight articles on primes**

1. A conjecture about a way in which the squares of primes can be written and five other related conjectures
2. A conjecture about an infinity of sets of integers, each one having an infinite number of primes
3. A trivial but notable observation about a relation between the twin primes and the number 14
4. An observation about the digital root of the twin primes, few conjectures and an open problem on primes
5. A conjecture on primes involving the pairs of sexy primes
6. A conjecture on the pairs of primes  $p, q$ , where  $q$  is equal to the sum of  $p$  and a primorial number
7. Two conjectures involving the sum of a prime and a factorial number
8. Seven conjectures on a certain way to write primes including two generalizations of the twin primes conjecture
9. An interesting formula for generating primes and five conjectures about a certain type of pairs of primes
10. Few possible infinite sets of triplets of primes related in a certain way and an open problem
11. Two types of pairs of primes that could be associated to Poulet numbers
12. A set of Poulet numbers and generalizations of the twin primes and de Polignac's conjectures inspired by this
13. A very exhaustive generalization of de Polignac's conjecture
14. A formula which conducts to primes or to a type of composites that could form a class themselves
15. Four sequences of numbers obtained through concatenation, rich in primes and semiprimes
16. A conjecture on the squares of primes of the form  $6k - 1$
17. A conjecture on the squares of primes of the form  $6k + 1$
18. Nine conjectures on the infinity of certain sequences of primes
19. Five conjectures on primes based on the observation of Poulet and Carmichael numbers
20. Six conjectures on primes based on the study of 3-Carmichael numbers and a question about primes
21. Ten conjectures on primes based on the study of Carmichael numbers, involving the multiples of 30
22. Two sequences of primes whose formulas contain the number 360
23. Two sequences of primes whose formulas contain the powers of the number 2
24. Conjectures about a way to express a prime as a sum of three other primes of a certain type
25. A bold conjecture about a way in which any prime can be written
26. Two conjectures, on the primes of the form  $6k + 1$  respectively of the form  $6k - 1$
27. A possible way to write any prime, using just another prime and the powers of the numbers 2, 3 and 5
28. Two conjectures about the pairs of primes separated by a certain distance

29. Five conjectures on a diophantine equation involving two primes and a square of prime
30. An amazing formula for producing big primes based on the numbers 25 and 906304
31. Four unusual conjectures on primes involving Egyptian fractions
32. Three formulas that generate easily certain types of triplets of primes
33. A new bold conjecture about a way in which any prime can be written
34. A bold conjecture about a way in which any square of prime can be written
35. Statements on the infinity of few sequences or types of duplets or triplets of primes
36. An interesting relation between the squares of primes and the number 96 and two conjectures
37. A formula that seems to generate easily big numbers that are primes or products of very few primes
38. Four conjectures based on the observation of a type of recurrent sequences involving semiprimes
39. Conjecture that states that a Mersenne number with odd exponent is either prime either divisible by a 2-Poulet number
40. Conjecture that states that a Fermat number is either prime either divisible by a 2-Poulet number
41. Two exciting classes of odd composites defined by a relation between their prime factors
42. A formula for generating a certain kind of semiprimes based on the two known Wieferich primes
43. Formula that uses primes as input values for obtaining larger primes as output, based on the numbers 7 and 186
44. Conjectures on Smarandache generalized Fermat numbers
45. An interesting class of Smarandache generalized Fermat numbers
46. Conjecture which states that an iterative operation of any pair of two odd primes conducts to a larger prime
47. A probably infinite sequence of primes formed using Carmichael numbers, the number 584 and concatenation
48. Recurrent formulas which conduct to probably infinite sequences of primes and a generalization of a Cunningham chain

## Part one. One hundred and eighty conjectures on primes

**Conjecture 1:** The square of any prime  $p$ ,  $p \geq 5$ , can be written at least in one way as  $p^2 = 3q - r - 1$ , where  $q$  and  $r$  are distinct primes,  $q \geq 5$  and  $r \geq 5$ .

**Conjecture 2:** There exist an infinity of primes  $p$  that can be written as  $p = (q^2 + q + 1)/3$ , where  $q$  is also a prime.

**Conjecture 3:** The square of any prime  $p$ ,  $p \geq 5$ , can be written at least in one way as  $p^2 = 3q - r - 1$ , where  $q$  is a Poulet number and  $r$  a prime,  $r \geq 5$ .

**Conjecture 4:** For any prime  $p$ ,  $p \geq 5$ , there exist an infinity of pairs of distinct primes  $[q, r]$  such that  $p = \sqrt{3q - r - 1}$ .

**Conjecture 5:** For any prime  $p$ ,  $p \geq 5$ , there exist at least a pair of distinct primes  $[q, r]$  such that  $p = (q^2 + r + 1)/3$ .

**Conjecture 6:** For any prime  $p$  of the form  $p = 6k + 1$  there exist an infinity of pairs of distinct primes  $[q, r]$  such that  $p = 3q - p^2 - 1$ .

**Conjecture 7:** For an infinity of odd positive integers  $m$  there is an infinite set of primes with the property that the sum of their digits is equal to  $m + 1$ .

**Conjecture 8:** For an infinity of primes  $p$  there is an infinite set of primes with the property that the sum of their digits is equal to  $p + 1$ .

**Conjecture 9:** There is an infinite number of values the sum of the digits of the numbers  $p + 1$ , where  $p$  is odd prime, may have.

**Conjecture 10:** There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that the sum of its digits it's equal to 14.

**Conjecture 11:** There is an infinity of primes with the property that the sum of their digits is equal to 14.

**Conjecture 12:** There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 2.

**Conjecture 13:** There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 5.

**Conjecture 14:** There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 8.

**Conjecture 15:** Let  $a_i$  be the sequence of the lesser of twin primes whose digital root is equal to 2,  $b_i$  be the sequence of the lesser of twin primes whose digital root is equal to 5 and  $c_i$  be the sequence of the lesser of twin primes whose digital root is equal to 8. Than:

: there exist an infinity of terms  $n$  of  $a_i$  for which the number of the terms of  $b_i$  smaller than  $n$  is equal to the number of the terms of  $c_i$  smaller than  $n$ ;

: there exist an infinity of terms  $n$  of  $b_i$  for which the number of the terms of  $a_i$  smaller than  $n$  is equal to the number of the terms of  $c_i$  smaller than  $n$ ;

: there exist an infinity of terms  $n$  of  $c_i$  for which the number of the terms of  $a_i$  smaller than  $n$  is equal to the number of the terms of  $b_i$  smaller than  $n$ .

**Conjecture 16:** There is an infinity of primes with the property that the sum of their digits is equal to 14.

**Conjecture 17:** If  $n$  and  $n + 6$  are both primes (in other words if  $[n, n + 6]$  is a pair of sexy primes), where  $n \geq 7$ , then the number  $m = n + 3$  can be written at least in one way as  $m = p + q$ , where  $p$  and  $q$  are primes,  $q = p + 6*r$  and  $r$  is positive integer.

**Conjecture 18:** If  $p$  and  $p + p(n)\#$  are both primes, where  $p > p(n)\#$ ,  $n \geq 2$  and  $p(n)\#$  is a primorial number (which means the product of first  $n$  primes), then the number  $m = p + p(n)\#/2$  can be written at least in one way as  $m = x + y$ , where  $x$  and  $y$  are primes or squares of primes,  $y = x + p(n)\#*r$  and  $r$  is positive integer.

**Conjecture 19:** For any odd prime  $p$  there exist at least one prime  $q$  such that  $p + n! = q$ , where  $n$  is a positive integer,  $n < p$ .

**Conjecture 20:** For any odd prime  $p$ ,  $p \geq 5$ , there exist an infinity of primes  $q$  of the form  $q = (p + n!)/n^k$ , where  $n$  and  $k$  are positive integers and  $n \geq p$ .

**Conjecture 21:** Any odd prime  $p$  can be written in an infinity of distinct ways like  $p = q - r + 1$ , where  $q$  and  $r$  are also primes; in other words, there exist an infinity of pairs of primes  $(q, r)$  such that  $q - r = p - 1$ , for any odd prime  $p$  (it can be seen that for  $p = 3$  the conjecture states the same thing with the twin primes conjecture).

**Conjecture 22:** Any prime  $p$  of the form  $p = 6*k + 1$ , where  $k$  is positive integer, can be written in an infinity of distinct ways like  $p = q - r + 1$ , where  $q$  is a prime of the form  $q = 6*h - 1$  and  $r$  is a prime of the form  $q = 6*i - 1$  and, where  $h$  and  $i$  are positive integers.

**Conjecture 23:** Any prime  $p$  of the form  $p = 6*k + 1$ , where  $k$  is positive integer, can be written in an infinity of distinct ways like  $p = q - r + 1$ , where  $q$  is a prime of the form  $q = 6*h + 1$  and  $r$  is a prime of the form  $q = 6*i + 1$  and, where  $h$  and  $i$  are positive integers.

**Conjecture 24:** Any prime  $p$  of the form  $p = 6*k - 1$ , where  $k$  is positive integer, can be written in an infinity of distinct ways like  $p = q - r + 1$ , where  $q$  is a prime of the form  $q = 6*h - 1$  and  $r$  is a prime of the form  $q = 6*i + 1$  and, where  $h$  and  $i$  are positive integers.

**Conjecture 25:** There exist an infinity of pairs of primes  $(p, q)$ , where  $p$  is of the form  $6*k - 1$  and  $q$  is of the form  $6*h + 1$ , such that  $q - p + 1 = 3^n$ , for any  $n$  non-null positive integer (it can be seen that for  $n = 1$  the conjecture states the same thing with the twin primes conjecture).

**Conjecture 26:** Any square of prime  $p^2$ ,  $p \geq 5$ , can be written in an infinity of distinct ways like  $p^2 = q - r + 1$ , where  $q$  is a prime of the form  $q = 6*h + 1$  and  $r$  is a prime of the form  $q = 6*i + 1$ .

**Conjecture 27:** Any square of prime  $p^2$ ,  $p \geq 5$ , can be written in an infinity of distinct ways like  $p^2 = q - r + 1$ , where  $q$  is a prime of the form  $q = 6*h - 1$  and  $r$  is a prime of the form  $q = 6*i - 1$ .

**Conjecture 28:** For any  $r$  prime,  $r \geq 5$ , there exist an infinity of pairs of primes  $(p, q)$  such that the numbers  $(q^2 - p^2 - 2*r)/2$  and  $(q^2 - p^2 + 2*r)/2$  are both primes.

**Conjecture 29:** For any pair of primes  $(p, r)$ ,  $p \geq 5$ ,  $r \geq 5$ , there exist an infinity of primes  $q$  such that the numbers  $(q^2 - p^2 - 2*r)/2$  and  $(q^2 - p^2 + 2*r)/2$  are both primes.

**Conjecture 30:** For any  $p$  prime,  $p \geq 5$ , there exist an infinity of primes  $q$  such that the numbers  $(q^2 - p^2 - 2*p)/2$  and  $(q^2 - p^2 + 2*p)/2$  are both primes.

**Conjecture 31:** If  $x, y$  and  $r$  are odd primes such that  $y = x + 2*r$ , where  $r \geq 5$ , then there exist  $p$  and  $q$  also primes such that  $x = (q^2 - p^2 - 2*r)/2$  and  $y = (q^2 - p^2 + 2*r)/2$ .

**Conjecture 32:** For any  $p$  prime,  $p \geq 7$ , there exist a pair of smaller primes  $(q, r)$  such that the numbers  $x = (p^2 - q^2 - 2*r)/2$  and  $y = (p^2 - q^2 + 2*r)/2$  are both primes.

**Conjecture 33:** There exist an infinity of primes  $p$  such that  $2*p^2 - 1 = q*r$ , where  $q$  and  $r$  are also primes.

**Conjecture 34:** If  $p$  is prime and  $2*p^2 - 1 = q*r$ , where  $q$  and  $r$  are also primes, there exist an infinity of pairs of even positive integers  $[m, n]$  such that  $2*(p + m)^2 - 1 = (q + n)*(r + n)$ , such that  $p + m, q + n$  and  $r + n$  are also primes.

**Conjecture 35:** If  $p$  is prime and  $2*p^2 - 1 = q^2$ , where  $q$  is also prime, there exist an infinity of pairs of even positive integers  $[m, n]$  such that  $2*(p + m)^2 - 1 = (q + n)^2$ , such that  $p + m$  and  $q + n$  are also primes.

**Conjecture 36:** Any Poulet number of the form  $10*n + 1$  or  $10*n + 9$  can be written at least in one way as  $p*q + 10*k*h$ , where  $p$  and  $q$  are primes or powers of primes of the same form from the following four ones:  $10*m + 1, 10*m + 3, 10*m + 7$  or  $10*m + 9$ ,  $k$  and  $h$  are non-null positive integers and  $q - p = 10*k$ .

**Conjecture 37:** For any Poulet number  $N$  not divisible by 3 there exist at least a pair of numbers  $[p, q]$ , where  $p$  is prime and  $q$  is prime or square of prime, such that  $N = p^2 + q - 1$ .

**Conjecture 38:** There exist an infinity of Poulet numbers of the form  $n^2 + 120*n$ , where  $n$  is prime or a composite positive integer.

**Conjecture 39:** There exist an infinity of duplets of primes  $[p, q]$  such that  $p - q = 120$ ; there also exist an infinity of triplets of primes  $[p1, p2, q]$  such that  $p1*p2 - q = 120$ ; there also exist an infinity of quadruplets of primes  $[p1, p2, p3, q]$  such that  $p1*p2*p3 - q = 120$ ; generally, for any non-null positive integer  $i$  there exist  $i$  primes  $p1, p2, \dots, pi$  and a prime  $q$  such that  $p1*p2*...*pi - q = 120$ .

**Conjecture 40:** For any non-null positive integer  $i$  there exist an infinity of sets of  $i + 1$  primes  $p1, p2, \dots, pi, q$  such that  $p1*p2*...*pi - q = 2$ .

**Conjecture 41:** For any  $n$  even positive integer and for any  $i$  non-null positive integer there exist an infinity of sets of  $i + 1$  primes  $p_1, p_2, \dots, p_i, q$  such that  $p_1 * p_2 * \dots * p_i - q = n$ .

**Conjecture 42:** For any  $n$  even positive integer and for any  $i$  and  $j$  non-null positive integers there exist an infinity of distinct sets of  $i$  primes  $p_1, p_2, \dots, p_i$  and also an infinity of distinct sets of  $j$  primes  $q_1, q_2, \dots, q_j$  such that  $p_1 * p_2 * \dots * p_i - q_1 * q_2 * \dots * q_j = n$ .

**Conjecture 43:** For any  $n$  even positive integer and for any  $i, j, k, l$  non-null positive integers, for any  $k$  given primes  $a_1, a_2, \dots, a_k$  and for any  $l$  given primes  $b_1, b_2, \dots, b_l$ , there exist an infinity of distinct sets of  $i$  primes  $p_1, p_2, \dots, p_i$  and also an infinity of distinct sets of  $j$  primes  $q_1, q_2, \dots, q_j$  such that  $p_1 * p_2 * \dots * p_i * a_1 * a_2 * \dots * a_k - q_1 * q_2 * \dots * q_j * b_1 * b_2 * \dots * b_l = n$ .

**Conjecture 44:** For any square of a prime  $p$  of the form  $p = 6*k - 1$  is true at least one of the following six statements:

- (1)  $p^2$  can be deconcatenated into a prime and a number congruent to 2, 3 or 5 modulo 6;
- (2)  $p^2$  can be deconcatenated into a semiprime  $q*r$  where  $r - q = 8*k$  and a number congruent to 2, 3 or 5 modulo 6;
- (3)  $p^2$  can be deconcatenated into a semiprime  $3*q$ , where  $q$  is of the form  $10*k + 7$ , and a number congruent to 1 modulo 6;
- (4)  $p^2$  can be deconcatenated into a number of the form  $49 + 120*k$  and a number congruent to 0 modulo 6;
- (5)  $p^2$  can be deconcatenated into a number of the form  $121 + 48*k$  and a number congruent to 0 modulo 6;
- (6)  $p^2$  is a palindromic number.

**Conjecture 45:** For any square of a prime  $p$  of the form  $p = 6*k + 1$  is true at least one of the following six statements:

- (1)  $p^2$  can be deconcatenated into a prime and a number congruent to 2, 3 or 5 modulo 6;
- (2)  $p^2$  can be deconcatenated into a semiprime  $3^n*q$  and a number congruent to 1 modulo 6;
- (3)  $p^2$  can be deconcatenated into a number  $n$  such that  $n + 1$  is prime or power of prime and the digit 1;
- (4)  $p^2$  can be deconcatenated into a number  $n$  such that  $n + 1$  is prime or power of prime and the digit 9;
- (5)  $p^2$  can be deconcatenated into a number of the form  $49 + 120*k$  and a number congruent to 0 modulo 6;
- (6)  $p^2$  can be deconcatenated into a number of the form  $121 + 24*k$  and a number congruent to 0 modulo 6.

**Conjecture 46:** For any prime  $p$  there exist an infinity of positive integers  $n$  such that the number  $n*p - n + 1$  is prime.

**Conjecture 47:** For any prime  $p$  there exist an infinity of positive integers  $n$  such that the number  $n*p + n - 1$  is prime.

**Conjecture 48:** For any prime  $p$  there exist an infinity of positive integers  $n$  such that the number  $n^2*p - n + 1$  is prime.



**Conjecture 49:** For any prime  $p$  there exist an infinity of positive integers  $n$  such that the number  $n^{2p} + n - 1$  is prime.

**Conjecture 50:** For any prime  $p$  there exist an infinity of positive integers  $n$  such that the number  $n^p - p + n$  is prime.

**Conjecture 51:** For any prime  $p$  there exist an infinity of positive integers  $n$  such that the number  $n^p - p - n$  is prime.

**Conjecture 52:** For any prime  $p$  there exist an infinity of positive integers  $n$  such that the number  $(n - 1)^{2p} + n$  is prime.

**Conjecture 53:** For any prime  $p$  there exist an infinity of positive integers  $n$  such that the number  $(n - 1)^{2p} - n$  is prime.

**Conjecture 54:** For any two distinct primes greater than three  $p$  and  $q$  there exist an infinity of positive integers  $n$  such that the number  $(p^2 - 1)^n + q^2$  is prime, also an infinity of positive integers  $m$  such that the number  $(q^2 - 1)^m + p^2$  is prime.

**Conjecture 55:** For any  $p, q$  distinct primes,  $p > 30$ , there exist  $n$  positive integer such that  $p - 30^n$  and  $q + 30^n$  are both primes.

**Conjecture 56:** For any  $p, q, r$  distinct primes there exist  $n$  positive integer such that the numbers  $30^n - p, 30^n - q$  and  $30^n - r$  are all three primes.

**Conjecture 57:** There exist an infinity of pairs of distinct primes  $(p, q)$ , where  $p < q$ , both of the same form from the following eight ones:  $30^k + 1, 30^k + 7, 30^k + 11, 30^k + 13, 30^k + 17, 30^k + 19, 30^k + 23$  and  $30^k + 29$  such that the number  $p^q + (q - p)$  is prime.

**Conjecture 58:** There exist an infinity of pairs of distinct primes  $(p, q)$ , where  $p < q$ , both of the same form from the following eight ones:  $30^k + 1, 30^k + 7, 30^k + 11, 30^k + 13, 30^k + 17, 30^k + 19, 30^k + 23$  and  $30^k + 29$  such that the number  $p^q - (q - p)$  is prime.

**Conjecture 59:** For any  $p$  prime there exist an infinity of primes  $q, q > p$ , where  $p$  and  $q$  are both of the same form from the following eight ones:  $30^k + 1, 30^k + 7, 30^k + 11, 30^k + 13, 30^k + 17, 30^k + 19, 30^k + 23$  and  $30^k + 29$  such that the number  $p^q - (q - p)$  is prime.

**Conjecture 60:** For any pair of odd primes  $[p, q]$  there exist an infinity of pairs of distinct positive integers  $[m, n]$  such that the numbers  $x = p^{(m + 1)} - n$  and  $y = q^{(n + 1)} - m$  are both primes.

**Conjecture 61:** For any pair of odd primes  $[p, q]$  there exist an infinity of pairs of distinct positive integers  $[m, n]$  such that the numbers  $x = p^{(m - 1)} + n$  and  $y = q^{(n - 1)} + m$  are both primes.

**Conjecture 62:** For any pair of odd primes  $[p, q]$  there exist an infinity of pairs of distinct positive integers  $[m, n]$  such that the numbers  $x = p + (m + 1)^n$  and  $y = q + m^n$  are both primes.

**Conjecture 63:** For any pair of odd primes  $[p, q]$  there exist an infinity of pairs of distinct positive integers  $[m, n]$  such that the numbers  $x = p^*m - 2^*n$  and  $y = q^*n + 2^*m$  are both primes.

**Conjecture 64:** For any pair of odd primes  $[p, q]$  there exist an infinity of pairs of distinct positive integers  $[m, n]$  such that the numbers  $x = p^*m - 2^*n$  and  $y = q^*n - 2^*m$  are both primes.

**Conjecture 65:** For any pair of odd primes  $[p, q]$  there exist an infinity of pairs of distinct positive integers  $[m, n]$  such that the numbers  $x = p^*m + 2^*n$  and  $y = q^*n + 2^*m$  are both primes.

**Conjecture 66:** There exist an infinity of positive integers  $n$  such that the numbers  $30^*n + 7$ ,  $60^*n + 13$  and  $150^*n + 31$  are all three primes.

**Conjecture 67:** There exist an infinity of positive integers  $n$  such that the numbers  $30^*n - 23$ ,  $60^*n - 47$  and  $90^*n - 71$  are all three primes.

**Conjecture 68:** There exist an infinity of positive integers  $n$  such that the numbers  $30^*n - 29$ ,  $60^*n - 59$  and  $90^*n - 89$  are all three primes.

**Conjecture 69:** There exist an infinity of positive integers  $n$  such that the numbers  $30^*n - 23$ ,  $60^*n - 47$  and  $90^*n - 71$  are all three primes.

**Conjecture 70:** There exist an infinity of positive integers  $n$  such that the numbers  $30^*n - 7$ ,  $90^*n - 23$  and  $300^*n - 79$  are all three primes.

**Conjecture 71:** There exist an infinity of positive integers  $n$  such that the numbers  $30^*n - 17$ ,  $90^*n - 53$  and  $150^*n - 89$  are all three primes.

**Conjecture 72:** There exist an infinity of positive integers  $n$  such that the numbers  $60^*n + 13$ ,  $180^*n + 37$  and  $300^*n + 61$  are all three primes.

**Conjecture 73:** There exist an infinity of positive integers  $n$  such that the numbers  $330^*n + 7$ ,  $660^*n + 13$ ,  $990^*n + 19$  and  $1980^*n + 37$  are all four primes.

**Conjecture 74:** There exist an infinity of positive integers  $n$  such that the numbers  $90^*n + 1$ ,  $180^*n + 1$ ,  $270^*n + 1$  and  $540^*n + 1$  are all four primes.

**Conjecture 75:** There exist an infinity of pairs of primes  $[p, q]$  such that the numbers  $p + 30^*n$ ,  $q + 30^*n$  and  $p^*q + 30^*n$  are all three primes.

**Conjecture 76:** There exist an infinity of primes  $p$  such that the numbers  $x = 30^*n + p$  and  $y = 30^*m^*n + m^*p - m + 1$ , where  $m, n$  are non-null positive integers, are both primes.

**Conjecture 77:** There exist an infinity of primes of the form  $360^*p^*q + 1$ , where  $p, q$  are primes, both greater than or equal to 7.

**Conjecture 79:** There exist an infinity of primes of the form  $360^*p^*q + r$ , where  $p, q, r$  are primes, all of them greater than or equal to 7.

**Conjecture 80:** There exist an infinity of primes of the form  $2^m + n^2$ , where  $m$  is non-null positive integer and  $n$  odd integer.

**Conjecture 81:** There exist an infinity of primes of the form  $(2^n)^k + 2^n + 1$ , where  $n$  is non-null positive integer and  $k$  positive integer.

**Conjecture 82:** Any prime  $p$  of the form  $10^k + 1$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:  $10^x + 1$ ,  $10^y + 1$  respectively  $10^z + 1$ .

**Conjecture 83:** Any prime  $p$  of the form  $10^k + 1$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:  $10^x + 1$ ,  $10^y + 3$  respectively  $10^z + 7$ .

**Conjecture 84:** Any prime  $p$  of the form  $10^k + 1$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:  $10^x + 7$ ,  $10^y + 7$  respectively  $10^z + 7$ .

**Conjecture 85:** Any prime  $p$  of the form  $10^k + 1$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:  $10^x + 3$ ,  $10^y + 9$  respectively  $10^z + 9$ .

**Conjecture 86:** Any prime  $p$  of the form  $10^k + 3$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:  $10^x + 1$ ,  $10^y + 1$  respectively  $10^z + 1$ .

**Conjecture 87:** Any prime  $p$  of the form  $10^k + 3$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:  $10^x + 1$ ,  $10^y + 3$  respectively  $10^z + 9$ .

**Conjecture 88:** Any prime  $p$  of the form  $10^k + 3$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:  $10^x + 3$ ,  $10^y + 3$  respectively  $10^z + 7$ .

**Conjecture 89:** Any prime  $p$  of the form  $10^k + 3$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:  $10^x + 7$ ,  $10^y + 7$  respectively  $10^z + 9$ .

**Conjecture 90:** Any prime  $p$  of the form  $10^k + 7$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:  $10^x + 1$ ,  $10^y + 3$  respectively  $10^z + 3$ .

**Conjecture 91:** Any prime  $p$  of the form  $10^k + 7$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:  $10^x + 3$ ,  $10^y + 7$  respectively  $10^z + 7$ .

**Conjecture 92:** Any prime  $p$  of the form  $10^k + 7$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:  $10^x + 1$ ,  $10^y + 7$  respectively  $10^z + 9$ .

**Conjecture 93:** Any prime  $p$  of the form  $10^k + 7$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:  $10^x + 9$ ,  $10^y + 9$  respectively  $10^z + 9$ .

**Conjecture 94:** Any prime  $p$  of the form  $10^k + 9$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:  $10^x + 1$ ,  $10^y + 1$  respectively  $10^z + 7$ .

**Conjecture 95:** Any prime  $p$  of the form  $10^k + 9$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:  $10^x + 3$ ,  $10^y + 3$  respectively  $10^z + 3$ .

**Conjecture 96:** Any prime  $p$  of the form  $10^k + 9$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:  $10^x + 3$ ,  $10^y + 7$  respectively  $10^z + 9$ .

**Conjecture 97:** Any prime  $p$  of the form  $10^k + 9$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:  $10^x + 1$ ,  $10^y + 9$  respectively  $10^z + 9$ .

**Conjecture 98:** Any prime greater than or equal to 5 can be written at least in one way as  $(9 \cdot p^2 - q^2)/(2^n)$ , where  $p$  and  $q$  are primes and  $n$  non-null positive integer.

**Conjecture 99:** Any prime  $p$  of the form  $6^k + 1$  greater than or equal to 13 can be written as  $(q^2 - q + r)/3$ , where  $q$  is prime of the form  $6^k - 1$  and  $r$  is prime or power of prime or number 1.

**Conjecture 100:** Any prime  $p$  of the form  $6^k - 1$  greater than or equal to 11 can be written as  $(q^2 - q + r)/3$ , where  $q$  is prime of the form  $6^k - 1$  and  $r$  is prime or power of prime or number 1.

**Conjecture 101:** Any odd prime  $p$  can be written at least in one way as  $p = (q^{2^a \cdot 3^b \cdot 5^c \pm 1}) \cdot 2^n \pm 1$ , where  $q$  is an odd prime or is equal to 1, where  $a$ ,  $b$  and  $c$  are non-negative integers and  $n$  is non-null positive integer.

**Conjecture 102:** Any pair of twin primes  $[p_1, p_2]$  can be written as  $[p_1 = (q^{2^a \cdot 3^b \cdot 5^c \pm 1}) \cdot 2^n - 1, p_2 = (q^{2^a \cdot 3^b \cdot 5^c \pm 1}) \cdot 2^n + 1]$ , where  $q$  is prime or is equal to 1, where  $a$ ,  $b$  and  $c$  are non-negative integers and  $n$  is non-null positive integer.

**Conjecture 103:** For any pair of primes, greater than 3,  $[p_1, q_1]$ , where  $q_1 - p_1 = d$ , there exist at least a pair of positive integers  $[m, n]$ , where  $n - m = d$ , such that the numbers  $p_2 = p_1 \cdot q_1 - n + 1$  and  $q_2 = p_1 \cdot q_1 - m + 1$  are both primes.

**Conjecture 104:** For any even number  $d$  there exist an infinity of pairs of primes  $[p_1, q_1]$ , where  $q_1 - p_1 = d$ , such that the numbers  $p_2 = p_1 \cdot q_1 - p_1 + 1$  and  $q_2 = p_1 \cdot q_1 - q_1 + 1$  are both primes.

**Conjecture 105:** For any  $n$  non-null positive integer there exist  $q, r$  primes such that  $120^n \cdot q^r + 1 = p^2$ , where  $p$  is prime or a power of prime.

**Conjecture 106:** For any  $q$  odd prime there exist  $n$  non-null positive integer and  $r$  prime such that  $120^n \cdot q^r + 1 = p^2$ , where  $p$  is prime or a power of prime.

**Conjecture 107:** For any  $q, r$  odd primes there exist  $n$  non-null positive integer such that  $120^n \cdot q^r + 1 = p^2$ , where  $p$  is prime or a power of prime.

**Conjecture 108:** For any  $n$  non-null positive integer and any  $q$  prime there exist  $r$  prime such that  $120^n \cdot q^r + 1 = p^2$ , where  $p$  is prime or a power of prime.

**Conjecture 109:** For any  $n$  non-null positive integer there exist  $q$  prime such that  $120^n \cdot q^2 + 1 = p^2$ , where  $p$  is prime or a power of prime.

**Conjecture 110:** There exist an infinity of primes  $p$  of the form  $p = 25^n + 906304$ .

**Conjecture 111:** There exist an infinity of infinite sequences of the form  $a(1) = 1/p(1) + 1/p(2)$ ,  $a(2) = 1/p(1) + 1/p(2) + 1/p(3)$ , ..., where  $p(1) < p(2) < p(3)$ ..., such that the period of the rational

number  $a(1)$  is equal to  $p(2) - 1$ , the period of the rational number  $a(2)$  is equal to  $p(3) - 1$ , the period of the rational number  $a(n)$  is equal to  $a(n) - 1$ .

**Conjecture 112:** For any  $p(1)$  odd prime there exist infinite sequences of the form  $a(1) = 1/p(1) + 1/p(2)$ ,  $a(2) = 1/p(1) + 1/p(2) + 1/p(3)$ , ..., where  $p(1) < p(2) < p(3)$ ..., such that the period of the rational number  $a(1)$  is equal to  $p(2) - 1$ , the period of the rational number  $a(2)$  is equal to  $p(3) - 1$ , the period of the rational number  $a(n)$  is equal to  $a(n) - 1$ .

**Conjecture 113:** For any  $p(1)$  odd prime there exist infinite sequences of the form  $a(1) = 1/p(1) + 1/p(2)$ ,  $a(2) = 1/p(1) + 1/p(2) + 1/p(3)$ , ..., where  $p(1) < p(2) < p(3)$ ..., such that the period of the rational number  $a(1)$  is a multiple of  $p(2) - 1$ , the period of the rational number  $a(2)$  is a multiple of  $p(3) - 1$ , the period of the rational number  $a(n)$  is equal to  $a(n) - 1$ .

**Conjecture 114:** For any  $p(1)$  odd prime there exist infinite sequences of the form  $a(1) = 1/p(1) + 1/p(2)$ ,  $a(2) = 1/p(1) + 1/p(2) + 1/p(3)$ , ..., where  $p(1) < p(2) < p(3)$ ..., such that the period of the rational number  $a(1)$  divides  $p(2) - 1$ , the period of the rational number  $a(2)$  divides  $p(3) - 1$ , the period of the rational number  $a(n)$  divides  $a(n) - 1$ .

**Conjecture 115:** For any Poulet number  $P$  there exist a rational number  $r$  equal to a sum of unit fractions  $1/p(1) + 1/p(2) + 1/p(3)$ , ..., where  $p(1), p(2), p(3)$ ... are distinct odd primes, such that the period of  $r$  is equal to  $P - 1$ .

**Conjecture 116:** Any prime greater than or equal to 53 can be written at least in one way as a sum of three odd primes, not necessarily distinct, of the same form from the following four ones:  $10k + 1$ ,  $10k + 3$ ,  $10k + 7$  or  $10k + 9$ .

**Conjecture 117:** There exist an infinity of primes  $p$  that can be written as  $p = 2^*m + n$ , where  $m$  and  $n$  are distinct primes of the form  $10k + 1$ .

**Conjecture 118:** There exist an infinity of primes  $p$  that can be written as  $p = 2^*m + n$ , where  $m$  and  $n$  are distinct primes of the form  $10k + 3$ .

**Conjecture 119:** There exist an infinity of primes  $p$  that can be written as  $p = 2^*m + n$ , where  $m$  and  $n$  are distinct primes of the form  $10k + 7$ .

**Conjecture 120:** There exist an infinity of primes  $p$  that can be written as  $p = 2^*m + n$ , where  $m$  and  $n$  are distinct primes of the form  $10k + 9$ .

**Conjecture 121:** Any square of a prime greater than or equal to 7 can be written at least in one way as a sum of three odd primes, not necessarily distinct, but all three of the form  $10k + 3$  or all three of the form  $10k + 7$ .

**Conjecture 122.** Any square of a prime  $p^2$ , where  $p$  is greater than or equal to 7, can be written as  $p^2 = 2^*m + n$ , where  $m$  and  $n$  are distinct primes, both of the form  $10k + 3$  or both of the form  $10k + 7$ .

**Conjecture 123.** There exist an infinity of positive integers  $k$  such that  $6^*k - 1$  and  $18^*k - 5$  are both primes.

**Conjecture 124.** There exist an infinity of positive integers  $k$  such that  $6^k + 1$  and  $12^k + 1$  are both primes.

**Conjecture 125.** There exist an infinity of positive integers  $k$  such that  $6^k + 1$  and  $18^k + 1$  are both primes.

**Conjecture 126.** There exist an infinity of positive integers  $k$  such that  $6^k - 5$  and  $24^k - 5$  are both primes.

**Conjecture 127.** There exist an infinity of positive integers  $k$  such that  $6^k + 1$ ,  $12^k + 1$  and  $18^k + 1$  are all three primes.

**Conjecture 128.** There exist an infinity of positive integers  $k$  such that  $6^k + 1$ ,  $12^k + 1$  and  $18^k + 13$  are all three primes.

**Conjecture 129.** There exist an infinity of positive integers  $k$  such that  $k$ ,  $2^k - 1$  and  $5^k - 4$  are all three primes.

**Conjecture 130.** There exist an infinity of positive integers  $k$  such that  $k$ ,  $2^k - 1$  and  $3^k - 2$  are all three primes.

**Conjecture 131.** There exist an infinity of positive integers  $k$  such that  $k$ ,  $3^k - 2$  and  $4^k - 3$  are all three primes.

**Conjecture 132.** There exist an infinity of positive integers  $k$  such that  $40^k + 1$ ,  $60^k + 1$  and  $100^k + 1$  are all three primes.

**Conjecture 133.** There exist an infinity of positive integers  $k$  such that  $k$ ,  $2^k - 1$ ,  $7^k - 6$  and  $14^k - 13$  are all four primes.

**Conjecture 134.** There exist an infinity of positive integers  $k$  such that  $k$ ,  $2^k - 1$ ,  $6^k - 5$  and  $12^k - 11$  are all four primes.

**Conjecture 135.** There exist an infinity of pairs of distinct non-null positive integers  $m, n$  such that  $60^m \cdot n - 29$  and  $60^m \cdot n - (60^m + 29)$  are both primes.

**Conjecture 136.** There exist an infinity of pairs of distinct non-null positive integers  $[m, n]$  such that  $40^m - 10^n - 29$  and  $40^m - 10^n - 129$  are both primes.

**Conjecture 137.** For any pair of twin primes  $[q, r]$  there exist an infinity of primes  $p$  of the form  $p = 7200 \cdot q^r \cdot n + 1$ , where  $n$  is positive integer.

**Conjecture 138.** There exist an infinity of primes  $p$  of the form  $p = n \cdot s(p) - n + 1$ , where  $n$  is positive integer and  $s(p)$  is the sum of the digits of  $p$ .

**Conjecture 139.** There exist an infinity of primes  $p$  of the form  $p = n \cdot s(p) + n - 1$ , where  $n$  is positive integer and  $s(p)$  is the sum of the digits of  $p$ .

**Conjecture 140.** There exist an infinity of primes  $p$  of the form  $p = n \cdot s(p) + n - 1$ , where  $n$  is positive integer and  $s(p)$  is the sum of the digits of  $p$ .

**Conjecture 141.** There exist an infinity of primes  $p$  of the form  $p = m*n + m - n$ , where  $m$  and  $n$  are distinct odd primes.

**Conjecture 142.** There exist an infinity of primes  $p$  of the form  $p = m^2 - m*n + n$ , where  $m$  and  $n$  are distinct odd primes.

**Conjecture 143.** There exist an infinity of primes  $p$  of the form  $p = (q + 5^k)/10$ , where  $q$  is prime and  $k$  positive integer.

**Conjecture 144.** There exist an infinity of primes  $p$  of the form  $p = (q + 5^k)/30$ , where  $q$  is prime and  $k$  positive integer.

**Conjecture 145.** If  $p$  is a prime greater than or equal to 5, then the sequence  $q = p^2 + 96*k$ , where  $k$  is positive integer, contains an infinity of numbers which are primes or squares of primes.

**Conjecture 146.** If  $p$  is a prime greater than or equal to 5, then the sequence  $q = p^2 + 96*k$ , where  $k$  is positive integer, contains an infinity of semiprimes  $q = m*n$ , where  $m < n$ , with the following property: the number  $n - m + 1$  is a prime or a square of a prime.

**Conjecture 147.** Let  $a(i)$  be the general term of the sequence formed in the following way:  $a(i) = (((p*q - p)^2 - p)^2 - p)...$  and  $b(i)$  be the general term of the sequence formed in the following way:  $b(i) = (((p*q - q)^2 - q)^2 - q)...$ , where  $p, q$  are distinct odd primes. Then there exist an infinity of primes of the form  $a(i)/p$  as well as an infinity of primes of the form  $b(i)/q$  for any pair  $[p, q]$ .

**Conjecture 148.** Let  $a(i)$  be the general term of the sequence formed in the following way:  $a(i) = (((p*q - p)^2 - p)^2 - p)...$  and  $b(i)$  be the general term of the sequence formed in the following way:  $b(i) = (((p*q - q)^2 - q)^2 - q)...$ , where  $p, q$  are distinct odd primes. Then there exist an infinity of pairs  $[p, q]$  such that the sequence of primes  $a(i)/p$  is the same with the sequence of primes  $b(i)/q$ .

**Conjecture 149.** There exist an infinity of primes, for  $k$  positive integer, of the form  $n*2^k + 1$ , for  $n$  equal to 5, 6, 29, 49 or 99 (note that this conjecture is a consequence of Conjecture 1 and the examples observed above).

**Conjecture 150.** There exist an infinity of positive integers  $n$  such that the sequence  $n*2^k + 1$ , where  $k$  is positive integer, contains an infinity of primes.

**Conjecture 151.** Any Mersenne number  $2^n - 1$  with odd exponent  $n$ , where  $n$  is greater than or equal to 3, also  $n$  is not a power of 3, is either prime either divisible by a 2-Poulet number.

**Conjecture 152.** Any Mersenne-Coman number of the form  $P = ((2^m)^n - 1)/3^k$ , where  $m$  is non-null positive integer,  $n$  is odd, greater than or equal to 5, also  $n$  is not a power of 3, and  $k$  is equal to 0 or is equal to the greatest positive integer such that  $P$  is integer, is either a prime either divisible by at least a 2-Poulet number.

**Conjecture 153.** For any prime  $p$  greater than or equal to 5 the number  $(4^p - 1)/3$  is either prime either a product of primes  $p_1*p_2*...*p_n$  such that all the numbers  $p_i*p_j$  are 2-Poulet numbers for  $1 \leq i < j \leq n$ .

**Conjecture 154.** Any Fermat number  $F = 2^{2^n} + 1$  is either prime or divisible by a 2-Poulet number.

**Conjecture 155.** Any Fermat-Coman number of the form  $N = ((2^m)^p + 1)/3^k$ , where  $m$  is non-null positive integer,  $p$  is prime, greater than or equal to 7, and  $k$  is equal to 0 or is equal to the greatest positive integer such that  $N$  is integer, is either a prime or divisible by at least a 2-Poulet number.

**Conjecture 156.** For any prime  $p$  greater than or equal to 7 the number  $(4^p + 1)/5$  is either prime or a product of primes  $p_1 * p_2 * \dots * p_n$  such that all the numbers  $p_i * p_j$  are 2-Poulet numbers for  $1 \leq i < j \leq n$ .

**Conjecture 157.** For every prime  $p$ ,  $p > 5$ , there exist an infinity of primes  $q$ ,  $q = p + 30 * n$ , where  $n$  positive integer, such that the number  $p * q + 1092$  is equal to a semiprime  $p_i * q_i$ , where  $q_i - p_i = 30 * m$ , where  $m$  positive integer.

**Conjecture 158.** For every prime  $p$ ,  $p > 5$ , there exist an infinity of primes  $q$ ,  $q = p + 30 * n$ , where  $n$  positive integer, such that the number  $p * q + 1092$  is equal to a prime.

**Conjecture 159.** For every prime  $p$ ,  $p > 5$ , there exist an infinity of primes  $q$ ,  $q = p + 30 * n$ , where  $n$  positive integer, such that the number  $p * q + 3510$  is equal to a semiprime  $p_i * q_i$ , where  $q_i - p_i = 30 * m$ , where  $m$  positive integer.

**Conjecture 160.** For every prime  $p$ ,  $p > 5$ , there exist an infinity of primes  $q$ ,  $q = p + 30 * n$ , where  $n$  positive integer, such that the number  $p * q + 3510$  is equal to a prime.

**Conjecture 161.** There exist an infinity of the primes of the form  $30 * k + n$ , for any  $k$  positive integer and  $n$  equal to 1, 7, 11, 13, 17, 19, 23 or 29.

**Conjecture 162.** There exist an infinity of primes  $p$  such that the number  $7 * p^2 + 186$  is prime.

**Conjecture 163.** There exist an infinity of primes  $p$  such that the number  $7 * p^2 + 186$  is square of prime.

**Conjecture 164.** Let be  $F(k) = 2^{2^k} + n$ , where  $k$  is positive integer and  $n$  is an odd number. Then, there exist an infinity of numbers  $n$  such that  $F(k)$  is prime for  $k = 0$ ,  $k = 1$  and  $k = 2$ .

**Conjecture 165.** There exist an infinity of quadruplets of primes of the form  $[30 * k + 17, 30 * k + 19, 30 * k + 31, 30 * k + 43]$ . Such primes are, as can be seen above, for instance,  $[17, 19, 31, 43]$  or  $[137, 139, 151, 163]$ . There exist an infinity of quadruplets of primes of the form  $[30 * k + 17, 30 * k + 19, 30 * k + 31, 30 * k + 43]$ . Such primes are, as can be seen above, for instance,  $[17, 19, 31, 43]$  or  $[137, 139, 151, 163]$ .

**Conjecture 166.** Let be  $F(k) = 4^{4^k} + 3$ , where  $k$  is positive integer. Then, there exist an infinity of numbers  $k$  such that  $F(k)$  is equal to  $7 * p$ , where  $p$  is prime.

**Conjecture 167.** Let be  $F(k) = m^{(m^k)} + n$ , where  $m$  is even and  $n$  is odd, such that  $m + n = p$ , where  $p$  is prime. Then, there exist at least a  $k$ , beside of course  $k = 0$ , for which  $F(k)$  has as a prime factor the number  $p$ .



**Conjecture 168.** Let be  $F(k) = m^{(m^k)} + n$ , where  $m$  is odd and  $n$  is even, such that  $m + n = p$ , where  $p$  is prime. Then, there exist at least a  $k$ , beside of course  $k = 0$ , for which  $F(k)$  has as a prime factor the number  $p$ .

**Conjecture 169.** Let be  $F(k) = m^{(n^k)} + n$ , where  $k$  is positive integer and  $m$  and  $n$  are coprime positive integers, not both of them odd or both of them even. Then, there exist at least a  $k$ , beside of course  $k = 0$ , for which  $F(k)$  has as a prime factor the number  $p$ .

**Conjecture 170.** Let  $p$  and  $q$  be two odd primes; then, through the iterative operation  $((((p + 1)*q + 1) + 1)*q + 1\dots)$  finally it will be obtained a prime.

**Conjecture 171.** Let  $p$  be an odd prime; then the formula  $n^*p + n - 1$ , where  $n$  integer,  $n > 1$ , conducts to an infinity of prime numbers (note that, for  $n = 2$ , the conjecture states the infinity of Sophie Germain primes).

**Conjecture 172.** There exist an infinity of semiprimes  $p*q = N/3^m$ , where  $p < q$ ,  $m$  is positive integer and  $N$  is the number formed concatenating to the left a Carmichael number with the number 584, with the property that  $q - p + 1$  is a prime number.

**Conjecture 173.** Let  $p$  be an odd prime; then the formula  $((((n^*p + n - 1)^*p + n - 1))^*p + n - 1\dots)$  conducts to an infinity of prime numbers for any  $n$  integer,  $n > 1$ .

**Conjecture 174.** Let  $p$  and  $q$  be distinct odd primes; then the formula  $((((n^*p + (n - 1)^*q)^*p + (n - 1)^*q)^*p + (n - 1)^*q)\dots)$  conducts to an infinity of prime numbers for any  $n$  integer,  $n > 1$ .

**Conjecture 175.** Let  $p$  be an odd prime; then the formula  $((((n^*p + n - 1)^*p + n - 1))^*p + n - 1\dots)$  conducts, for any  $n$  integer,  $n > 1$ , to an infinity of Coman semiprimes.

**Conjecture 176.** Let  $p$  and  $q$  be distinct odd primes; then the formula  $((((n^*p + (n - 1)^*q)^*p + (n - 1)^*q)^*p + (n - 1)^*q)\dots)$  conducts, for any  $n$  integer,  $n > 1$ , to an infinity of Coman semiprimes.

**Conjecture 177.** Let  $p$  be an odd prime; then the formula  $((((n^*p - n + 1)^*p - n + 1))^*p - n + 1\dots)$  conducts to an infinity of prime numbers for any  $n$  integer,  $n > 1$ .

**Conjecture 178.** Let  $p$  and  $q$  be distinct odd primes; then the formula  $((((n^*p - (n + 1)^*q)^*p - (n + 1)^*q)^*p - (n + 1)^*q)\dots)$  conducts to an infinity of prime numbers for any  $n$  integer,  $n > 1$ .

**Conjecture 179.** Let  $p$  be an odd prime; then the formula  $((((n^*p - n + 1)^*p - n + 1))^*p - n + 1\dots)$  conducts, for any  $n$  integer,  $n > 1$ , to an infinity of Coman semiprimes.

**Conjecture 180.** Let  $p$  and  $q$  be distinct odd primes; then the formula  $((((n^*p - (n + 1)^*q)^*p - (n + 1)^*q)^*p - (n + 1)^*q)\dots)$  conducts, for any  $n$  integer,  $n > 1$ , to an infinity of Coman semiprimes.

## Part two. Forty-eight articles on primes

### 1. A conjecture about a way in which the squares of primes can be written and five other related conjectures

**Abstract.** I was playing with randomly formed formulas based on two distinct primes and the difference of them, when I noticed that the formula  $p + q + 2*(q - p) - 1$ , where  $p, q$  primes, conducts often to a result which is prime, semiprime, square of prime or product of very few primes. Starting from here, I made a conjecture about a way in which any square of a prime seems that can be written. Following from there, I made a conjecture about a possible infinite set of primes, a conjecture regarding the squares of primes and Poulet numbers and yet three other related conjectures.

#### Conjecture 1:

The square of any prime  $p, p \geq 5$ , can be written at least in one way as  $p^2 = 3*q - r - 1$ , where  $q$  and  $r$  are distinct primes,  $q \geq 5$  and  $r \geq 5$ .

#### Comment:

I really have no idea yet how it could be proved the conjecture or what implications it could have if it were true, so I'll just check it for the first few squares of primes.

#### Verifying the conjecture:

(for the first few squares of primes)

- :  $5^2 = 3*11 - 7 - 1$ , so  $[p, q, r] = [5, 11, 7]$ ;
- :  $7^2 = 3*19 - 7 - 1$ , so  $[p, q, r] = [7, 19, 7]$ ;
- :  $11^2 = 3*43 - 7 - 1$ , so  $[p, q, r] = [11, 43, 7]$ ;
- :  $13^2 = 3*59 - 7 - 1$ , so  $[p, q, r] = [13, 59, 7]$ ;
- :  $17^2 = 3*101 - 13 - 1$ , so  $[p, q, r] = [17, 101, 13]$ ;
- :  $19^2 = 3*127 - 19 - 1$ , so  $[p, q, r] = [19, 127, 19]$ ;
- :  $23^2 = 3*179 - 7 - 1$ , so  $[p, q, r] = [23, 179, 7]$ ;
- :  $29^2 = 3*283 - 7 - 1$ , so  $[p, q, r] = [29, 283, 7]$ ;
- :  $31^2 = 3*331 - 31 - 1$ , so  $[p, q, r] = [31, 331, 31]$ ;
- :  $37^2 = 3*461 - 13 - 1$ , so  $[p, q, r] = [37, 461, 13]$ ;
- :  $41^2 = 3*563 - 7 - 1$ , so  $[p, q, r] = [41, 563, 7]$ ;
- :  $43^2 = 3*619 - 7 - 1$ , so  $[p, q, r] = [43, 619, 7]$ ;
- :  $47^2 = 3*739 - 7 - 1$ , so  $[p, q, r] = [47, 739, 7]$ .

#### Note:

It can be seen that in few cases from the ones above we have  $p = r$ , so we make yet another conjecture:

### Conjecture 2:

There exist an infinity of primes  $p$  that can be written as  $p = (q^2 + q + 1)/3$ , where  $q$  is also a prime.

### Examples of such primes:

- :  $19 = (7^2 + 7 + 1)/3$ , so  $[p, q] = [19, 7]$ ;
- :  $127 = (19^2 + 19 + 1)/3$ , so  $[p, q] = [127, 19]$ ;
- :  $331 = (31^2 + 31 + 1)/3$ , so  $[p, q] = [331, 31]$ .

### Conjecture 3:

The square of any prime  $p$ ,  $p \geq 5$ , can be written at least in one way as  $p^2 = 3q - r - 1$ , where  $q$  is a Poulet number and  $r$  a prime,  $r \geq 5$ .

### Verifying the conjecture:

(for the first few squares of primes)

- :  $5^2 = 3 \cdot 341 - 997 - 1$ , so  $[p, q, r] = [5, 341, 997]$ ;
- :  $7^2 = 3 \cdot 1387 - 4111 - 1$ , so  $[p, q, r] = [7, 1387, 4111]$ ;
- :  $11^2 = 3 \cdot 4371 - 13921 - 1$ , so  $[p, q, r] = [11, 4371, 13921]$ ;
- :  $13^2 = 3 \cdot 341 - 853 - 1$ , so  $[p, q, r] = [13, 341, 853]$ .

### Comment:

Considering the results from the three conjectures above, I make three other related conjectures.

### Conjecture 4:

For any prime  $p$ ,  $p \geq 5$ , there exist an infinity of pairs of distinct primes  $[q, r]$  such that  $p = \sqrt{3q - r - 1}$ .

### Example:

(for  $p = 7$ )

- :  $7 = \sqrt{3 \cdot 19 - 7 - 1}$ , so  $[q, r] = [19, 7]$ ;
- :  $7 = \sqrt{3 \cdot 23 - 19 - 1}$ , so  $[q, r] = [23, 19]$ ;
- :  $7 = \sqrt{3 \cdot 29 - 37 - 1}$ , so  $[q, r] = [29, 37]$ ;
- :  $7 = \sqrt{3 \cdot 31 - 43 - 1}$ , so  $[q, r] = [31, 43]$ ;
- :  $7 = \sqrt{3 \cdot 37 - 61 - 1}$ , so  $[q, r] = [37, 61]$ ;
- :  $7 = \sqrt{3 \cdot 41 - 73 - 1}$ , so  $[q, r] = [41, 73]$  (...).

### Conjecture 5:

For any prime  $p$ ,  $p \geq 5$ , there exist at least a pair of distinct primes  $[q, r]$  such that  $p = (q^2 + r + 1)/3$ .

### Verifying the conjecture:

(for the first few primes)

- :  $5 = (3^2 + 5 + 1)/3$ , so  $[p, q, r] = [5, 3, 5]$ ;
- :  $7 = (3^2 + 11 + 1)/3$ , so  $[p, q, r] = [7, 3, 11]$ ;
- :  $11 = (3^2 + 5 + 1)/3$ , so  $[p, q, r] = [5, 3, 5]$ ;
- :  $13 = (5^2 + 13 + 1)/3$ , so  $[p, q, r] = [13, 5, 13]$ ;
- :  $17 = (3^2 + 41 + 1)/3$ , so  $[p, q, r] = [17, 3, 41]$ ;
- :  $19 = (5^2 + 31 + 1)/3$ , so  $[p, q, r] = [19, 5, 31]$ .

**Conjecture 6:**

For any prime  $p$  of the form  $p = 6k + 1$  there exist an infinity of pairs of distinct primes  $[q, r]$  such that  $p = 3q - r^2 - 1$ .

**Example:**

(for  $p = 37$ )

- :  $37 = 3 \cdot 29 - 7^2 - 1$ , so  $[q, r] = [29, 7]$ ;
- :  $37 = 3 \cdot 53 - 11^2 - 1$ , so  $[q, r] = [53, 11]$ ;
- :  $37 = 3 \cdot 109 - 17^2 - 1$ , so  $[q, r] = [109, 17]$ ;
- :  $37 = 3 \cdot 293 - 29^2 - 1$ , so  $[q, r] = [293, 29]$ ;
- :  $37 = 3 \cdot 1693 - 71^2 - 1$ , so  $[q, r] = [1693, 71]$ ;
- :  $37 = 3 \cdot 1789 - 73^2 - 1$ , so  $[q, r] = [1789, 73]$  (...).

## 2. A conjecture about an infinity of sets of integers, each one having an infinite number of primes

**Abstract.** In this paper, inspired by one of my previous papers posted on Vixra, I make, considering the sum of the digits of an odd integer, a conjecture about an infinity of sets of integers, each one having an infinite number of primes and I also make, considering the sum of the digits of a prime number, two other conjectures.

### Conjecture 1:

For an infinity of odd positive integers  $m$  there is an infinite set of primes with the property that the sum of their digits is equal to  $m + 1$ .

### Conjecture 2:

For an infinity of primes  $p$  there is an infinite set of primes with the property that the sum of their digits is equal to  $p + 1$ .

### Comment:

Such a prime  $p$  I conjectured to be, in a previous paper posted on Vixra, the number 13.

### Conjecture 3:

There is an infinite number of values the sum of the digits of the numbers  $p + 1$ , where  $p$  is odd prime, may have.

### Note:

For a list with prime numbers with the property that the sum of their digits is equal to an even number see the sequence A119449 in OEIS.

### Note:

We will refer hereinafter with  $D(m)$  to the set of primes with the property that the sum of their digits is equal to  $m + 1$ , where  $m$  is an odd integer.

### The sequence $D(1)$ :

: 101 (...).

### The sequence $D(3)$ :

: 13, 31, 103, 211, 1021, 1201 (...).

### The sequence $D(5)$ :

: (...).

**The sequence D(7):**

: 17, 53, 71, 107, 233, 251, 431, 503, 521, 701, 1061, 1151, 1223 (...).

**The sequence D(9):**

: 19, 37, 73, 109, 127, 163, 181, 271, 307, 433, 523, 541, 613, 631, 811, 1009, 1063, 1117, 1153, 1171 (...).

**The sequence D(11):**

: (...).

**The sequence D(13):**

: 59, 149, 167, 239, 257, 293, 347, 419, 491, 563, 617, 653, 743, 761, 941, 1049, 1193, 1229, 1283, 1319 (...).

**The sequence D(15):**

: 79, 97, 277, 349, 367, 383, 439, 457, 547, 619, 673, 691, 709, 727, 853, 907, 1069, 1087, 1249 (...).

**The sequence D(17):**

: (...).

**The sequence D(19):**

: 389, 479, 569, 587, 659, 677, 839, 857, 929, 947, 983, 1289 (...).

**The sequence D(21):**

: 499, 769, 787, 859, 877, 967 (...).

**Note:**

It can easily be seen that for some values of odd integers  $m$  were obtained much more primes with the sum of the digits equal to  $m + 1$  than for other values of  $m$ ; for instance were obtained, from the first hundred of primes having the sum of digits equal to an even number, 20 such primes for which  $m = 9$ , 21 such primes for which  $m = 13$ , 19 such primes for which  $m = 15$ , but no such primes at all for which  $m = 5$ ,  $m = 11$  or  $m = 17$ .

### 3. A trivial but notable observation about a relation between the twin primes and the number 14

**Abstract.** There are known few interesting properties which distinguish twin primes from the general set of primes, like for instance that 46% of primes smaller than 19000 are Ramanujan primes while about 78% of the lesser of twin primes smaller than 19000 are Ramanujan primes. But seems that a much more trivial observation about the lesser of twin primes escaped attention: from the first 500 numbers which are lesser in a pair of twin primes, 66 of them have the following remarkable property: the sum of their digits is equal to 14.

#### **Note:**

For a list of lesser of twin primes and also for the property mentioned in Abstract regarding Ramanujan primes see the sequence A001359 in OEIS.

#### **Observation:**

Like I mentioned in abstract, this paper is a trivial observation of a fact: from the first 100 numbers which are lesser in a pair of twin primes, 20 of them have the property that the sum of their digits is equal to 14; from the first 500 numbers which are lesser in a pair of twin primes, 66 of them have this property; these numbers are:

: 59, 149, 239, 347, 419, 617, 1049, 1229, 1319, 1427, 1481, 1607, 2129, 2237, 2309, 2381, 3119, 3371, 3461, 3821, 4019, 4091, 4127, 4217, 4271, 4721, 5009, 5441, 6701, 7331, 8231, 9041, 10067, 10139, 10427, 11057, 12821, 13217, 13721, 13901, 14009, 14081, 16061, 18041, 18131, 18311, 19211, 20147, 20507, 21191, 21317, 22037, 22091, 22109, 22271, 22541, 23027, 24107, 25601, 29021, 30137, 31181, 31541, 31721, 32027, 32117.

#### **Conjecture:**

There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that the sum of its digits it's equal to 14.

#### **Note:**

The conjecture above implies the following one:

#### **Conjecture:**

There is an infinity of primes with the property that the sum of their digits is equal to 14.

#### 4. An observation about the digital root of the twin primes, few conjectures and an open problem on primes

**Abstract.** Few interesting properties which distinguish twin primes from the general set of primes there are already known. I wrote myself an article regarding an interesting property of a set of (pairs of) twin primes based on the sum of the digits of the lesser (implicitly greater) prime from a pair of twin primes. This paper notes a property regarding twin primes based on their digital root.

##### **Observation:**

The digital root of the lesser prime  $p$  from a pair of twin primes  $[p, q]$ , under the condition that  $p > 3$ , is, for the first such 100 primes (for a list of lesser of twin primes see the sequence A001359 in OEIS):

: 5, 2, 8, 2, 5, 5, 8, 2, 8, 2, 5, 8, 2, 8, 2, 5, 8, 2, 5, 5, 8, 2, 8, 2, 5, 5, 2, 2, 8, 2, 8, 2, 8, 2, 5, 5, 8, 2, 8, 5, 8, 2, 5, 5, 5, 2, 5, 2, 5, 8, 2, 5, 2, 5, 8, 5, 5, 5, 8, 2, 2, 8, 5, 5, 8, 5, 8, 5, 8, 5, 2, 8, 2, 5, 2, 2, 8, 2, 8, 2, 5, 8, 2, 8, 5, 8, 2, 5, 5, 5, 2, 8, 2, 2, 8, 8, 5, 5.

##### **Note:**

Obviously the digital root of a lesser from a pair of twin primes can never be equal to 3, 6 or 9 (it would be then a number divisible by 3 not a prime) or 1, 4 or 7 (the greater from the pair of twin primes would be in this case divisible by 3).

##### **Conjecture 1:**

There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 2.

##### **Conjecture 2:**

There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 5.

##### **Conjecture 3:**

There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 8.

##### **Note:**

Is remarkable that the three subsets of the set of (lesser of) twin primes seem to have (of course, for a given term great enough) an approximately equal number of terms; for instance, from the first 100 from the lesser of twin primes, 33 of them have the digital root equal to 2, 35 have the digital root equal to 5 and 32 have the digital root equal to 7.



**Conjecture 4:**

Let  $a_i$  be the sequence of the lesser of twin primes whose digital root is equal to 2,  $b_i$  be the sequence of the lesser of twin primes whose digital root is equal to 5 and  $c_i$  be the sequence of the lesser of twin primes whose digital root is equal to 8. Then:

- : there exist an infinity of terms  $n$  of  $a_i$  for which the number of the terms of  $b_i$  smaller than  $n$  is equal to the number of the terms of  $c_i$  smaller than  $n$ ;
- : there exist an infinity of terms  $n$  of  $b_i$  for which the number of the terms of  $a_i$  smaller than  $n$  is equal to the number of the terms of  $c_i$  smaller than  $n$ ;
- : there exist an infinity of terms  $n$  of  $c_i$  for which the number of the terms of  $a_i$  smaller than  $n$  is equal to the number of the terms of  $b_i$  smaller than  $n$ .

**Conjecture 5:**

There is an infinity of integers  $n$  for which the set of the lesser (greater) of the twin primes smaller than  $n$  is divided in three subsets with an equal number of terms,  $a_i$  with the property that the digital root of its terms is equal to 2 (4),  $b_i$  with the property that the digital root of its terms is equal to 5 (7) and  $c_i$  with the property that the digital root of its terms is equal to 8 (1).

**Open problem:**

Is there any other prime  $p$  beside  $p = 23$  with the property that the following six subsets of odd primes have an equal number of terms smaller than  $p$ ? The terms of the six subsets are the primes whose digital root is equal to 1, 2, 4, 5, 7 respectively 8 (it can be seen that, for  $p = 23$ , we have the following odd primes smaller than 23 that belong to the six subsets: 5, 7, 11, 13, 17, 19, whose digital root is 5, 7, 2, 4, 8, 1).

## 5. A conjecture on primes involving the pairs of sexy primes

**Abstract.** This paper states a conjecture on primes involving two types of pairs of primes: the pairs of sexy primes, which are the two primes that differ from each other by six and the pairs of primes of the form  $[p, q]$ , where  $q = p + 6*r$ , where  $r$  is positive integer.

### Conjecture:

If  $n$  and  $n + 6$  are both primes (in other words if  $[n, n + 6]$  is a pair of sexy primes), where  $n \geq 7$ , then the number  $m = n + 3$  can be written at least in one way as  $m = p + q$ , where  $p$  and  $q$  are primes,  $q = p + 6*r$  and  $r$  is positive integer.

### Verifying the conjecture:

(for the first fifteen pairs of sexy primes)

- : for  $[n, n + 6] = [7, 13]$  we have  $[p, q, r] = [5, 5, 0]$ ;
- : for  $[n, n + 6] = [11, 17]$  we have  $[p, q, r] = [7, 7, 0]$ ;
- : for  $[n, n + 6] = [13, 19]$  we have  $[p, q, r] = [5, 11, 1]$ ;
- : for  $[n, n + 6] = [17, 23]$  we have  $[p, q, r] = [7, 13, 1]$ ;
- : for  $[n, n + 6] = [23, 29]$  we have  $[p, q, r] = [13, 13, 0]$ ;
- : for  $[n, n + 6] = [31, 37]$  we have  $[p, q, r] = [5, 29, 1]$  or  $[17, 17, 0]$ ;
- : for  $[n, n + 6] = [37, 43]$  we have  $[p, q, r] = [11, 29, 3]$  or  $[17, 23, 1]$ ;
- : for  $[n, n + 6] = [41, 47]$  we have  $[p, q, r] = [7, 37, 1]$  or  $[13, 31, 3]$ ;
- : for  $[n, n + 6] = [47, 53]$  we have  $[p, q, r] = [7, 37, 1]$  or  $[13, 37, 4]$  or  $[19, 31, 2]$ ;
- : for  $[n, n + 6] = [53, 59]$  we have  $[p, q, r] = [13, 43, 5]$  or  $[19, 37, 3]$ ;
- : for  $[n, n + 6] = [61, 67]$  we have  $[p, q, r] = [17, 47, 5]$  or  $[23, 41, 3]$ ;
- : for  $[n, n + 6] = [67, 73]$  we have  $[p, q, r] = [11, 59, 8]$  or  $[17, 53, 6]$  or  $[23, 47, 4]$  or  $[29, 41, 2]$ ;
- : for  $[n, n + 6] = [73, 79]$  we have  $[p, q, r] = [17, 59, 7]$  or  $[23, 53, 5]$  or  $[29, 47, 3]$ ;
- : for  $[n, n + 6] = [83, 89]$  we have  $[p, q, r] = [7, 79, 12]$  or  $[13, 73, 10]$  or  $[19, 67, 8]$  or  $[43, 43, 0]$ ;
- : for  $[n, n + 6] = [97, 103]$  we have  $[p, q, r] = [11, 89, 13]$ ;  
or  $[17, 83, 11]$  or  $[29, 71, 7]$  or  $[47, 53, 1]$ .

## 6. A conjecture on the pairs of primes $p$ , $q$ , where $q$ is equal to the sum of $p$ and a primorial number

**Abstract.** In a previous paper I stated a conjecture on primes involving the pairs of sexy primes, which are the two primes that differ from each other by six. In this paper I extend that conjecture on the pairs of primes  $[p, q]$ , where  $q$  is of the form  $p + p(n)\#$ , where  $p(n)\#$  is a primorial number, which means the product of first  $n$  primes.

### Conjecture:

If  $p$  and  $p + p(n)\#$  are both primes, where  $p > p(n)\#$ ,  $n \geq 2$  and  $p(n)\#$  is a primorial number (which means the product of first  $n$  primes), then the number  $m = p + p(n)\#/2$  can be written at least in one way as  $m = x + y$ , where  $x$  and  $y$  are primes or squares of primes,  $y = x + p(n)\#*r$  and  $r$  is positive integer.

### Note:

For a list of primorial numbers, see the sequence A002110 in OEIS; the first ten primorial numbers are: 1, 2, 6, 30, 210, 2310, 30030, 510510, 9699690, 223092870.

### Note:

Because  $p(0)\#$  is, by convention or justified through the concept of “empty product”, equal to 1, then  $p(1)\#$  is equal to 2,  $p(2)\#$  is equal to 6,  $p(3)\#$  is equal to 30,  $p(4)\#$  is equal to 210 and so on.

### Comment:

The conjecture it will be then formulate:

: for  $p(2)\# = 6$ :

If  $p$  and  $p + 6$  are both primes, where  $p > 6$ , then the number  $m = p + 3$  can be written at least in one way as  $m = x + y$ , where  $x$  and  $y$  are primes or squares of primes,  $y = x + 6*r$  and  $r$  is positive integer (this is the conjecture which I made in a previous paper, where I also verified it for the first fifteen pairs of sexy primes, with the difference that in the formulation from there  $x$  and  $y$  were “primes” not “primes or squares of primes”);

: for  $p(3)\# = 30$ :

If  $p$  and  $p + 30$  are both primes, where  $p > 30$ , then the number  $m = p + 15$  can be written at least in one way as  $m = x + y$ , where  $x$  and  $y$  are primes or squares of primes,  $y = x + 30*r$  and  $r$  is positive integer;

: for  $p(4)\# = 210$ :

If  $p$  and  $p + 210$  are both primes, where  $p > 210$ , then the number  $m = p + 105$  can be written at least in one way as  $m = x + y$ , where  $x$  and  $y$  are primes or squares of primes,  $y = x + 210*r$  and  $r$  is positive integer;

: for  $p(5)\# = 2310$ :

If  $p$  and  $p + 2310$  are both primes, where  $p > 2310$ , then the number  $m = p + 1155$  can be written at least in one way as  $m = x + y$ , where  $x$  and  $y$  are primes,  $y = x + 2310*r$  and  $r$  is positive integer.

**Verifying the conjecture:**

(for the first ten pairs of primes  $[p, p + 30]$ , where  $p > 30$ )

- : for  $[p, p + 30] = [31, 61]$  we have  $[x, y, r] = [23, 23, 0]$ ;
- : for  $[p, p + 30] = [37, 67]$  we have  $[p, q, r] = [11, 41, 1]$ ;
- : for  $[p, p + 30] = [41, 71]$  we have  $[p, q, r] = [13, 43, 1]$ ;
- : for  $[p, p + 30] = [43, 73]$  we have  $[p, q, r] = [29, 29, 0]$ ;
- : for  $[p, p + 30] = [53, 83]$  we have  $[p, q, r] = [19, 49, 1]$ ;
- : for  $[p, p + 30] = [59, 89]$  we have  $[p, q, r] = [37, 37, 0]$  or  $[7, 67, 2]$ ;
- : for  $[p, p + 30] = [67, 97]$  we have  $[p, q, r] = [41, 41, 0]$  or  $[11, 71, 2]$ ;
- : for  $[p, p + 30] = [71, 101]$  we have  $[p, q, r] = [43, 43, 0]$  or  $[13, 73, 2]$ ;
- : for  $[p, p + 30] = [73, 103]$  we have  $[p, q, r] = [29, 59, 1]$ ;
- : for  $[p, p + 30] = [79, 109]$  we have  $[p, q, r] = [47, 47, 0]$ .

**Verifying the conjecture:**

(for the first eight pairs of primes  $[p, p + 210]$ , where  $p > 210$ )

- : for  $[p, p + 210] = [13, 223]$  we have  $[x, y, r] = [59, 59, 0]$  or  $[29, 89, 2]$ ;
- : for  $[p, p + 210] = [17, 227]$  we have  $[x, y, r] = [61, 61, 0]$ ;
- : for  $[p, p + 210] = [19, 229]$  we have  $[x, y, r] = [17, 107, 3]$ ;
- : for  $[p, p + 210] = [23, 233]$  we have  $[x, y, r] = [19, 109, 3]$  or  $[49, 79, 1]$ ;
- : for  $[p, p + 210] = [29, 239]$  we have  $[x, y, r] = [67, 67, 0]$  or  $[7, 127, 4]$  or  $[37, 97, 2]$ ;
- : for  $[p, p + 210] = [31, 241]$  we have  $[x, y, r] = [23, 113, 3]$  or  $[53, 83, 1]$ ;
- : for  $[p, p + 210] = [41, 251]$  we have  $[x, y, r] = [73, 73, 0]$  or  $[43, 103, 2]$ ;
- : for  $[p, p + 210] = [47, 257]$  we have  $[x, y, r] = [31, 121, 3]$ .

**Verifying the conjecture:**

(for the first pair of primes  $[p, p + 2310]$ , where  $p > 2310$ )

- : for  $[p, p + 2310] = [23, 2333]$  we have  $[x, y, r] = [49, 1129, 36]$  or  $[109, 1069, 32]$  or  $[139, 1039, 30]$  or  $[169, 1009, 28]$  or  $[349, 829, 16]$  or  $[469, 709, 12]$  or  $[439, 739, 10]$ .

## 7. Two conjectures involving the sum of a prime and a factorial number

**Abstract.** In this paper I state two conjectures about the sum of a prime and a factorial.

### Note:

For a list of factorial numbers see the sequence A000142 in OEIS.

### Conjecture 1:

For any odd prime  $p$  there exist at least one prime  $q$  such that  $p + n! = q$ , where  $n$  is a positive integer,  $n < p$ .

### Verifying the conjecture:

(for the first five odd primes  $p$ )

- :  $3 + 2! = 5$ , so  $[p, q, n] = [3, 5, 2]$ ;
- :  $5 + 2! = 7$  and  $5 + 3! = 11$  and  $5 + 4! = 29$ , so  $[p, q, n] = [5, 7, 2]$  or  $[5, 11, 3]$  or  $[5, 29, 4]$ ;
- :  $7 + 3! = 13$  and  $7 + 4! = 31$  and  $7 + 5! = 127$  and  $7 + 6! = 727$  so  $[p, q, n] = [7, 13, 3]$  or  $[7, 31, 4]$  or  $[7, 127, 5]$  or  $[7, 727, 6]$ ;
- :  $11 + 2! = 13$  and  $11 + 6! = 17$  and  $11 + 5! = 131$  and  $11 + 7! = 5051$  and  $11 + 10! = 3628811$  so  $[p, q, n] = [11, 13, 2]$  or  $[11, 17, 6]$  or  $[11, 131, 10]$  or  $[11, 5051, 7]$  or  $[11, 3628811, 10]$ ;
- :  $13 + 3! = 19$  and  $13 + 4! = 37$  and  $13 + 6! = 733$  so  $[p, q, n] = [13, 19, 3]$  or  $[13, 37, 4]$  or  $[13, 733, 6]$ .

### Note:

From the primes  $q$ ,  $q \geq 5$ ,  $q \leq 401$ , just three primes can't be written as  $p + n!$ , where  $p$  is a lesser odd prime and  $n$  is a positive integer, *i.e.* the primes 41, 101, 367 (but, interesting, 367 can be written as  $7^3 + 5!$ ); indeed:

- :  $q$  can be written as  $p + 2!$  for  $q = 5, 7, 31, 43, 61, 73, 103, 109, 139, 151, 181, 193, 199, 229, 241, 271, 283, 313, 349$  [...];
- :  $q$  can be written as  $p + 3!$  for  $q = 11, 13, 17, 23, 29, 37, 47, 53, 59, 67, 79, 89, 107, 113, 157, 163, 173, 179, 197, 239, 251, 257, 263, 269, 277, 337, 359, 373, 379, 389$  [...];
- :  $q$  can be written as  $p + 4!$  for  $q = 71, 83, 97, 127, 131, 191, 223, 251, 281, 293, 307, 317, 331, 383, 397$  [...];
- :  $q$  can be written as  $p + 5!$  for  $q = 149, 167, 227, 233, 311, 347, 353, 401$  [...].

### Conjecture 2:

For any odd prime  $p$ ,  $p \geq 5$ , there exist an infinity of primes  $q$  of the form  $q = (p + n!)/n^k$ , where  $n$  and  $k$  are positive integers and  $n \geq p$ .

## Examples:

- : for  $p = 5$ , we have:
  - :  $q = 29 = (6! + 5)/5^2$ ;
  - :  $q = 1009 = (7! + 5)/5^1$ ;
  - :  $q = 1613 = (8! + 5)/5^2$ ;
  - :  $q = 72577 = (9! + 5)/5^1$ ;
  - [...]
- : for  $p = 7$ , we have:
  - :  $q = 103 = (7! + 7)/7^2$ ;
  - :  $q = 823 = (8! + 7)/7^2$ ;
  - [...]
- : for  $p = 11$ , we have:
  - :  $q = 329891 = (11! + 11)/11^2$ ;
  - [...]
- : for  $p = 13$ , we have:
  - :  $q = 2834329 = (13! + 13)/13^3$ ;
  - :  $q = 515847877 = (13! + 13)/13^2$ ;
  - [...].

## 8. Seven conjectures on a certain way to write primes including two generalizations of the twin primes conjecture

**Abstract.** In this paper I make few conjectures about a way to write an odd prime  $p$ , id est  $p = q - r + 1$ , where  $q$  and  $r$  are also primes; two of these conjectures can be regarded as generalizations of the twin primes conjecture, which states that there exist an infinity of pairs of twin primes.

### Conjecture 1

(Which can be regarded as a generalization of the twin primes conjecture)

Any odd prime  $p$  can be written in an infinity of distinct ways like  $p = q - r + 1$ , where  $q$  and  $r$  are also primes; in other words, there exist an infinity of pairs of primes  $(q, r)$  such that  $q - r = p - 1$ , for any odd prime  $p$  (it can be seen that for  $p = 3$  the conjecture states the same thing with the twin primes conjecture).

### Conjecture 2

Any prime  $p$  of the form  $p = 6*k + 1$ , where  $k$  is positive integer, can be written in an infinity of distinct ways like  $p = q - r + 1$ , where  $q$  is a prime of the form  $q = 6*h - 1$  and  $r$  is a prime of the form  $q = 6*i - 1$  and, where  $h$  and  $i$  are positive integers.

Example: the prime  $p = 7$  can be written as  $11 - 5 + 1$ ;  $17 - 11 + 1$ ;  $23 - 17 + 1$  etc.; in fact, for  $p = 7$  the conjecture states that there exist an infinity of pairs of sexy primes  $(q, r)$ , both of the form  $6*k - 1$  (sexy primes are the primes that differ by each other by six).

### Conjecture 3

Any prime  $p$  of the form  $p = 6*k + 1$ , where  $k$  is positive integer, can be written in an infinity of distinct ways like  $p = q - r + 1$ , where  $q$  is a prime of the form  $q = 6*h + 1$  and  $r$  is a prime of the form  $q = 6*i + 1$  and, where  $h$  and  $i$  are positive integers.

Example: the prime  $p = 7$  can be written as  $13 - 7 + 1$ ;  $19 - 13 + 1$ ;  $37 - 31 + 1$  etc.; in fact, for  $p = 7$  the conjecture states that there exist an infinity of pairs of sexy primes  $(q, r)$ , both of the form  $6*k + 1$ .

### Conjecture 4

Any prime  $p$  of the form  $p = 6*k - 1$ , where  $k$  is positive integer, can be written in an infinity of distinct ways like  $p = q - r + 1$ , where  $q$  is a prime of the form  $q = 6*h - 1$  and  $r$  is a prime of the form  $q = 6*i + 1$  and, where  $h$  and  $i$  are positive integers.

### Conjecture 5

(Which can be regarded as a generalization of the twin primes conjecture)

There exist an infinity of pairs of primes  $(p, q)$ , where  $p$  is of the form  $6*k - 1$  and  $q$  is of the form  $6*h + 1$ , such that  $q - p + 1 = 3^n$ , for any  $n$  non-null positive integer (it can be seen that for  $n = 1$  the conjecture states the same thing with the twin primes conjecture).

Example: for  $n = 2$  we have the pairs of primes  $(p, q)$ :  $(11, 19)$ ;  $(23, 31)$  etc.; for  $n = 3$  we have the pairs of primes  $(5, 31)$ ;  $(11, 37)$  etc.

### **Conjecture 6**

Any square of prime  $p^2$ ,  $p \geq 5$ , can be written in an infinity of distinct ways like  $p^2 = q - r + 1$ , where  $q$  is a prime of the form  $q = 6 \cdot h + 1$  and  $r$  is a prime of the form  $q = 6 \cdot i + 1$ .

Example: the number  $49 = 7^2$  can be written as  $61 - 13 + 1$ ;  $67 - 19 + 1$ ;  $79 - 31 + 1$  etc.

### **Conjecture 7**

Any square of prime  $p^2$ ,  $p \geq 5$ , can be written in an infinity of distinct ways like  $p^2 = q - r + 1$ , where  $q$  is a prime of the form  $q = 6 \cdot h - 1$  and  $r$  is a prime of the form  $q = 6 \cdot i - 1$ .

Example: the number  $49 = 7^2$  can be written as  $53 - 5 + 1$ ;  $59 - 11 + 1$ ;  $71 - 23 + 1$  etc.



## 9. An interesting formula for generating primes and five conjectures about a certain type of pairs of primes

**Abstract.** In this paper I just enunciate a formula which often leads to primes and products of very few primes and I state five conjectures about the pairs of primes of the form  $[(q^2 - p^2 - 2*r)/2, (q^2 - p^2 + 2*r)/2]$ , where  $p, q, r$  are odd primes.

### Conjecture 1:

For any  $r$  prime,  $r \geq 5$ , there exist an infinity of pairs of primes  $(p, q)$  such that the numbers  $(q^2 - p^2 - 2*r)/2$  and  $(q^2 - p^2 + 2*r)/2$  are both primes.

### Conjecture 2:

For any pair of primes  $(p, r)$ ,  $p \geq 5, r \geq 5$ , there exist an infinity of primes  $q$  such that the numbers  $(q^2 - p^2 - 2*r)/2$  and  $(q^2 - p^2 + 2*r)/2$  are both primes.

### Note:

The numbers  $m = (q^2 - p^2 - 2*r)/2$  and  $n = (q^2 - p^2 + 2*r)/2$ , where  $p, q, r$  are odd primes, seems to be often primes and generally products of very few primes.

### Examples:

- : For  $(p, q, r) = (13, 11, 5)$  we have  $(m, n) = (19, 29)$ ;
- : For  $(p, q, r) = (17, 11, 5)$  we have  $(m, n) = (79, 89)$ ;
- : For  $(p, q, r) = (37, 11, 5)$  we have  $(m, n) = (1039, 1049)$ ;
  
- : For  $(p, q, r) = (13, 11, 7)$  we have  $(m, n) = (17, 31)$ ;
- : For  $(p, q, r) = (19, 11, 7)$  we have  $(m, n) = (113, 127)$ ;
- : For  $(p, q, r) = (23, 11, 7)$  we have  $(m, n) = (197, 211)$ ;
  
- : For  $(p, q, r) = (17, 13, 7)$  we have  $(m, n) = (53, 67)$ ;
- : For  $(p, q, r) = (19, 13, 7)$  we have  $(m, n) = (89, 103)$ ;
- : For  $(p, q, r) = (37, 13, 7)$  we have  $(m, n) = (593, 607)$ ;
  
- : For  $(p, q, r) = (19, 17, 7)$  we have  $(m, n) = (29, 43)$ ;
- : For  $(p, q, r) = (23, 17, 7)$  we have  $(m, n) = (113, 127)$ ;
- : For  $(p, q, r) = (29, 17, 7)$  we have  $(m, n) = (269, 283)$ ;
- : For  $(p, q, r) = (11, 7, 7)$  we have  $(m, n) = (29, 43)$ ;
- : For  $(p, q, r) = (13, 7, 7)$  we have  $(m, n) = (53, 67)$ ;
- : For  $(p, q, r) = (17, 7, 7)$  we have  $(m, n) = (113, 127)$ ;
  
- : For  $(p, q, r) = (19, 11, 11)$  we have  $(m, n) = (109, 131)$ ;
- : For  $(p, q, r) = (31, 11, 11)$  we have  $(m, n) = (409, 431)$ ;
- : For  $(p, q, r) = (61, 11, 11)$  we have  $(m, n) = (1789, 1811)$ .

**Conjecture 3:**

For any  $p$  prime,  $p \geq 5$ , there exist an infinity of primes  $q$  such that the numbers  $(q^2 - p^2 - 2*p)/2$  and  $(q^2 - p^2 + 2*p)/2$  are both primes.

**Conjecture 4:**

If  $x$ ,  $y$  and  $r$  are odd primes such that  $y = x + 2*r$ , where  $r \geq 5$ , then there exist  $p$  and  $q$  also primes such that  $x = (q^2 - p^2 - 2*r)/2$  and  $y = (q^2 - p^2 + 2*r)/2$ .

**Examples:**

- : For  $(x, y, r) = (17, 31, 7)$ , we have  $(p, q) = (11, 13)$ ;
- : For  $(x, y, r) = (29, 43, 7)$ , we have  $(p, q) = (17, 19)$ ;
- : For  $(x, y, r) = (53, 67, 7)$ , we have  $(p, q) = (13, 17)$ .

**Conjecture 5:**

For any  $p$  prime,  $p \geq 7$ , there exist a pair of smaller primes  $(q, r)$  such that the numbers  $x = (p^2 - q^2 - 2*r)/2$  and  $y = (p^2 - q^2 + 2*r)/2$  are both primes.

**Examples:**

- : For  $p = 7$ ,  $(q, r) = (5, 5)$  and  $(x, y) = (7, 17)$ ;
- : For  $p = 11$ ,  $(q, r) = (5, 5)$  and  $(x, y) = (43, 53)$  and also  $(q, r) = (7, 7)$  and  $(x, y) = (43, 53)$  and also  $(q, r) = (7, 5)$  and  $(x, y) = (31, 41)$ ;
- : For  $p = 13$ ,  $(q, r) = (7, 7)$  and  $(x, y) = (53, 67)$  and also  $(q, r) = (11, 5)$  and  $(x, y) = (19, 29)$  and also  $(q, r) = (11, 7)$  and  $(x, y) = (17, 31)$ .

## 10. Few possible infinite sets of triplets of primes related in a certain way and an open problem

**Abstract.** In this paper I make three conjectures about a type of triplets of primes related in a certain way, i.e. the triplets of primes  $[p, q, r]$ , where  $2 \cdot p^2 - 1 = q \cdot r$  and I raise an open problem about the primes of the form  $q = (2 \cdot p^2 - 1)/r$ , where  $p, r$  are also primes.

### Conjecture 1:

There exist an infinity of primes  $p$  such that  $2 \cdot p^2 - 1 = q \cdot r$ , where  $q$  and  $r$  are also primes.

Examples: such primes are: 5, 19, 23, 29, 31, 47, 53, 61, 67, 71, 79, 83, 97 (...).

### Conjecture 2:

If  $p$  is prime and  $2 \cdot p^2 - 1 = q \cdot r$ , where  $q$  and  $r$  are also primes, there exist an infinity of pairs of even positive integers  $[m, n]$  such that  $2 \cdot (p + m)^2 - 1 = (q + n) \cdot (r + n)$ , such that  $p + m, q + n$  and  $r + n$  are also primes.

Examples:

- : for  $p = 5$ ,  $[q, r] = [7, 7]$ ; for  $[m, n] = [24, 34]$ ,  $[p + n, q + n, r + n] = [29, 41, 41]$ ;
- : for  $p = 19$ ,  $[q, r] = [7, 103]$ ; for  $[m, n] = [34, 34]$ ,  $[p + n, q + n, r + n] = [53, 41, 137]$ ;
- : for  $p = 23$ ,  $[q, r] = [7, 151]$ ; for  $[m, n] = [44, 40]$ ,  $[p + n, q + n, r + n] = [67, 47, 191]$ ;
- : for  $p = 31$ ,  $[q, r] = [17, 113]$ ; for  $[m, n] = [22, 24]$ ,  $[p + n, q + n, r + n] = [53, 41, 137]$ ;
- : for  $p = 71$ ,  $[q, r] = [17, 593]$ ; for  $[m, n] = [26, 13]$ ,  $[p + n, q + n, r + n] = [97, 31, 607]$ ;
- : for  $p = 83$ ,  $[q, r] = [23, 599]$ ; for  $[m, n] = [254, 210]$ ,  $[p + n, q + n, r + n] = [307, 233, 809]$ ; also for  $[m, n] = [198, 258]$ ,  $[p + n, q + n, r + n] = [347, 281, 857]$ ;
- : for  $p = 139$ ,  $[q, r] = [17, 2273]$ ; for  $[m, n] = [250, 110]$ ,  $[p + n, q + n, r + n] = [389, 127, 2383]$ .

### Conjecture 3:

If  $p$  is prime and  $2 \cdot p^2 - 1 = q^2$ , where  $q$  is also prime, there exist an infinity of pairs of even positive integers  $[m, n]$  such that  $2 \cdot (p + m)^2 - 1 = (q + n)^2$ , such that  $p + m$  and  $q + n$  are also primes.

Example: for  $p = 5$ ,  $q = 7$ ; for  $[m, n] = [24, 34]$ ,  $[p + n, q + n] = [29, 41]$ .

### Open problem:

Which primes  $q$  can be written as  $q = (2 \cdot p^2 - 1)/r$ , where  $p, r$  are also primes?

## 11. Two types of pairs of primes that could be associated to Poulet numbers

**Abstract.** In this paper I combine two of my objects of study, the Poulet numbers and the different types of pairs of primes and I state two conjectures about few ways in which types of Poulet numbers could be associated with types of pairs of primes.

### Conjecture 1:

Any Poulet number of the form  $10^n + 1$  or  $10^n + 9$  can be written at least in one way as  $p^*q + 10^*k^*h$ , where  $p$  and  $q$  are primes or powers of primes of the same form from the following four ones:  $10^*m + 1$ ,  $10^*m + 3$ ,  $10^*m + 7$  or  $10^*m + 9$ ,  $k$  and  $h$  are non-null positive integers and  $q - p = 10^*k$ .

### Verifying the conjecture:

(for the first six such Poulet numbers)

- :  $341 = 9*(9 + 20) + 4*20 = 9*(9 + 10) + 17*10$ , so  $[p, q] = [3^2, 29]$  or  $[3^2, 19]$ ;
- :  $561 = 19*(29 + 10) + 1*10 = 9*(9 + 50) + 3*10$ , so  $[p, q] = [19, 29]$  or  $[3^2, 59]$ ;
- :  $1729 = 23*(23 + 50) + 1*50 = 17*(17 + 80) + 1*80 = 23*(23 + 30) + 17*30 = 27*(27 + 10) + 73*10 = 23*(23 + 20) + 37*20 = 13*(13 + 60) + 13*60 = 7*(7 + 120) + 7*120 = 17*(17 + 30) + 31*30 = 13*(13 + 40) + 26*40$ , so  $[p, q] = [23, 73]$  or  $[17, 97]$  or  $[23, 53]$  or  $[3^3, 37]$  or  $[23, 43]$  or  $[13, 73]$  or  $[7, 127]$  or  $[17, 47]$  etc.;
- :  $2701 = 29*(29 + 60) + 2*60$ , so  $[p, q] = [29, 89]$  etc.;
- :  $2821 = 29*(29 + 60) + 4*60$ , so  $[p, q] = [29, 89]$  etc.;
- :  $4369 = 27*(27 + 130) + 1*130$ , so  $[p, q] = [3^3, 157]$  etc.

### Note 1:

Some such Poulet numbers can be written as  $p^*q + (q - p)$ , where  $p, q$  primes; for instance, the Hardy-Ramanujan number 1729 can be written in two different ways like this:  $1729 = 23*53 + (53 - 23) = 17*97 + (97 - 17)$ .

### Note 2:

Probably this conjecture can stipulate for  $h$  to be equal to 1 or prime or power of prime (in the examples above, we found that  $h$  is equal to:  $2^2$  or 17; 1 or 3; 1 or 17 or 73 or 37 or 13 or 7 or 31; 2;  $2^2$ ; 1).

### Conjecture 2:

For any Poulet number  $N$  not divisible by 3 there exist at least a pair of numbers  $[p, q]$ , where  $p$  is prime and  $q$  is prime or square of prime, such that  $N = p^2 + q - 1$ .

### Verifying the conjecture:

(for the first six such Poulet numbers)

- :  $341 = 7^2 + 293 - 1 = 13^2 + 173 - 1 = 17^2 + 53 - 1$ , so  $[p, q] = [7, 293]$  or  $[13, 173]$  or  $[17, 53]$ ;

- :  $1105 = 13^2 + 937 - 1 = 23^2 + 577 - 1$ , so  $[p, q] = [13, 937]$  or  $[23, 577]$ ;
- :  $1387 = 23^2 + 859 - 1 = 29^2 + 547 - 1 = 37^2 + 19 - 1$ , so  $[p, q] = [23, 859]$  or  $[29, 547]$  or  $[37, 19]$ ;
- :  $1729 = 7^2 + 41^2 - 1 = 11^2 + 1609 - 1 = 19^2 + 37^2 - 1 = 23^2 + 1201 - 1 = 31^2 + 769 - 1$ , so  $[p, q] = [7, 41^2]$  or  $[41, 7^2]$  or  $[11, 1609]$  or  $[19, 37^2]$  or  $[37, 19^2]$  or  $[23, 1201]$  or  $[31, 769]$ .

**Note:**

Some such Poulet numbers can be written as  $p^2 + q^2 - 1$ , where  $p, q$  are primes; for instance, the Hardy-Ramanujan number 1729 can be written in two different ways like this:  $1729 = 7^2 + 41^2 - 1 = 19^2 + 37^2 - 1$ .

## 12. A set of Poulet numbers and generalizations of the twin primes and de Polignac's conjectures inspired by this

**Abstract.** In this paper I show a set of Poulet numbers, each one of them having the same interesting relation between its prime factors, and I make four conjectures, one about the infinity of this set, one about the infinity of a certain type of duplets respectively triplets respectively quadruplets and so on of primes and finally two generalizations, of the twin primes conjecture respectively of de Polignac's conjecture.

### Conjecture 1:

There exist an infinity of Poulet numbers of the form  $n^2 + 120*n$ , where  $n$  is prime or a composite positive integer.

#### Note:

In the first case, obviously  $n$  is a prime factor of such a Poulet number and the product of the other prime factors is equal to  $n + 120$ ; for instance, the number 1729 is a part of this set of Poulet numbers because  $1729 = 7*13*19$  can be written as  $13^2 + 13*120$  and implicitly  $7*19 = 13 + 120$ . First few such Poulet numbers are:

:  $1729 = 7*13*19 = 13^2 + 13*120$ ;  
:  $4681 = 31*151 = 31^2 + 31*120$ ;  
:  $6601 = 7*23*41 = 41^2 + 41*120$ .

#### Note:

In the second case, obviously  $n$  is a product of few prime factors of such a Poulet number and the product of the other prime factors is equal to  $n + 120$ . Such a Poulet number is  $75361 = 11*13*17*31 = 221^2 + 221*120$  and implicitly  $11*31 = 13*17 + 120$ .

### Conjecture 2:

There exist an infinity of duplets of primes  $[p, q]$  such that  $p - q = 120$ ; there also exist an infinity of triplets of primes  $[p1, p2, q]$  such that  $p1*p2 - q = 120$ ; there also exist an infinity of quadruplets of primes  $[p1, p2, p3, q]$  such that  $p1*p2*p3 - q = 120$ ; generally, for any non-null positive integer  $i$  there exist  $i$  primes  $p1, p2, \dots, pi$  and a prime  $q$  such that  $p1*p2*...*pi - q = 120$ .

#### Examples:

:  $151 - 31 = 120$ ;  
:  $7*19 - 13 = 120$ ;  
:  $7*17*37 - 4283 = 120$ .

### Conjecture 3:

(generalization of the twin primes conjecture)

For any non-null positive integer  $i$  there exist an infinity of sets of  $i + 1$  primes  $p_1, p_2, \dots, p_i, q$  such that  $p_1 * p_2 * \dots * p_i - q = 2$ .

**Conjecture 4:**

(generalization of de Polignac's conjecture)

For any  $n$  even positive integer and for any  $i$  non-null positive integer there exist an infinity of sets of  $i + 1$  primes  $p_1, p_2, \dots, p_i, q$  such that  $p_1 * p_2 * \dots * p_i - q = n$ .

### 13. A very exhaustive generalization of de Polignac's conjecture

**Abstract.** In a previous paper I made a generalization of de Polignac's conjecture. In this paper I extend that generalization as much as is possible.

#### **Conjecture:**

For any  $n$  even positive integer and for any  $i$  and  $j$  non-null positive integers there exist an infinity of distinct sets of  $i$  primes  $p_1, p_2, \dots, p_i$  and also an infinity of distinct sets of  $j$  primes  $q_1, q_2, \dots, q_j$  such that  $p_1 * p_2 * \dots * p_i - q_1 * q_2 * \dots * q_j = n$ .

#### **Case $[i, j, n] = [1, 1, 2]$ :**

In this case we have  $p - q = 2$ , which gave us the twin primes conjecture.

#### **Case $[i, j, n] = [1, 1, n]$ :**

In this case we have  $p - q = n$ , which gave us de Polignac's conjecture.

#### **Case $[i, j, n] = [2, 1, 2]$ :**

In this case we have  $p_1 * p_2 - q = 2$ .

Such triplets of primes  $[p_1, p_2, q]$ , are:  $[7, 13, 89]$ ,  $[7, 19, 131]$ ,  $[7, 37, 257]$ ... Note that the conjecture can be further extended in this case to: for any  $p_1$  odd prime there exist an infinity of pairs of primes  $[p_2, q]$  such that  $p_1 * p_2 - q = 2$ .

#### **Case $[i, j, n] = [1, 2, 2]$ :**

In this case we have  $p - q_1 * q_2 = 2$ .

Such triplets of primes  $[p, q_1, q_2]$ , are:  $[79, 11, 7]$ ,  $[163, 23, 7]$ ,  $[331, 47, 7]$ ... Note that the conjecture can be further extended in this case to: for any  $q_1$  odd prime there exist an infinity of pairs of primes  $[p, q_2]$  such that  $p - q_1 * q_2 = 2$ .

#### **Conjecture:**

(the most exhaustive generalization of de Polignac's conjecture)

For any  $n$  even positive integer and for any  $i, j, k, l$  non-null positive integers, for any  $k$  given primes  $a_1, a_2, \dots, a_k$  and for any  $l$  given primes  $b_1, b_2, \dots, b_l$ , there exist an infinity of distinct sets of  $i$  primes  $p_1, p_2, \dots, p_i$  and also an infinity of distinct sets of  $j$  primes  $q_1, q_2, \dots, q_j$  such that  $p_1 * p_2 * \dots * p_i * a_1 * a_2 * \dots * a_k - q_1 * q_2 * \dots * q_j * b_1 * b_2 * \dots * b_l = n$ .



## 14. A formula which conducts to primes or to a type of composites that could form a class themselves

**Abstract.** In this paper I present a very simple formula which conducts often to primes or composites with very few prime factors; for instance, for the first 27 consecutive values introduced as “input” in this formula were obtained 10 primes, 4 squares of primes and 12 semiprimes; just 2 from the numbers obtained have three prime factors; but the most interesting thing is that the composites obtained have a special property that make them form a class of numbers themselves.

### Observation:

The numbers  $C = 3^3(3^3 + n \cdot 10) + n \cdot 10$ , where  $n$  is a positive integer of the form  $4 + 9 \cdot k$ , or in other words  $C = 2520 \cdot k + 1849$ , are very often primes or numbers with very few prime factors, composites that have certain very interesting properties. Let's see the case of the first 27 consecutive such numbers  $C$ ; we will consider all 27 numbers but we will list them separately in three different lists: the case  $C$  is prime or square of prime, the case  $C$  is Coman semiprime and the case of the other numbers  $C$  (note that a Coman semiprime is a semiprime  $p \cdot q$  with the property that  $p - q + 1$  is a prime or a square of prime; this is a class of numbers that I met it often in my research, for instance in the study of 2-Poulet numbers, many of these semiprimes having this property, but as well in the study of the prime factors of Carmichael numbers):

### The case $C$ is prime or square of prime:

- : for  $k = 0$  we have  $C = 43^2$  where 43 prime;
- : for  $k = 1$  we have  $C = 4369$  prime;
- : for  $k = 2$  we have  $C = 83^2$  where 83 prime;
- : for  $k = 3$  we have  $C = 97^2$  where 97 prime;
- : for  $k = 5$  we have  $C = 14449$  prime;
- : for  $k = 7$  we have  $C = 19489$  prime;
- : for  $k = 11$  we have  $C = 29569$  prime;
- : for  $k = 12$  we have  $C = 32089$  prime;
- : for  $k = 16$  we have  $C = 42169$  prime;
- : for  $k = 19$  we have  $C = 223^2$  where 223 prime;
- : for  $k = 20$  we have  $C = 52249$  prime;
- : for  $k = 23$  we have  $C = 59809$  prime;

### The case $C$ is Coman semiprime:

- : for  $k = 4$  we have  $C = 79 \cdot 151$  and  $151 - 79 + 1 = 73$  prime;
- : for  $k = 6$  we have  $C = 71 \cdot 239$  and  $239 - 71 + 1 = 13^2$ , where 13 prime;
- : for  $k = 8$  we have  $C = 13 \cdot 1693$  and  $1693 - 13 + 1 = 41^2$ , where 41 prime;
- : for  $k = 13$  we have  $C = 53 \cdot 653$  and  $653 - 53 + 1 = 601$  prime;
- : for  $k = 14$  we have  $C = 107 \cdot 347$  and  $347 - 107 + 1 = 241$  prime;
- : for  $k = 15$  we have  $C = 31 \cdot 1279$  and  $1279 - 31 + 1 = 1249$  prime;
- : for  $k = 24$  we have  $C = 157 \cdot 397$  and  $397 - 157 + 1 = 241$  prime.

- : for  $k = 25$  we have  $C = 64849$  prime;
- : for  $k = 26$  we have  $C = 67369$  prime.

**The other numbers C:**

- : for  $k = 9$  we have  $C = 19*1291$  and  $1291 - 19 + 1 = 19*67$  and  $67 - 19 + 1 = 7^2$ , where 7 prime;
- : for  $k = 10$  we have  $C = 11*2459$  and  $2459 - 11 + 1 = 31*79$  and  $79 - 31 + 1 = 7^2$ , where 7 prime;
- : for  $k = 17$  we have  $C = 23*29*67$  and  $23*29 - 67 + 1 = 601$  prime,  $29*67 - 23 + 1 = 17*113$  where  $113 - 17 + 1 = 97$  prime and  $23*67 - 28 = 17*89$  where  $89 - 17 + 1 = 73$  prime;
- : for  $k = 18$  we have  $C = 17*2777$  and  $2777 - 17 + 1 = 11*251$  and  $251 - 11 + 1 = 241$  prime;
- : for  $k = 21$  we have  $C = 11*13*383$  and  $11*13 - 383 + 1 = -239$  prime in absolute value,  $11*383 - 13 + 1 = 4201$  prime,  $13*383 - 11 + 1 = 4969$  prime;
- : for  $k = 22$  we have  $C = 59*971$  and  $971 - 59 + 1 = 11*83$  and  $83 - 11 + 1 = 73$  prime;
- : for  $k = 27$  we have  $C = 47*1487$  and  $1487 - 47 + 1 = 11*131$  and  $131 - 11 + 1 = 11^2$ , where 11 prime.

**Note:**

It can be seen that also “the other numbers C” have special properties; for instance, the semiprimes can be considered a kind of “extended Coman semiprimes” because of the iterative process that ends also in a prime or in a square of prime: let  $N = p_1*q_1$ ; then  $p_1 - q_1 + 1 = p_2*q_2$  then  $p_2 - q_2 + 1 = p_3*q_3$  and so on until is obtained a prime. On the other side, the numbers with three prime factors obtained  $p*q*r$  have the property that  $p*q - r + 1$ ,  $p*r - q + 1$  and  $q*r - p + 1$  are primes or (extended) Coman semiprimes.

## 15. Four sequences of numbers obtained through concatenation, rich in primes and semiprimes

**Abstract.** In this paper I will define four sequences of numbers obtained through concatenation, definitions which also use the notion of “sum of the digits of a number”, sequences that have the property to produce many primes, semiprimes and products of very few prime factors.

### Observation 1:

Let  $x$  be a number with the sum of the digits equal to  $p$ , where  $p$  is prime, and  $y$  the number obtained through concatenation of  $x$  and  $p$ ; then  $y$  is often a prime, a semiprime or a product of very few prime factors.

Numbers that belong to this sequence are:

: 2911 (semiprime), 9211 (semiprime), 4913 (cube of prime), 9413 (prime), 8917 (semiprime), 9817 (prime), 99119 (prime), 91919 (semiprime), 19919 (prime), 99523 (prime), 95923 (prime), 59923 (semiprime), 999431 (prime), 949931 (prime), 499931 (semiprime) etc.

### Observation 2:

Let  $x$  be a number equal to the sum of the digits of  $p$ , where  $p$  is prime, and  $y$  the number obtained through concatenation of  $x$  and  $p$ ; then  $y$  is often a prime, a semiprime or a product of very few prime factors.

Numbers that belong to this sequence are:

: 211 (prime), 413 (semiprime), 817 (semiprime), 1019 (prime), 523 (prime), 1129 (prime), 431 (prime), 1037 (semiprime), 541 (prime), 743 (prime), 1147 (semiprime), 853 (prime), 1459 (prime), 761 (prime), 1367 (prime), 871 (semiprime), 1073 (semiprime) etc.

### Observation 3:

Let  $x$  be a number whose sum of the digits is equal to the sum of the digits of  $p$ , where  $p$  is prime, and  $y$  the number obtained through concatenation of  $x$  and  $p$ ; then  $y$  is often a prime, a semiprime or a product of very few prime factors.

Numbers that belong to this sequence are:

: 1111 (semiprime), 10111 (prime), 2011 (prime), 20011 (prime), 200011 (semiprime), 2213 (prime), 22013 (prime), 3113 (semiprime), 4013 (prime), 40013 (prime), 400013 (semiprime), 4000013 (semiprime) etc.

### Observation 4:

Let  $x$  be a number with the sum of the digits equal to  $p$ , where  $p$  is prime, let  $y = 6*x + 5$  and  $z$  the number obtained through concatenation of  $y$  and  $p$ ; then  $z$  is often a prime, a semiprime or a product of very few prime factors.

Numbers that belong to this sequence are:

- : For  $[p, x, y] = [11, 29, 179]$  we have  $z = 17911$  prime;
- : For  $[p, x, y] = [11, 92, 557]$  we have  $z = 55711$  prime;
- : For  $[p, x, y] = [11, 902, 5417]$  we have  $z = 541711$  prime;
- : For  $[p, x, y] = [29, 9299, 55799]$  we have  $z$  semiprime;
- : For  $[p, x, y] = [29, 9929, 59579]$  we have  $z$  semiprime;
- : For  $[p, x, y] = [29, 9992, 59957]$  we have  $z = 5995729$  prime;
- : For  $[p, x, y] = [29, 2999, 17999]$  we have  $z = 1799929$  prime;
- : For  $[p, x, y] = [29, 9299, 55799]$  we have  $z$  semiprime;
- : For  $[p, x, y] = [29, 9929, 59579]$  we have  $z$  semiprime;
- : For  $[p, x, y] = [29, 9992, 59957]$  we have  $z = 5995729$  prime.

**Note:**

In order to see wherefrom the idea of this sequence originate see my previous paper “A conjecture about a large subset of Carmichael numbers related to concatenation”.

## 16. A conjecture on the squares of primes of the form $6k - 1$

**Abstract.** In this paper I make a conjecture on the squares of primes of the form  $6k - 1$ , conjecture that states that by a certain deconcatenation of those numbers (each one in other two numbers) it will be obtained similar results.

### Conjecture:

For any square of a prime  $p$  of the form  $p = 6k - 1$  is true at least one of the following six statements:

- (1)  $p^2$  can be deconcatenated into a prime and a number congruent to 2, 3 or 5 modulo 6;
- (2)  $p^2$  can be deconcatenated into a semiprime  $q*r$  where  $r - q = 8*k$  and a number congruent to 2, 3 or 5 modulo 6;
- (3)  $p^2$  can be deconcatenated into a semiprime  $3*q$ , where  $q$  is of the form  $10*k + 7$ , and a number congruent to 1 modulo 6;
- (4)  $p^2$  can be deconcatenated into a number of the form  $49 + 120*k$  and a number congruent to 0 modulo 6;
- (5)  $p^2$  can be deconcatenated into a number of the form  $121 + 48*k$  and a number congruent to 0 modulo 6;
- (6)  $p^2$  is a palindromic number.

### Examples for case (1):

- : for  $5^2 = 25$  we got 5 prime and  $2 \equiv 2 \pmod{6}$ ;
- : for  $17^2 = 289$  we got 89 prime and  $2 \equiv 2 \pmod{6}$ ;
- : for  $23^2 = 529$  we got 29 prime and  $5 \equiv 5 \pmod{6}$ ;
- : for  $29^2 = 841$  we got 41 prime and  $8 \equiv 2 \pmod{6}$ ;
- : for  $53^2 = 2809$  we got 809 prime and  $2 \equiv 2 \pmod{6}$ ;
- : for  $71^2 = 5041$  we got 41 prime and  $50 \equiv 2 \pmod{6}$ ;
- : for  $83^2 = 6889$  we got 89 prime and  $68 \equiv 2 \pmod{6}$ ;
- : for  $107^2 = 11449$  we got 449 prime and  $11 \equiv 5 \pmod{6}$ ;
- : for  $167^2 = 27889$  we got 89 prime and  $278 \equiv 2 \pmod{6}$ ;
- : for  $173^2 = 29929$  we got 29 prime and  $29 \equiv 5 \pmod{6}$  also 929 prime and  $2 \equiv 2 \pmod{6}$ ;
- : for  $179^2 = 32041$  we got 41 prime and  $320 \equiv 2 \pmod{6}$ ;
- : for  $191^2 = 36481$  we got 6481 prime and  $3 \equiv 3 \pmod{6}$ ;
- : for  $197^2 = 38809$  we got 809 prime and  $38 \equiv 2 \pmod{6}$ ;
- : for  $227^2 = 51529$  we got 29 prime and  $515 \equiv 5 \pmod{6}$ ;
- : for  $233^2 = 54289$  we got 89 prime and  $542 \equiv 2 \pmod{6}$ ;
- : for  $239^2 = 57121$  we got 7121 prime and  $5 \equiv 5 \pmod{6}$ ;
- : for  $269^2 = 72361$  we got 61 prime and  $723 \equiv 3 \pmod{6}$ .

### Examples for case (2):

- : for  $47^2 = 2209$  we got  $209 = 11*19$  where  $19 - 11 = 8*1$  and  $2 \equiv 2 \pmod{6}$ ;
- : for  $59^2 = 3481$  we got  $481 = 13*37$  where  $37 - 13 = 8*3$  and  $3 \equiv 3 \pmod{6}$ ;

- : for  $131^2 = 17161$  we got  $161 = 7*23$  where  $23 - 7 = 8*2$  and  $17 \equiv 5 \pmod{6}$ ;
- : for  $149^2 = 22201$  we got  $2201 = 31*71$  where  $71 - 31 = 8*5$  and  $2 \equiv 2 \pmod{6}$ .

**Examples for case (3):**

- : for  $41^2 = 1681$  we got  $681 = 3*227$  and  $1 \equiv 1 \pmod{6}$ ;
- : for  $89^2 = 7921$  we got  $921 = 3*307$  and  $7 \equiv 1 \pmod{6}$ .

**Examples for case (4):**

- : for  $83^2 = 6889$  we got  $889 = 49 + 120*7$  and  $6 \equiv 0 \pmod{6}$ ;
- : for  $113^2 = 12769$  we got  $769 = 49 + 120*6$  and  $12 \equiv 0 \pmod{6}$ ;
- : for  $137^2 = 18769$  we got  $769 = 49 + 120*6$  and  $18 \equiv 0 \pmod{6}$ ;
- : for  $257^2 = 66049$  we got  $6049 = 49 + 120*50$  and  $6 \equiv 0 \pmod{6}$ ;
- : for  $263^2 = 69169$  we got  $9169 = 49 + 120*76$  and  $6 \equiv 0 \pmod{6}$ .

**Examples for case (5):**

- : for  $251^2 = 63001$  we got  $3001 = 121 + 48*60$  and  $6 \equiv 0 \pmod{6}$ .

**Examples for case (6):**

- :  $11^2 = 121$ ;
- :  $101^2 = 10201$ .

**Note:**

This conjecture is verified up to  $p = 269$ .

## 17. A conjecture on the squares of primes of the form $6k + 1$

**Abstract.** In this paper I make a conjecture on the squares of primes of the form  $6k + 1$ , conjecture that states that by a certain deconcatenation of those numbers (each one in other two numbers) it will be obtained similar results.

### Conjecture:

For any square of a prime  $p$  of the form  $p = 6k + 1$  is true at least one of the following six statements:

- (1)  $p^2$  can be deconcatenated into a prime and a number congruent to 2, 3 or 5 modulo 6;
- (2)  $p^2$  can be deconcatenated into a semiprime  $3^n \cdot q$  and a number congruent to 1 modulo 6;
- (3)  $p^2$  can be deconcatenated into a number  $n$  such that  $n + 1$  is prime or power of prime and the digit 1;
- (4)  $p^2$  can be deconcatenated into a number  $n$  such that  $n + 1$  is prime or power of prime and the digit 9;
- (5)  $p^2$  can be deconcatenated into a number of the form  $49 + 120 \cdot k$  and a number congruent to 0 modulo 6;
- (6)  $p^2$  can be deconcatenated into a number of the form  $121 + 24 \cdot k$  and a number congruent to 0 modulo 6.

### Examples for case (1):

- : for  $67^2 = 4489$  we got 89 prime and  $44 \equiv 2 \pmod{6}$ ;
- : for  $73^2 = 5329$  we got 29 prime and  $53 \equiv 5 \pmod{6}$ ;
- : for  $79^2 = 6241$  we got 41 prime and  $62 \equiv 2 \pmod{6}$ ;
- : for  $109^2 = 11881$  we got 881 prime and  $11 \equiv 2 \pmod{6}$ ;
- : for  $163^2 = 26569$  we got 569 prime and  $26 \equiv 2 \pmod{6}$ ;
- : for  $181^2 = 32761$  we got 761 prime and  $32 \equiv 5 \pmod{6}$ ;
- : for  $199^2 = 39601$  we got 601 prime and  $39 \equiv 3 \pmod{6}$ .

### Examples for case (2):

- : for  $13^2 = 169$  we got  $69 = 3 \cdot 23$  and  $1 \equiv 1 \pmod{6}$ ;
- : for  $37^2 = 1369$  we got  $369 = 3^2 \cdot 41$  and  $1 \equiv 1 \pmod{6}$ ;
- : for  $43^2 = 1849$  we got  $849 = 3 \cdot 283$  and  $1 \equiv 1 \pmod{6}$ ;
- : for  $61^2 = 3721$  we got  $21 = 3 \cdot 7$  and  $37 \equiv 1 \pmod{6}$ ;
- : for  $127^2 = 16129$  we got  $6129 = 3^3 \cdot 227$  and  $1 \equiv 1 \pmod{6}$ ;
- : for  $193^2 = 37249$  we got  $249 = 3 \cdot 83$  and  $37 \equiv 1 \pmod{6}$ .

### Examples for case (3):

- : for  $19^2 = 361$  we got  $36 + 1 = 37$  prime;
- : for  $31^2 = 961$  we got  $96 + 1 = 97$  prime;
- : for  $79^2 = 6241$  we got  $624 + 1 = 625$  power of prime.

- : for  $139^2 = 19321$  we got  $1932 + 1 = 1933$  prime;
- : for  $151^2 = 22801$  we got  $2280 + 1 = 2281$  prime.

**Examples for case (4):**

- : for  $7^2 = 49$  we got  $4 + 1 = 5$  prime;
- : for  $97^2 = 9409$  we got  $940 + 1 = 941$  prime;
- : for  $103^2 = 10609$  we got  $1060 + 1 = 1061$  prime.

**Examples for case (5):**

- : for  $157^2 = 24649$  we got  $649 = 49 + 120 \cdot 5$  and  $24 \equiv 0 \pmod{6}$ .

**Examples for case (6):**

- : for  $79^2 = 6241$  we got  $241 = 121 + 24 \cdot 5$  and  $6 \equiv 0 \pmod{6}$ .

**Note:**

This conjecture is verified up to  $p = 199$ .

**Note:**

I mention that this conjecture and the one from my previous paper “A conjecture on the squares of primes of the form  $6k - 1$ ” were made with the title of *jocandi causa*. It is not relevant if they are not true if they raise interesting questions about squares of primes.



## 18. Nine conjectures on the infinity of certain sequences of primes

**Abstract.** In this paper I enunciate nine conjectures on primes, all of them on the infinity of certain sequences of primes.

### Conjecture 1:

For any prime  $p$  there exist an infinity of positive integers  $n$  such that the number  $n^*p - n + 1$  is prime.

#### Examples:

: For  $p = 19$  we have the following primes:  $2^*19 - 1 = 37$ ;  $4^*19 - 3 = 73$ ;  $6^*19 - 5 = 109$ ;  $7^*19 - 6 = 127$ ;  $9^*19 - 8 = 163$ ;  $10^*19 - 9 = 181$  etc.

### Conjecture 2:

For any prime  $p$  there exist an infinity of positive integers  $n$  such that the number  $n^*p + n - 1$  is prime.

#### Examples:

: For  $p = 11$  we have the following primes:  $2^*11 + 1 = 23$ ;  $4^*11 + 3 = 47$ ;  $5^*11 + 4 = 59$ ;  $6^*11 + 5 = 71$ ;  $7^*11 + 6 = 83$ ;  $9^*11 + 8 = 107$  etc.

### Conjecture 3:

For any prime  $p$  there exist an infinity of positive integers  $n$  such that the number  $n^{\wedge 2} * p - n + 1$  is prime.

#### Examples:

: For  $p = 7$  we have the following primes:  $3^{\wedge 2} * 7 - 2 = 61$ ;  $4^{\wedge 2} * 7 - 3 = 109$ ;  $7^{\wedge 2} * 7 - 6 = 337$ ;  $10^{\wedge 2} * 7 - 9 = 691$ ;  $12^{\wedge 2} * 7 - 11 = 997$  etc.

### Conjecture 4:

For any prime  $p$  there exist an infinity of positive integers  $n$  such that the number  $n^{\wedge 2} * p + n - 1$  is prime.

#### Examples:

: For  $p = 11$  we have the following primes:  $3^{\wedge 2} * 11 + 2 = 101$ ;  $4^{\wedge 2} * 11 + 3 = 179$ ;  $6^{\wedge 2} * 11 + 5 = 401$ ;  $10^{\wedge 2} * 11 + 9 = 1109$ ;  $13^{\wedge 2} * 11 + 12 = 1871$  etc.

### Conjecture 5:

For any prime  $p$  there exist an infinity of positive integers  $n$  such that the number  $n^*p - p + n$  is prime.

**Examples:**

: For  $p = 5$  we have the following primes:  $1*5 + 2 = 7$ ;  $2*5 + 3 = 13$ ;  $3*5 + 4 = 19$ ;  $5*5 + 6 = 31$ ;  $6*5 + 7 = 37$ ;  $7*5 + 8 = 43$  etc.

**Conjecture 6:**

For any prime  $p$  there exist an infinity of positive integers  $n$  such that the number  $n*p - p - n$  is prime.

**Examples:**

: For  $p = 5$  we have the following primes:  $1*5 - 2 = 3$ ;  $2*5 - 3 = 7$ ;  $5*5 - 6 = 19$ ;  $6*5 - 7 = 23$ ;  $8*5 - 9 = 31$ ;  $11*5 - 12 = 43$  etc.

**Conjecture 7:**

For any prime  $p$  there exist an infinity of positive integers  $n$  such that the number  $(n - 1)^2*p + n$  is prime.

**Examples:**

: For  $p = 7$  we have the following primes:  $2^2*7 + 3 = 31$ ;  $3^2*7 + 4 = 67$ ;  $5^2*7 + 4 = 179$ ;  $6^2*7 + 5 = 257$ ;  $7^2*7 + 6 = 349$  etc.

**Conjecture 8:**

For any prime  $p$  there exist an infinity of positive integers  $n$  such that the number  $(n - 1)^2*p - n$  is prime.

**Examples:**

: For  $p = 7$  we have the following primes:  $3^2*7 - 4 = 59$ ;  $4^2*7 - 5 = 107$ ;  $8^2*7 - 9 = 439$ ;  $9^2*7 - 10 = 557$ ;  $15^2*7 - 16 = 1559$  etc.

**Conjecture 9:**

For any two distinct primes greater than three  $p$  and  $q$  there exist an infinity of positive integers  $n$  such that the number  $(p^2 - 1)*n + q^2$  is prime, also an infinity of positive integers  $m$  such that the number  $(q^2 - 1)*n + p^2$  is prime.

**Examples:**

: For  $(p, q) = (7, 11)$  we have the following primes of the form  $48*n + 121$ : 313, 409, 457, 601, 937, 1033 etc. and the following primes of the form  $120*n + 49$ : 409, 769, 1009 etc.

**Note:**

The idea of these sequences didn't come to me from "nowhere". Many from the types of primes presented in this paper are met in the study of Fermat pseudoprimes.

## 19. Five conjectures on primes based on the observation of Poulet and Carmichael numbers

**Abstract.** In this paper I enunciate five conjectures on primes, based on the study of Fermat pseudoprimes and on the author's believe in the importance of multiples of 30 in the study of primes.

### Conjecture 1:

For any  $p, q$  distinct primes,  $p > 30$ , there exist  $n$  positive integer such that  $p - 30*n$  and  $q + 30*n$  are both primes.

#### Note:

This conjecture is based on the observation of 2-Poulet numbers (see my paper "A conjecture about 2-Poulet numbers and a question about primes").

### Conjecture 2:

For any  $p, q, r$  distinct primes there exist  $n$  positive integer such that the numbers  $30*n - p$ ,  $30*n - q$  and  $30*n - r$  are all three primes.

#### Note:

This enunciation is obviously equivalent to the enunciation that there exist  $m$  such that  $p + 30*m$ ,  $q + 30*m$  and  $r + 30*m$  are all three primes (take  $x = 30*n - p$ ,  $y = 30*n - q$  and  $z = 30*n - r$ . Then there exist  $k$  such that  $30*k - 30*n + p$ ,  $30*k - 30*n + q$  and  $30*k - 30*n + r$  are all three primes).

#### Note:

This conjecture implies of course that for any pair of twin primes  $(p, q)$  there exist a pair of primes  $(30*n - p, 30*n - q)$  so that there are infinitely many pairs of twin primes.

#### Note:

This conjecture is based on the observation of 3-Carmichael numbers (see my paper "A conjecture about primes based on heuristic arguments involving Carmichael numbers).

### Conjecture 3:

There exist an infinity of pairs of distinct primes  $(p, q)$ , where  $p < q$ , both of the same form from the following eight ones:  $30*k + 1$ ,  $30*k + 7$ ,  $30*k + 11$ ,  $30*k + 13$ ,  $30*k + 17$ ,  $30*k + 19$ ,  $30*k + 23$  and  $30*k + 29$  such that the number  $p*q + (q - p)$  is prime.

#### Note:

This conjecture is based on the observation of Carmichael numbers.

**Examples:**

- :  $31 \cdot 151 + (151 - 31) = 4801$  prime;
- :  $37 \cdot 127 + (127 - 37) = 4789$  prime;
- :  $41 \cdot 101 + (101 - 41) = 4201$  prime;
- :  $13 \cdot 103 + (103 - 13) = 1429$  prime;
- :  $17 \cdot 47 + (47 - 17) = 829$  prime;
- :  $19 \cdot 109 + (109 - 19) = 2161$  prime;
- :  $23 \cdot 53 + (53 - 23) = 1249$  prime.

**Conjecture 4:**

There exist an infinity of pairs of distinct primes  $(p, q)$ , where  $p < q$ , both of the same form from the following eight ones:  $30 \cdot k + 1$ ,  $30 \cdot k + 7$ ,  $30 \cdot k + 11$ ,  $30 \cdot k + 13$ ,  $30 \cdot k + 17$ ,  $30 \cdot k + 19$ ,  $30 \cdot k + 23$  and  $30 \cdot k + 29$  such that the number  $p \cdot q - (q - p)$  is prime.

**Note:**

This conjecture is based on the observation of Carmichael numbers.

**Examples:**

- :  $31 \cdot 61 - (61 - 31) = 1861$  prime;
- :  $7 \cdot 37 - (37 - 7) = 229$  prime;
- :  $11 \cdot 41 - (41 - 11) = 421$  prime;
- :  $13 \cdot 73 - (73 - 13) = 919$  prime;
- :  $17 \cdot 47 - (47 - 17) = 769$  prime;
- :  $19 \cdot 139 - (139 - 19) = 2521$  prime;
- :  $23 \cdot 293 - (293 - 23) = 6469$  prime.

**Conjecture 5:**

For any  $p$  prime there exist an infinity of primes  $q$ ,  $q > p$ , where  $p$  and  $q$  are both of the same form from the following eight ones:  $30 \cdot k + 1$ ,  $30 \cdot k + 7$ ,  $30 \cdot k + 11$ ,  $30 \cdot k + 13$ ,  $30 \cdot k + 17$ ,  $30 \cdot k + 19$ ,  $30 \cdot k + 23$  and  $30 \cdot k + 29$  such that the number  $p \cdot q - (q - p)$  is prime.

## 20. Six conjectures on primes based on the study of 3-Carmichael numbers and a question about primes

**Abstract.** In this paper are stated six conjectures on primes, more precisely on the infinity of some types of pairs of primes, all of them met in the study of 3-Carmichael numbers.

### Conjecture 1:

For any pair of odd primes  $[p, q]$  there exist an infinity of pairs of distinct positive integers  $[m, n]$  such that the numbers  $x = p*(m + 1) - n$  and  $y = q*(n + 1) - m$  are both primes.

Examples:

- : for  $[p, q] = [3, 3]$  we have  $[x, y] = [5, 13]$  for  $[m, n] = [2, 4]$ ;
- : for  $[p, q] = [7, 11]$  we have  $[x, y] = [29, 73]$  for  $[m, n] = [4, 6]$ .

### Conjecture 2:

For any pair of odd primes  $[p, q]$  there exist an infinity of pairs of distinct positive integers  $[m, n]$  such that the numbers  $x = p*(m - 1) + n$  and  $y = q*(n - 1) + m$  are both primes.

Examples:

- : for  $[p, q] = [7, 7]$  we have  $[x, y] = [11, 23]$  for  $[m, n] = [2, 4]$ ;
- : for  $[p, q] = [5, 13]$  we have  $[x, y] = [11, 67]$  for  $[m, n] = [2, 6]$ .

### Conjecture 3:

For any pair of odd primes  $[p, q]$  there exist an infinity of pairs of distinct positive integers  $[m, n]$  such that the numbers  $x = p + (m + 1)*n$  and  $y = q + m*n$  are both primes.

Examples:

- : for  $[p, q] = [5, 5]$  we have  $[x, y] = [17, 13]$  for  $[m, n] = [2, 4]$ ;
- : for  $[p, q] = [5, 7]$  we have  $[x, y] = [29, 23]$  for  $[m, n] = [2, 8]$ .

### Conjecture 4:

For any pair of odd primes  $[p, q]$  there exist an infinity of pairs of distinct positive integers  $[m, n]$  such that the numbers  $x = p*m - 2*n$  and  $y = q*n + 2*m$  are both primes.

Examples:

- : for  $[p, q] = [11, 11]$  we have  $[x, y] = [23, 61]$  for  $[m, n] = [3, 5]$ ;
- : for  $[p, q] = [11, 13]$  we have  $[x, y] = [23, 71]$  for  $[m, n] = [3, 5]$ .

### Conjecture 5:

For any pair of odd primes  $[p, q]$  there exist an infinity of pairs of distinct positive integers  $[m, n]$  such that the numbers  $x = p*m - 2*n$  and  $y = q*n - 2*m$  are both primes.

Examples:

- : for  $[p, q] = [3, 3]$  we have  $[x, y] = [7, 17]$  for  $[m, n] = [11, 13]$ ;
- : for  $[p, q] = [3, 5]$  we have  $[x, y] = [13, 61]$  for  $[m, n] = [17, 19]$ .

**Conjecture 6:**

For any pair of odd primes  $[p, q]$  there exist an infinity of pairs of distinct positive integers  $[m, n]$  such that the numbers  $x = p^*m + 2*n$  and  $y = q*n + 2*m$  are both primes.

Examples:

- : for  $[p, q] = [5, 5]$  we have  $[x, y] = [29, 41]$  for  $[m, n] = [3, 7]$ ;
- : for  $[p, q] = [5, 11]$  we have  $[x, y] = [19, 79]$  for  $[m, n] = [1, 7]$ .

**Question:**

Are there an infinity of primes with the property that can be written as  $p^*m + n - q$  as well as  $q^*n + m - p$ , where  $p, q$  are distinct primes and  $m, n$  are distinct positive integers? But under the condition that  $m, n, p, q$  are all four primes? Such number is, for instance,  $397 = 13*31 + 7 - 13 = 61*7 + 31 - 61$ .

**Note:**

Like I already said in Abstract, I met these pairs of primes in the study of 3-Carmichael numbers: see my previous paper "Connections between the three prime factors of a 3-Carmichael number".

## 21. Ten conjectures on primes based on the study of Carmichael numbers, involving the multiples of 30

**Abstract.** In this paper are stated ten conjectures on primes, more precisely on the infinity of some types of triplets and quadruplets of primes, all of them using the multiples of the number 30 and also all of them met on the study of Carmichael numbers.

### Conjecture 1:

There exist an infinity of positive integers  $n$  such that the numbers  $30*n + 7$ ,  $60*n + 13$  and  $150*n + 31$  are all three primes.

The sequence of these numbers  $n$  is: 0, 1, 2, 3 (...), corresponding to the triplets of primes [7, 13, 31], [37, 73, 181], [67, 73, 181], [97, 193, 481]...

### Conjecture 2:

There exist an infinity of positive integers  $n$  such that the numbers  $30*n - 23$ ,  $60*n - 47$  and  $90*n - 71$  are all three primes.

The sequence of these numbers  $n$  is: 1, 2, 3 (...), corresponding to the triplets of primes [7, 13, 19], [37, 73, 109], [67, 133, 199]...

### Conjecture 3:

There exist an infinity of positive integers  $n$  such that the numbers  $30*n - 29$ ,  $60*n - 59$  and  $90*n - 89$  are all three primes.

The sequence of these numbers  $n$  is: 8, 10 (...), corresponding to the triplets of primes [211, 421, 691], [271, 541, 811]...

### Conjecture 4:

There exist an infinity of positive integers  $n$  such that the numbers  $30*n - 7$ ,  $90*n - 23$  and  $300*n - 79$  are all three primes.

The sequence of these numbers  $n$  is: 2, 9 (...), corresponding to the triplets of primes [53, 157, 521], [263, 787, 2621]...

### Conjecture 5:

There exist an infinity of positive integers  $n$  such that the numbers  $30*n - 17$ ,  $90*n - 53$  and  $150*n - 89$  are all three primes.

The sequence of these numbers  $n$  is: 1, 2 (...), corresponding to the triplets of primes [13, 37, 61], [43, 127, 211]...

**Conjecture 6:**

There exist an infinity of positive integers  $n$  such that the numbers  $60*n + 13$ ,  $180*n + 37$  and  $300*n + 61$  are all three primes.

The sequence of these numbers  $n$  is: 2, 6 (...), corresponding to the triplets of primes [133, 397, 661], [373, 1117, 1861]...

**Conjecture 7:**

There exist an infinity of positive integers  $n$  such that the numbers  $330*n + 7$ ,  $660*n + 13$ ,  $990*n + 19$  and  $1980*n + 37$  are all four primes.

The sequence of these numbers  $n$  is: 1 (...), corresponding to the quadruplets of primes [133, 397, 661]...

**Conjecture 8:**

There exist an infinity of positive integers  $n$  such that the numbers  $90*n + 1$ ,  $180*n + 1$ ,  $270*n + 1$  and  $540*n + 1$  are all four primes.

The sequence of these numbers  $n$  is: 3 (...), corresponding to the quadruplets of primes [271, 541, 811, 1621]...

**Conjecture 9:**

There exist an infinity of pairs of primes  $[p, q]$  such that the numbers  $p + 30*n$ ,  $q + 30*n$  and  $p*q + 30*n$  are all three primes.

Examples:  $[p, q] = [7, 7]$ ,  $[7, 11]$ ,  $[11, 7]$  etc. corresponding to the triplets  $[37, 67, 137]$ ,  $[37, 71, 167]$ ,  $[41, 67, 167]$  etc.

**Conjecture 10:**

There exist an infinity of primes  $p$  such that the numbers  $x = 30*n + p$  and  $y = 30*m*n + m*p - m + 1$ , where  $m, n$  are non-null positive integers, are both primes.

Examples:

- : for  $p = 7$  we have  $[x, y] = [30*n + 7, 30*m*n + 6*m + 1]$ ; for  $[m, n] = [2, 1]$  we have  $[x, y] = [37, 73]$ ;
- : for  $p = 11$  we have  $[x, y] = [30*n + 11, 30*m*n + 10*m + 1]$ ; for  $[m, n] = [2, 1]$  we have  $[x, y] = [41, 101]$ ;

**Note:**

Like I already said in Abstract, I met these triplets and quadruplets of primes in the study of Carmichael numbers: see my previous paper "A list of 13 sequences of Carmichael numbers based on the multiples of the number 30".



## 22. Two sequences of primes whose formulas contain the number 360

**Abstract.** In this paper I present two possible infinite sequences of primes, having in common the fact that their formulas contain the number 360.

### Conjecture 1:

There exist an infinity of primes of the form  $360 \cdot p \cdot q + 1$ , where  $p, q$  are primes, both greater than or equal to 7.

The first few such primes:

- :  $360 \cdot 7 \cdot 17 + 1 = 42841$ ;
- :  $360 \cdot 7 \cdot 19 + 1 = 47881$ ;
- :  $360 \cdot 11 \cdot 13 + 1 = 51481$ ;
- :  $360 \cdot 13 \cdot 17 + 1 = 79561$ ;
- :  $360 \cdot 11 \cdot 23 + 1 = 91081$ ;
- :  $360 \cdot 13 \cdot 23 + 1 = 107641$ .

### Conjecture 2:

There exist an infinity of primes of the form  $360 \cdot p \cdot q + r$ , where  $p, q, r$  are primes, all of them greater than or equal to 7.

The first few such primes for  $p = q = 7$ :

- :  $360 \cdot 7 \cdot 7 + 17 = 17657$ ;
- :  $360 \cdot 7 \cdot 7 + 19 = 17659$ ;
- :  $360 \cdot 7 \cdot 7 + 29 = 17669$ ;
- :  $360 \cdot 7 \cdot 7 + 41 = 17681$ ;
- :  $360 \cdot 7 \cdot 7 + 41 = 17683$ .

The first few such primes for  $p = 7, q = 11$ :

- :  $360 \cdot 7 \cdot 11 + 13 = 27733$ ;
- :  $360 \cdot 7 \cdot 11 + 17 = 27737$ ;
- :  $360 \cdot 7 \cdot 11 + 19 = 27739$ ;
- :  $360 \cdot 7 \cdot 11 + 23 = 27743$ ;
- :  $360 \cdot 7 \cdot 11 + 29 = 27749$ ;
- :  $360 \cdot 7 \cdot 11 + 31 = 27751$ .

Note the six consecutive primes obtained above!

## 23. Two sequences of primes whose formulas contain the powers of the number 2

**Abstract.** In this paper I present two possible infinite sequences of primes, having in common the fact that their formulas contain the powers of the number 2.

### Conjecture 1:

There exist an infinity of primes of the form  $2^m + n^2$ , where  $m$  is non-null positive integer and  $n$  odd integer.

The first few such primes for  $[m, n] = [2, n]$ :

- :  $3^2 + 4 = 13$  for  $n = 3$ ;
- :  $5^2 + 4 = 29$  for  $n = 5$ ;
- :  $7^2 + 4 = 53$  for  $n = 7$ ;
- :  $13^2 + 4 = 173$  for  $n = 13$ ;
- :  $17^2 + 4 = 293$  for  $n = 17$ .

The first few such primes for  $[m, n] = [4, n]$ :

- :  $5^2 + 16 = 41$  for  $n = 5$ ;
- :  $11^2 + 16 = 137$  for  $n = 11$ ;
- :  $29^2 + 16 = 857$  for  $n = 29$ ;
- :  $31^2 + 16 = 977$  for  $n = 31$ ;
- :  $41^2 + 16 = 1697$  for  $n = 41$ .

The first few such primes for  $[m, n] = [8, n]$ :

- :  $5^2 + 256 = 281$  for  $n = 5$ ;
- :  $19^2 + 256 = 617$  for  $n = 19$ ;
- :  $29^2 + 256 = 1097$  for  $n = 29$ ;
- :  $31^2 + 256 = 1217$  for  $n = 31$ ;
- :  $71^2 + 256 = 5297$  for  $n = 71$ .

The first few such primes for  $[m, n] = [m, 1]$ :

- :  $2^1 + 1 = 3$  for  $m = 1$ ;
- :  $2^2 + 1 = 5$  for  $m = 2$ ;
- :  $2^4 + 1 = 17$  for  $m = 4$ ;
- :  $2^8 + 1 = 257$  for  $m = 8$ ;
- :  $2^{16} + 1 = 65537$  for  $m = 16$ .

The first few such primes for  $[m, n] = [m, 3]$ :

- :  $2^1 + 9 = 11$  for  $m = 1$ ;
- :  $2^2 + 9 = 13$  for  $m = 2$ ;
- :  $2^3 + 9 = 17$  for  $m = 3$ ;
- :  $2^5 + 9 = 41$  for  $m = 5$ ;
- :  $2^6 + 9 = 73$  for  $m = 6$ .

## Conjecture 2:

There exist an infinity of primes of the form  $(2^n)^k + 2^n + 1$ , where  $n$  is non-null positive integer and  $k$  positive integer.

The first few such primes for  $[n, k] = [n, 1]$ :

: 5 for  $n = 1$ ;  
: 17 for  $n = 3$ ;  
: 257 for  $n = 7$ .

The first few such primes for  $[n, k] = [n, 2]$ :

: 7 for  $n = 1$ ;  
: 73 for  $n = 3$ ;  
: 262657 for  $n = 9$ .

The first few such primes for  $[n, k] = [n, 3]$ :

: 11 for  $n = 1$ ;  
: 521 for  $n = 3$ ;  
: 32801 for  $n = 5$ .

The first few such primes for  $[n, k] = [1, k]$ :

: 5 for  $k = 1$ ;  
: 7 for  $k = 2$ ;  
: 11 for  $k = 3$ .

The first few such primes for  $[n, k] = [3, k]$ :

: 17 for  $k = 1$ ;  
: 73 for  $k = 2$ ;  
: 521 for  $k = 3$ .

The first few such primes for  $[n, k] = [5, k]$ :

: 32801 for  $k = 3$ ;  
: 1048609 for  $k = 4$ ;  
: 1073741857 for  $k = 6$ .

## 24. Conjectures about a way to express a prime as a sum of three other primes of a certain type

**Abstract.** These conjectures state that any prime  $p$  greater than 60 can be written as a sum of three primes of a certain type from the following four ones:  $10k + 1$ ,  $10k + 3$ ,  $10k + 7$  and  $10k + 9$ .

### Conjecture 1a:

Any prime  $p$  of the form  $10^*k + 1$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:

:  $10^*x + 1$ ,  $10^*y + 1$  respectively  $10^*z + 1$ .

Examples:

:  $61 = 11 + 31 + 19$ ;

:  $71 = 11 + 31 + 29 = 11 + 41 + 19$ .

### Conjecture 1b:

Any prime  $p$  of the form  $10^*k + 1$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:

:  $10^*x + 1$ ,  $10^*y + 3$  respectively  $10^*z + 7$ .

Examples:

:  $61 = 41 + 13 + 7 = 31 + 23 + 7 = 31 + 13 + 17$ ;

:  $71 = 41 + 23 + 7 = 41 + 13 + 17 = 31 + 23 + 7$ .

### Conjecture 1c:

Any prime  $p$  of the form  $10^*k + 1$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:

:  $10^*x + 7$ ,  $10^*y + 7$  respectively  $10^*z + 7$ .

Examples:

:  $61 = 7 + 17 + 37 = 7 + 7 + 47$ ;

:  $71 = 17 + 17 + 37 = 7 + 17 + 47$ .

### Conjecture 1d:

Any prime  $p$  of the form  $10^*k + 1$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:

:  $10^*x + 3$ ,  $10^*y + 9$  respectively  $10^*z + 9$ .

Examples:

:  $61 = 13 + 19 + 29 = 23 + 19 + 19$ ;

:  $71 = 23 + 19 + 29 = 13 + 29 + 29$ .

**Conjecture 2a:**

Any prime  $p$  of the form  $10^*k + 3$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:

:  $10^*x + 1$ ,  $10^*y + 1$  respectively  $10^*z + 1$ .

**Conjecture 2b:**

Any prime  $p$  of the form  $10^*k + 3$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:

:  $10^*x + 1$ ,  $10^*y + 3$  respectively  $10^*z + 9$ .

**Conjecture 2c:**

Any prime  $p$  of the form  $10^*k + 3$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:

:  $10^*x + 3$ ,  $10^*y + 3$  respectively  $10^*z + 7$ .

**Conjecture 2d:**

Any prime  $p$  of the form  $10^*k + 3$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:

:  $10^*x + 7$ ,  $10^*y + 7$  respectively  $10^*z + 9$ .

**Conjecture 3a:**

Any prime  $p$  of the form  $10^*k + 7$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:

:  $10^*x + 1$ ,  $10^*y + 3$  respectively  $10^*z + 3$ .

**Conjecture 3b:**

Any prime  $p$  of the form  $10^*k + 7$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:

:  $10^*x + 3$ ,  $10^*y + 7$  respectively  $10^*z + 7$ .

**Conjecture 3c:**

Any prime  $p$  of the form  $10^*k + 7$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:

:  $10^*x + 1$ ,  $10^*y + 7$  respectively  $10^*z + 9$ .

**Conjecture 3d:**

Any prime  $p$  of the form  $10^*k + 7$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:

:  $10^*x + 9$ ,  $10^*y + 9$  respectively  $10^*z + 9$ .

**Conjecture 4a:**

Any prime  $p$  of the form  $10*k + 9$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:

:  $10*x + 1$ ,  $10*y + 1$  respectively  $10*z + 7$ .

**Conjecture 4b:**

Any prime  $p$  of the form  $10*k + 9$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:

:  $10*x + 3$ ,  $10*y + 3$  respectively  $10*z + 3$ .

**Conjecture 4c:**

Any prime  $p$  of the form  $10*k + 9$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:

:  $10*x + 3$ ,  $10*y + 7$  respectively  $10*z + 9$ .

**Conjecture 4d:**

Any prime  $p$  of the form  $10*k + 9$ ,  $p > 60$ , can be written as a sum of three primes of the following forms:

:  $10*x + 1$ ,  $10*y + 9$  respectively  $10*z + 9$ .

**Addenda**

In one of my previous papers, “Two conjectures that relates any Poulet number by a type of triplets respectively of duplets of primes” I made the following two conjectures:

**Conjecture:**

Any square of a prime of the form  $p^2 = 10*k + 1$  can be written as  $p^2 = x + y + z$ , where  $x, y, z$  are primes, not necessarily all three distinct, of the form  $10*k + 7$ .

Examples:

:  $11^2 = 121 = 37 + 37 + 47$ ;

:  $19^2 = 361 = 7 + 37 + 317$ .

**Conjecture:**

Any square of a prime of the form  $p^2 = 10*k + 9$  can be written as  $p^2 = x + y + z$ , where  $x, y, z$  are primes, not necessarily all three distinct, of the form  $10*k + 3$ .

Examples:

:  $7^2 = 49 = 13 + 13 + 23$ ;

:  $19^2 = 361 = 13 + 43 + 305$ .

## 25. A bold conjecture about a way in which any prime can be written

**Abstract.** In this paper I make a conjecture which states that any prime greater than or equal to 5 can be written in a certain way, in other words that any such prime can be expressed using just two other primes and a power of the number 2.

### Conjecture:

Any prime greater than or equal to 5 can be written at least in one way as  $(9*p^2 - q^2)/(2^n)$ , where p and q are primes and n non-null positive integer.

### Verifying the conjecture:

(For the first nine such primes)

- :  $5 = (9*7^2 - 11^2)/64$ , so  $[p, q, n] = [7, 11, 6]$  but also  $5 = (9*7^2 - 19^2)/16$  so  $[p, q, n] = [7, 19, 4]$ ;
- :  $7 = (9*5^2 - 13^2)/64$ , so  $[p, q, n] = [5, 13, 3]$ ;
- :  $11 = (9*5^2 - 7^2)/16$ , so  $[p, q, n] = [5, 7, 4]$ ;
- :  $13 = (9*5^2 - 11^2)/8$ , so  $[p, q, n] = [5, 11, 3]$  but also  $13 = (9*7^2 - 5^2)/32$  so  $[p, q, n] = [7, 5, 5]$ ;
- :  $17 = (9*7^2 - 13^2)/16$ , so  $[p, q, n] = [7, 13, 4]$ ;
- :  $19 = (9*7^2 - 17^2)/8$ , so  $[p, q, n] = [7, 17, 3]$  but also  $19 = (9*13^2 - 37^2)/8$  so  $[p, q, n] = [13, 37, 3]$  but also  $19 = (9*17^2 - 13^2)/128$  so  $[p, q, n] = [17, 11, 7]$ ;
- :  $23 = (9*13^2 - 7^2)/64$ , so  $[p, q, n] = [13, 7, 6]$ ;
- :  $31 = (9*11^2 - 29^2)/8$ , so  $[p, q, n] = [11, 29, 3]$  but also  $31 = (9*13^2 - 23^2)/32$  so  $[p, q, n] = [13, 23, 5]$ ;
- :  $37 = (9*23^2 - 5^2)/128$ , so  $[p, q, n] = [23, 5, 7]$ .

### Note:

For the prime 29 I didn't find primes solution  $[p, q]$  up to the denominator  $2^{12}$ , but surely I conjecture that there exist such solutions.

### Note:

For some of the primes we found that they verify also the formula  $(9*p^2 - q^4)/(2^n)$ .

## 26. Two conjectures, on the primes of the form $6k + 1$ respectively of the form $6k - 1$

**Abstract.** In this paper I make two conjectures, one about how could be expressed a prime of the form  $6k + 1$  and one about how could be expressed a prime of the form  $6k - 1$ .

### Conjecture 1:

Any prime  $p$  of the form  $6k + 1$  greater than or equal to 13 can be written as  $(q^2 - q + r)/3$ , where  $q$  is prime of the form  $6k - 1$  and  $r$  is prime or power of prime or number 1.

#### Note:

Because we have  $5^2 - 5 = 20$ ,  $11^2 - 11 = 110$ ,  $17^2 - 17 = 272$ ,  $23^2 - 23 = 506$  and so on, the conjecture is equivalent to the existence of a prime or power of prime among the numbers  $3p - 20$ ,  $3p - 110$ ,  $3p - 272$ ,  $3p - 506$  and so on.

### Verifying the conjecture:

(up to  $p = 229$ )

- :  $13 \cdot 3 - 20 = 19$ , prime, so  $[p, q, r] = [13, 5, 19]$ ;
- :  $19 \cdot 3 - 20 = 37$ , prime, so  $[p, q, r] = [19, 5, 37]$ ;
- :  $31 \cdot 3 - 20 = 73$ , prime, so  $[p, q, r] = [31, 5, 73]$ ;
- :  $37 \cdot 3 - 110 = 1$ , so  $[p, q, r] = [37, 11, 1]$ ;
- :  $43 \cdot 3 - 20 = 109$ , prime, so  $[p, q, r] = [43, 5, 109]$  and also  $43 \cdot 3 - 110 = 29$ , prime, so  $[p, q, r] = [43, 11, 29]$ ;
- :  $61 \cdot 3 - 20 = 163$ , prime, so  $[p, q, r] = [61, 5, 163]$  and also  $61 \cdot 3 - 110 = 73$ , prime, so  $[p, q, r] = [61, 11, 73]$ ;

We also found the following triplets  $[p, q, r]$ :  $[67, 5, 181]$ ,  $[73, 5, 199]$ ,  $[73, 11, 109]$ ,  $[79, 11, 127]$ ,  $[97, 5, 271]$ ,  $[97, 11, 181]$ ,  $[97, 17, 19]$ ,  $[103, 11, 199]$ ,  $[109, 5, 307]$ ,  $[127, 11, 271]$ ,  $[127, 17, 109]$ ,  $[139, 5, 397]$ ,  $[139, 11, 307]$ ,  $[151, 5, 433]$ ,  $[151, 17, 71]$ ,  $[157, 17, 199]$ ,  $[163, 1, 379]$ ,  $[181, 5, 523]$ ,  $[181, 11, 433]$ ,  $[181, 17, 271]$ ,  $[181, 23, 37]$ ,  $[193, 17, 307]$ ,  $[193, 23, 73]$ ,  $[199, 5, 577]$ ,  $[199, 11, 487]$ ,  $[211, 5, 613]$ ,  $[211, 11, 523]$ ,  $[211, 17, 19^2]$ ,  $[211, 23, 127]$ ,  $[223, 17, 397]$ ,  $[223, 506, 163]$ ,  $[229, 11, 577]$ ,  $[229, 23, 181]$ , so the conjecture is verified up to  $p = 229$ .

### Conjecture 2:

Any prime  $p$  of the form  $6k - 1$  greater than or equal to 11 can be written as  $(q^2 - q + r)/3$ , where  $q$  is prime of the form  $6k - 1$  and  $r$  is prime or power of prime or number 1.

#### Note:

Because we have  $5^2 - 5 = 20$ ,  $11^2 - 11 = 110$ ,  $17^2 - 17 = 272$ ,  $23^2 - 23 = 506$  and so on, the conjecture is equivalent to the existence of a prime or power of prime among the numbers  $3p - 20$ ,  $3p - 110$ ,  $3p - 272$ ,  $3p - 506$  and so on.



**Verifying the conjecture:**

(up to  $p = 179$ )

:  $11 \cdot 3 - 20 = 13$ , prime, so  $[p, q, r] = [11, 5, 13]$ ;

:  $17 \cdot 3 - 20 = 31$ , prime, so  $[p, q, r] = [17, 5, 31]$ ;

We also found the following triplets  $[p, q, r]$ :  $[29, 5, 67]$ ,  $[41, 5, 103]$ ,  $[47, 11, 31]$ ,  $[53, 5, 129]$ ,  $[59, 5, 157]$ ,  $[59, 11, 67]$ ,  $[71, 5, 193]$ ,  $[71, 11, 103]$ ,  $[83, 5, 229]$ ,  $[83, 11, 139]$ ,  $[89, 11, 157]$ ,  $[101, 5, 283]$ ,  $[101, 11, 193]$ ,  $[101, 17, 31]$ ,  $[107, 11, 211]$ ,  $[113, 11, 229]$ ,  $[113, 17, 67]$ ,  $[131, 5, 373]$ ,  $[131, 11, 283]$ ,  $[137, 17, 139]$ ,  $[149, 11, 337]$ ,  $[167, 17, 229]$ ,  $[173, 5, 449]$ ,  $[173, 11, 409]$ ,  $[173, 23, 13]$ ,  $[179, 23, 31]$  so the conjecture is verified up to  $p = 179$ .

**Comment:**

In the case that the conjectures are invalidated, still remain two open problems:

- (1) Which are the smallest primes that don't satisfy each from the two conjectures?;
- (2) Which is the maximum length of a chain formed in the following way:  $p_2 = 3 \cdot p_1 - (q^2 - q)$ ,  $p_3 = 3 \cdot p_2 - (q^2 - q)$ , ...,  $p_n = 3 \cdot p_{n-1} - (q^2 - q)$ ? For instance, such a chain of length 3 is  $[43, 109, 307]$  for  $q = 5$ .

## 27. A possible way to write any prime, using just another prime and the powers of the numbers 2, 3 and 5

**Abstract.** In this paper I make a conjecture which states that any odd prime can be written in a certain way, in other words that any such prime can be expressed using just another prime and the powers of the numbers 2, 3 and 5. I also make a related conjecture about twin primes.

### Conjecture:

Any odd prime  $p$  can be written at least in one way as  $p = (q \cdot 2^a \cdot 3^b \cdot 5^c \pm 1) \cdot 2^n \pm 1$ , where  $q$  is an odd prime or is equal to 1, where  $a$ ,  $b$  and  $c$  are non-negative integers and  $n$  is non-null positive integer.

### Verifying the conjecture:

(For the first five odd primes)

- :  $3 = (1 \cdot 2^1 \cdot 3^0 \cdot 5^0 - 1) \cdot 2^1 + 1$ , but also  $3 = (1 \cdot 2^0 \cdot 3^1 \cdot 5^0 - 1) \cdot 2^1 - 1$ ;
- :  $5 = (1 \cdot 2^1 \cdot 3^0 \cdot 5^0 + 1) \cdot 2^1 - 1$ , but also  $5 = (1 \cdot 2^0 \cdot 3^1 \cdot 5^0 - 1) \cdot 2^1 + 1$ , also  $5 = (1 \cdot 2^2 \cdot 3^0 \cdot 5^0 - 1) \cdot 2^1 - 1$ ;
- :  $7 = (1 \cdot 2^0 \cdot 3^1 \cdot 5^0 + 1) \cdot 2^1 - 1$ , but also  $7 = (1 \cdot 2^1 \cdot 3^0 \cdot 5^0 + 1) \cdot 2^1 + 1$ , also  $7 = (3 \cdot 2^0 \cdot 3^0 \cdot 5^0 + 1) \cdot 2^1 - 1$ , also  $7 = (5 \cdot 2^0 \cdot 3^0 \cdot 5^0 - 1) \cdot 2^1 - 1$ , also  $7 = (1 \cdot 2^2 \cdot 3^0 \cdot 5^0 - 1) \cdot 2^1 + 1$ ;
- :  $11 = (1 \cdot 2^1 \cdot 3^1 \cdot 5^0 - 1) \cdot 2^1 + 1$ , but also  $11 = (1 \cdot 2^0 \cdot 3^0 \cdot 5^1 + 1) \cdot 2^1 - 1$ , also  $11 = (3 \cdot 2^1 \cdot 3^0 \cdot 5^0 - 1) \cdot 2^1 + 1$ , also  $11 = (5 \cdot 2^0 \cdot 3^0 \cdot 5^0 + 1) \cdot 2^1 - 1$ , also  $11 = (7 \cdot 2^0 \cdot 3^0 \cdot 5^0 - 1) \cdot 2^1 - 1$ , also  $11 = (1 \cdot 2^2 \cdot 3^0 \cdot 5^0 + 1) \cdot 2^1 + 1$ ;
- :  $13 = (1 \cdot 2^1 \cdot 3^1 \cdot 5^0 + 1) \cdot 2^1 + 1$ , but also  $13 = (1 \cdot 2^0 \cdot 3^0 \cdot 5^1 + 1) \cdot 2^1 + 1$ , also  $13 = (3 \cdot 2^1 \cdot 3^0 \cdot 5^0 + 1) \cdot 2^1 + 1$ , also  $13 = (5 \cdot 2^0 \cdot 3^0 \cdot 5^0 + 1) \cdot 2^1 + 1$ , also  $13 = (7 \cdot 2^0 \cdot 3^0 \cdot 5^0 - 1) \cdot 2^1 + 1$ .

### Conjecture:

Any pair of twin primes  $[p_1, p_2]$  can be written as  $[p_1 = (q \cdot 2^a \cdot 3^b \cdot 5^c \pm 1) \cdot 2^n - 1, p_2 = (q \cdot 2^a \cdot 3^b \cdot 5^c \pm 1) \cdot 2^n + 1]$ , where  $q$  is prime or is equal to 1, where  $a$ ,  $b$  and  $c$  are non-negative integers and  $n$  is non-null positive integer.

### Verifying the conjecture:

(For the first three pairs of twin primes)

- :  $3 = (1 \cdot 2^0 \cdot 3^1 \cdot 5^0 - 1) \cdot 2^1 - 1$  and  $5 = (1 \cdot 2^0 \cdot 3^1 \cdot 5^0 - 1) \cdot 2^1 + 1$ ;
- :  $5 = (1 \cdot 2^1 \cdot 3^0 \cdot 5^0 + 1) \cdot 2^1 - 1$  and  $7 = (1 \cdot 2^1 \cdot 3^0 \cdot 5^0 + 1) \cdot 2^1 + 1$ , also

$$5 = (1 \cdot 2^2 \cdot 3^0 \cdot 5^0 - 1) \cdot 2^1 - 1 \text{ and}$$
$$7 = (1 \cdot 2^2 \cdot 3^0 \cdot 5^0 - 1) \cdot 2^1 + 1;$$

:

$$11 = (1 \cdot 2^0 \cdot 3^0 \cdot 5^1 + 1) \cdot 2^1 - 1 \text{ and}$$
$$13 = (1 \cdot 2^1 \cdot 3^1 \cdot 5^0 + 1) \cdot 2^1 + 1, \text{ also}$$

$$11 = (5 \cdot 2^0 \cdot 3^0 \cdot 5^0 + 1) \cdot 2^1 - 1 \text{ and}$$
$$13 = (5 \cdot 2^0 \cdot 3^0 \cdot 5^0 + 1) \cdot 2^1 + 1.$$

## 28. Two conjectures about the pairs of primes separated by a certain distance

**Abstract.** In this paper I make two conjectures about the pairs of primes  $[p_1, q_1]$ , where the difference between  $p_1$  and  $q_1$  is a certain even number  $d$ . I state that any such pair has at least one other corresponding, in a specified manner, pair of primes  $[p_2, q_2]$ , such that the difference between  $p_2$  and  $q_2$  is also equal to  $d$ .

### Conjecture 1:

For any pair of primes, greater than 3,  $[p_1, q_1]$ , where  $q_1 - p_1 = d$ , there exist at least a pair of positive integers  $[m, n]$ , where  $n - m = d$ , such that the numbers  $p_2 = p_1 * q_1 - n + 1$  and  $q_2 = p_1 * q_1 - m + 1$  are both primes.

### Examples:

- : For  $[p_1, q_1] = [5, 7]$  there exist the pair  $[m, n] = [5, 7]$  such that  $p_2 = 5*7 - 7 + 1 = 29$  and  $q_2 = 5*7 - 5 + 1 = 31$  are both primes;
- : For  $[p_1, q_1] = [5, 11]$  there exist the pair  $[m, n] = [3, 9]$  such that  $p_2 = 5*11 - 9 + 1 = 47$  and  $q_2 = 5*11 - 3 + 1 = 53$  are both primes;
- : For  $[p_1, q_1] = [5, 13]$  there exist the pair  $[m, n] = [5, 13]$  such that  $p_2 = 5*13 - 13 + 1 = 53$  and  $q_2 = 5*13 - 5 + 1 = 61$  are both primes;
- : For  $[p_1, q_1] = [7, 11]$  there exist the pair  $[m, n] = [7, 11]$  such that  $p_2 = 7*11 - 11 + 1 = 67$  and  $q_2 = 7*11 - 7 + 1 = 71$  are both primes;
- : For  $[p_1, q_1] = [7, 13]$  there exist the pair  $[m, n] = [7, 11]$  such that  $p_2 = 7*11 - 11 + 1 = 67$  and  $q_2 = 7*11 - 7 + 1 = 71$  are both primes;
- : For  $[p_1, q_1] = [11, 13]$  there exist the pair  $[m, n] = [5, 7]$  such that  $p_2 = 11*13 - 5 + 1 = 137$  and  $q_2 = 11*13 - 7 + 1 = 139$  are both primes.

### Conjecture 2:

For any even number  $d$  there exist an infinity of pairs of primes  $[p_1, q_1]$ , where  $q_1 - p_1 = d$ , such that the numbers  $p_2 = p_1 * q_1 - p_1 + 1$  and  $q_2 = p_1 * q_1 - q_1 + 1$  are both primes.

Note: See, for instance, from the examples to the Conjecture 1 from above, the pair  $[5, 7]$  for  $d = 2$ , the pair  $[7, 11]$  for  $d = 4$ , the pair  $[5, 13]$  for  $d = 8$ .

## 29. Five conjectures on a diophantine equation involving two primes and a square of prime

**Abstract.** In this paper I make five conjectures about the primes  $q$ ,  $r$  and the square of prime  $p^2$ , which appears as solutions in the diophantine equation  $120 \cdot n \cdot q \cdot r + 1 = p^2$ , where  $n$  is non-null positive integer.

### Conjecture 1:

For any  $n$  non-null positive integer there exist  $q$ ,  $r$  primes such that  $120 \cdot n \cdot q \cdot r + 1 = p^2$ , where  $p$  is prime or a power of prime.

### Conjecture 2:

For any  $q$  odd prime there exist  $n$  non-null positive integer and  $r$  prime such that  $120 \cdot n \cdot q \cdot r + 1 = p^2$ , where  $p$  is prime or a power of prime.

### Conjecture 3:

For any  $q$ ,  $r$  odd primes there exist  $n$  non-null positive integer such that  $120 \cdot n \cdot q \cdot r + 1 = p^2$ , where  $p$  is prime or a power of prime.

### Conjecture 4:

For any  $n$  non-null positive integer and any  $q$  prime there exist  $r$  prime such that  $120 \cdot n \cdot q \cdot r + 1 = p^2$ , where  $p$  is prime or a power of prime.

### Examples:

- : For  $[n, q] = [1, 5]$  there exist  $r = 17$  such that  $p = 101$  prime; also  $r = 37$  such that  $p = 149$  prime;
- : For  $[n, q] = [1, 7]$  there exist  $r = 23$  such that  $p = 139$  prime; also  $r = 53$  such that  $p = 211$  prime;
- : For  $[n, q] = [1, 11]$  there exist  $r = 13$  such that  $p = 131$  prime; also  $r = 83$  such that  $p = 331$  prime;
  
- : For  $[n, q] = [2, 5]$  there exist  $r = 19$  such that  $p = 151$  prime;
- : For  $[n, q] = [2, 7]$  there exist  $r = 3$  such that  $p = 71$  prime; also  $r = 17$  such that  $p = 169$  square of prime;
- : For  $[n, q] = [2, 11]$  there exist  $r = 3$  such that  $p = 89$  prime;
  
- : For  $[n, q] = [3, 7]$  there exist  $r = 13$  such that  $p = 181$  prime;
- : For  $[n, q] = [3, 11]$  there exist  $r = 3$  such that  $p = 109$  prime;
  
- : For  $[n, q] = [4, 5]$  there exist  $r = 67$  such that  $p = 401$  prime;
- : For  $[n, q] = [4, 7]$  there exist  $r = 17$  such that  $p = 239$  prime;
- : For  $[n, q] = [4, 11]$  there exist  $r = 11$  such that  $p = 241$  prime.

**Conjecture 5:**

For any  $n$  non-null positive integer there exist  $q$  prime such that  $120*n*q^2 + 1 = p^2$ , where  $p$  is prime or a power of prime.

Note, for instance, the case from the examples below:  $480*11^2 + 1 = 241^2$ .

### 30. An amazing formula for producing big primes based on the numbers 25 and 906304

**Abstract.** In this paper I present a formula for generating big primes and products of very few primes, based on the numbers 25 and 906304, formula equally extremely interesting and extremely simple, id est  $25^n + 906304$ . This formula produces for  $n$  from 1 to 30 (and for  $n = 30$  is obtained a number  $p$  with not less than 42 digits) only primes or products of maximum four prime factors.

#### Observation:

The number  $p = 25^n + 906304$  is often a prime or a product of very few primes.

#### Note:

I came to this formula more or less by chance, but the number 906304 has at least one other special property:  $906304 = 952^2 = 1105^2 - 561^2$ , where 561 and 1105 are the first and the second Carmichael numbers.

#### Examples:

- :  $p = 25^1 + 906304 = 906329$  prime;
- :  $p = 25^2 + 906304 = 906929$  prime;
- :  $p = 25^3 + 906304 = 921929 = 37 \cdot 24917$ ;
- :  $p = 25^4 + 906304 = 1296929$  prime;
- :  $p = 25^5 + 906304 = 10671929 = 421 \cdot 25349$ ;
- :  $p = 25^6 + 906304 = 245046929 = 97 \cdot 2526257$ ;
- :  $p = 25^7 + 906304 = 245046929 = 113 \cdot 2957 \cdot 18269$ ;
- :  $p = 25^8 + 906304 = 152588796929 = 36269 \cdot 4207141$ ;
- :  $p = 25^9 + 906304 = 3814698171929$  prime;
- :  $p = 25^{10} + 906304 = 95367432546929 = 41 \cdot 2326034940169$ ;
- :  $p = 25^{11} + 906304 = 2384185791921929 = 5573 \cdot 427810118773$ ;
- :  $p = 25^{12} + 906304 = 59604644776296929 = 61 \cdot 139361 \cdot 7011468949$ ;
- :  $p = 25^{13} + 906304 = 1490116119385671929 = 1097 \cdot 84389 \cdot 16096358813$ ;
- :  $p = 25^{14} + 906304 = 37252902984620046929$  prime;
- :  $p = 25^{15} + 906304 = 931322574615479421929 = 671477 \cdot 1386976135616677$ ;
- :  $p = 25^{16} + 906304 = 23283064365386963796929$   
 $= 1609 \cdot 1830341 \cdot 7905914013541$ ;
- :  $p = 25^{17} + 906304 = 582076609134674073171929$  prime;
- :  $p = 25^{18} + 906304 = 14551915228366851807546929$   
 $= 53^2 \cdot 5180461099454201426681$ ;
- :  $p = 25^{19} + 906304 = 363797880709171295166921929$  prime;
- :  $p = 25^{20} + 906304 = 9094947017729282379151296929$   
 $= 41 \cdot 237776289649 \cdot 932927233281481$ ;

**Notes:**

For n from 1 to 20, were obtained for p seven values which are primes, seven values which are semiprimes and six values which are products of three prime factors! Note also that the larger prime obtained in the examples above,  $p = 25^{19} + 906304 = 363797880709171295166921929$ , has 27 digits!

For n from 21 to 30 were also obtained products of maximum four primes; these are the following values of p:

: n = 21, p = 227373675443232059478760671929;  
: n = 22, p = 5684341886080801486968995046929;  
: n = 23, p = 142108547152020037174224854421929;  
: n = 24, p = 3552713678800500929355621338796929;  
: n = 25, p = 88817841970012523233890533448171929;  
: n = 26, p = 2220446049250313080847263336182546929;  
: n = 27, p = 55511151231257827021181583404541921929;  
: n = 28, p = 1387778780781445675529539585113526296929;  
: n = 29, p = 34694469519536141888238489627838135671929;  
: n = 30, p = 867361737988403547205962240695953370046929.

For n from 31 to 37 were also obtained products of maximum five primes; these are the following values of p:

: n = 31, p = 21684043449710088680149056017398834229421929  
: n = 32, p = 542101086242752217003726400434970855713796929  
: n = 33, p = 13552527156068805425093160010874271392823171929  
: n = 34, p = 338813178901720135627329000271856784820557546929  
: n = 35, p = 8470329472543003390683225006796419620513916921929  
: n = 36, p = 211758236813575084767080625169910490512847901296929  
: n = 37, p = 5293955920339377119177015629247762262821197510671929

Note that the number  $25^{34} + 906304$  is a prime with 48 digits!

**Conjecture:**

There exist an infinity of primes p of the form  $p = 25^n + 906304$ .



## 31. Four unusual conjectures on primes involving Egyptian fractions

**Abstract.** In this paper I make four conjectures on primes, conjectures which involve the sums of distinct unit fractions such as  $1/p(1) + 1/p(2) + (\dots)$ , where  $p(1)$ ,  $p(2)$ ,  $(\dots)$  are distinct primes, more specifically the periods of the rational numbers which are the results of the sums mentioned above.

### Conjecture 1:

There exist an infinity of infinite sequences of the form  $a(1) = 1/p(1) + 1/p(2)$ ,  $a(2) = 1/p(1) + 1/p(2) + 1/p(3)$ , ..., where  $p(1) < p(2) < p(3) \dots$ , such that the period of the rational number  $a(1)$  is equal to  $p(2) - 1$ , the period of the rational number  $a(2)$  is equal to  $p(3) - 1$ , the period of the rational number  $a(n)$  is equal to  $a(n) - 1$ .

### Examples:

: the period of  $a(1) = 1/3 + 1/7$  is equal to 6;  
: the period of  $a(2) = 1/3 + 1/7 + 1/19$  is equal to 18;  
: the period of  $a(3) = 1/3 + 1/7 + 1/19 + 73$  is equal to 72.  
(...)

The sequence of  $p(1)$ ,  $p(2)$ ,  $p(3) \dots$  is 3, 7, 19, 72...

: the period of  $a(1) = 1/5 + 1/29$  is equal to 28;  
: the period of  $a(2) = 1/5 + 1/29 + 1/113$  is equal to 112;  
: the period of  $a(3) = 1/5 + 1/29 + 1/113 + 1/337$  is equal to 336.  
(...)

The sequence of  $p(1)$ ,  $p(2)$ ,  $p(3) \dots$  is 5, 29, 113, 337...

### Conjecture 2:

For any  $p(1)$  odd prime there exist infinite sequences of the form  $a(1) = 1/p(1) + 1/p(2)$ ,  $a(2) = 1/p(1) + 1/p(2) + 1/p(3)$ , ..., where  $p(1) < p(2) < p(3) \dots$ , such that the period of the rational number  $a(1)$  is equal to  $p(2) - 1$ , the period of the rational number  $a(2)$  is equal to  $p(3) - 1$ , the period of the rational number  $a(n)$  is equal to  $a(n) - 1$ .

### Conjecture 3:

For any  $p(1)$  odd prime there exist infinite sequences of the form  $a(1) = 1/p(1) + 1/p(2)$ ,  $a(2) = 1/p(1) + 1/p(2) + 1/p(3)$ , ..., where  $p(1) < p(2) < p(3) \dots$ , such that the period of the rational number  $a(1)$  is a multiple of  $p(2) - 1$ , the period of the rational number  $a(2)$  is a multiple of  $p(3) - 1$ , the period of the rational number  $a(n)$  is equal to  $a(n) - 1$ .

### Example:

: the period of  $a(1) = 1/7 + 1/17$  is equal to 48 which is a multiple of 16;  
: the period of  $a(2) = 1/7 + 1/17 + 1/19$  is equal to 144 which is a multiple of 18;  
: the period of  $a(3) = 1/7 + 1/17 + 1/19 + 1/23$  is equal to 1584 a multiple of 22.  
(...)

The sequence of  $p(1)$ ,  $p(2)$ ,  $p(3) \dots$  is 7, 17, 19, 23...

**Conjecture 4:**

For any  $p(1)$  odd prime there exist infinite sequences of the form  $a(1) = 1/p(1) + 1/p(2)$ ,  $a(2) = 1/p(1) + 1/p(2) + 1/p(3)$ , ..., where  $p(1) < p(2) < p(3)$ ..., such that the period of the rational number  $a(1)$  divides  $p(2) - 1$ , the period of the rational number  $a(2)$  divides  $p(3) - 1$ , the period of the rational number  $a(n)$  divides  $a(n) - 1$ .

**Example:**

- : the period of  $a(1) = 1/3 + 1/11$  is equal to 2 which divides 10;
- : the period of  $a(2) = 1/3 + 1/11 + 1/13$  is equal to 6 which divides 12;
- : the period of  $a(3) = 1/3 + 1/11 + 1/13 + 1/37$  is equal to 6 which divides 36.
- (...)

The sequence of  $p(1), p(2), p(3)$ ... is 3, 11, 13, 37...

**Conjecture 5:**

For any Poulet number  $P$  there exist a rational number  $r$  equal to a sum of unit fractions  $1/p(1) + 1/p(2) + 1/p(3)$ , ..., where  $p(1), p(2), p(3)$ ... are distinct odd primes, such that the period of  $r$  is equal to  $P - 1$ .

Example: the period of  $r = 1/5 + 1/29 + 1/113 + 1/271$  is equal to 560, while 561 the second Poulet number and the first Carmichael number.

## 32. Three formulas that generate easily certain types of triplets of primes

**Abstract.** In this paper I present three formulas, each of them with the following property: starting from a given prime  $p$ , are obtained in many cases two other primes,  $q$  and  $r$ . I met the triplets of primes  $[p, q, r]$  obtained with these formulas in the study of Carmichael numbers; the three primes mentioned are often the three prime factors of a 3-Carmichael number.

### Note:

To refer to the three formulas easily I will name them the formula alpha, beta or gama and the triplets obtained the triplet alpha, beta or gama.

### Formula alpha:

The formula alpha is  $30 \cdot a^n - (a^p + a - 1)$ . The first prime of a triplet alpha is  $p$  and the other two ones are obtained giving to  $n$  values of integers, under the condition that  $a^p + a - 1$  is prime.

Examples:

- : For  $p = 11$  and  $a = 2$  the condition that  $a^p + a - 1$  is prime is met because  $2^{11} + 2 - 1 = 23$  which is prime; the formula alpha becomes  $60 \cdot n - 23$ ; it can be seen that for  $n = 1$  is obtained 47 (prime) and for  $n = 2$  is obtained 97 (prime) so we have the triplet alpha  $[11, 47, 97]$ ; also for  $n = 3$  is obtained 157 (prime) so other two triplets alpha are  $[11, 47, 157]$  and  $[11, 97, 157]$ ;
- : For  $p = 7$  and  $a = 3$  the condition that  $a^p + a - 1$  is prime is met because  $3^7 + 3 - 1 = 23$  which is prime; the formula alpha becomes  $90 \cdot n - 23$ ; it can be seen that for  $n = 1$  is obtained 67 (prime) and for  $n = 2$  is obtained 157 (prime) so we have the triplet alpha  $[7, 67, 157]$ ; also for  $n = 4$  is obtained 337 (prime) so other two triplets alpha are  $[7, 67, 337]$  and  $[7, 157, 337]$ .

Note: see the sequence A182416 in OEIS for the connection between Carmichael numbers and formula alpha.

### Formula beta:

The formula beta is  $30 \cdot a^n + (a^p + a - 1)$ . The first prime of a triplet beta is  $p$  and the other two ones are obtained giving to  $n$  values of integers, under the condition that  $a^p + a - 1$  is prime.

Examples:

- : For  $p = 11$  and  $a = 2$  the condition that  $a^p + a - 1$  is prime is met because  $2^{11} + 2 - 1 = 23$  which is prime; the formula beta becomes  $60 \cdot n + 23$ ; it can be seen that for  $n = 1$  is obtained 83 (prime) and for  $n = 4$  is obtained 263 (prime) so we

have the triplet beta [11, 83, 263]; also for  $n = 6$  is obtained 383 (prime) so other two triplets beta are [11, 83, 383] and [11, 263, 383];

: For  $p = 19$  and  $a = 3$  the condition that  $a^p + a - 1$  is prime is met because  $3^{19} + 3 - 1 = 59$  which is prime; the formula beta becomes  $90n + 59$ ; it can be seen that for  $n = 1$  is obtained 149 (prime) and for  $n = 2$  is obtained 239 (prime) so we have the triplet beta [59, 149, 239]; also for  $n = 4$  is obtained 419 (prime) so other two triplets beta are [59, 149, 419] and [59, 239, 419].

Note: see the sequence A182416 in OEIS for the connection between Carmichael numbers and formula beta.

### **Formula gama:**

The formula gama is  $2^p n - 2n + p$ . The first prime of a triplet gama is  $p$  and the other two ones are obtained giving to  $n$  values of integers, under the condition that  $2^p - 1$  is prime.

Example:

: For  $p = 7$  the condition that  $2^p - 1$  is prime is met; the formula gama becomes  $12n + 7$ ; for  $n = 1$  is obtained 19 (prime) and for  $n = 2$  is obtained 31 so we have the triplet gama [7, 19, 31]; also for  $n = 3$  is obtained 43 so other two triplets gama are [7, 19, 43] and [7, 31, 43].

Note: see the sequence A182207 in OEIS for the connection between Carmichael numbers and formula gama.

### 33. A new bold conjecture about a way in which any prime can be written

**Abstract.** In this paper I make a conjecture which states that any prime greater than or equal to 53 can be written at least in one way as a sum of three odd primes, not necessarily distinct, of the same form from the following four ones:  $10k + 1$ ,  $10k + 3$ ,  $10k + 7$  or  $10k + 9$ .

#### Conjecture:

Any prime greater than or equal to 53 can be written at least in one way as a sum of three odd primes, not necessarily distinct, of the same form from the following four ones:  $10k + 1$ ,  $10k + 3$ ,  $10k + 7$  or  $10k + 9$ .

#### Verifying the conjecture:

(For the first few primes greater than or equal to 53)

(Note that we will not show all ways in which a prime can be written in the way mentioned but only one way, enough to confirm the conjecture)

:  $53 = 11 + 11 + 31$ ;  
:  $59 = 13 + 23 + 23$ ;  
:  $61 = 7 + 17 + 37$ ;  
:  $67 = 19 + 19 + 29$ ;  
:  $71 = 17 + 17 + 37$ ;  
:  $73 = 11 + 31 + 31$ ;  
:  $79 = 13 + 23 + 43$ ;  
:  $83 = 11 + 31 + 41$ ;  
:  $89 = 23 + 23 + 23$ ;  
:  $97 = 19 + 19 + 59$ ;  
:  $101 = 17 + 17 + 67$ ;  
:  $103 = 11 + 31 + 61$ ;  
:  $107 = 19 + 29 + 59$ ;  
:  $109 = 13 + 13 + 83$ ;  
:  $113 = 11 + 31 + 71$ ;  
:  $127 = 19 + 29 + 79$ ;  
:  $131 = 7 + 17 + 107$ ;  
:  $137 = 19 + 29 + 89$ ;  
:  $139 = 13 + 13 + 113$ ;  
:  $149 = 13 + 23 + 113$ ;  
:  $151 = 7 + 7 + 137$ .

#### Conjecture:

There exist an infinity of primes  $p$  that can be written as  $p = 2*m + n$ , where  $m$  and  $n$  are distinct primes of the form  $10k + 1$ .

**Conjecture:**

There exist an infinity of primes  $p$  that can be written as  $p = 2^*m + n$ , where  $m$  and  $n$  are distinct primes of the form  $10k + 3$ .

**Conjecture:**

There exist an infinity of primes  $p$  that can be written as  $p = 2^*m + n$ , where  $m$  and  $n$  are distinct primes of the form  $10k + 7$ .

**Conjecture:**

There exist an infinity of primes  $p$  that can be written as  $p = 2^*m + n$ , where  $m$  and  $n$  are distinct primes of the form  $10k + 9$ .

### 34. A bold conjecture about a way in which any square of prime can be written

**Abstract.** In this paper I make a conjecture which states that any square of a prime greater than or equal to 7 can be written at least in one way as a sum of three odd primes, not necessarily distinct, but all three of the form  $10k + 3$  or all three of the form  $10k + 7$ .

#### Conjecture:

Any square of a prime greater than or equal to 7 can be written at least in one way as a sum of three odd primes, not necessarily distinct, but all three of the form  $10k + 3$  or all three of the form  $10k + 7$ .

#### Verifying the conjecture:

(For the first few primes greater than or equal to 7)

(Note that we will not show all ways in which a square of a prime can be written in the way mentioned but only one way, enough to confirm the conjecture)

$$\begin{aligned} : & 7^2 = 49 = 13 + 13 + 23; \\ : & 11^2 = 121 = 37 + 37 + 47; \\ : & 13^2 = 169 = 13 + 43 + 113; \\ : & 17^2 = 289 = 13 + 13 + 263; \\ : & 19^2 = 361 = 7 + 17 + 337; \\ : & 23^2 = 529 = 13 + 53 + 563. \end{aligned}$$

#### Conjecture:

Any square of a prime  $p^2$ , where  $p$  is greater than or equal to 7, can be written as  $p^2 = 2*m + n$ , where  $m$  and  $n$  are distinct primes, both of the form  $10k + 3$  or both of the form  $10k + 7$ .

#### Verifying the conjecture:

(For the first few primes greater than or equal to 7)

(Note that we will not show all ways in which a square of a prime can be written in the way mentioned but only one way, enough to confirm the conjecture)

$$\begin{aligned} : & 7^2 = 49 = 2*13 + 23; \\ : & 11^2 = 121 = 2*37 + 47; \\ : & 13^2 = 169 = 2*43 + 83; \\ : & 17^2 = 289 = 2*13 + 263; \\ : & 19^2 = 361 = 2*7 + 347; \\ : & 23^2 = 529 = 2*13 + 503. \end{aligned}$$

### 35. Statements on the infinity of few sequences or types of duplets or triplets of primes

**Abstract.** In this paper I make few statements on the infinity of few sequences or types of duplets and triplets of primes which, though could appear heterogenous, are all based on the observation of the prime factors of absolute Fermat pseudoprimes, Carmichael numbers, or of relative Fermat pseudoprimes to base two, Poulet numbers.

**Note:**

See, in my book “Two hundred conjectures and one hundred and fifty open problems on Fermat pseudoprimes”, Part Four, “One hundred and fifty open problems regarding Fermat pseudoprimes”.

**Conjecture 1:**

There exist an infinity of positive integers  $k$  such that  $6^k - 1$  and  $18^k - 5$  are both primes.

**Conjecture 2:**

There exist an infinity of positive integers  $k$  such that  $6^k + 1$  and  $12^k + 1$  are both primes.

**Conjecture 3:**

There exist an infinity of positive integers  $k$  such that  $6^k + 1$  and  $18^k + 1$  are both primes.

**Conjecture 4:**

There exist an infinity of positive integers  $k$  such that  $6^k - 5$  and  $24^k - 5$  are both primes.

**Conjecture 5:**

There exist an infinity of positive integers  $k$  such that  $6^k + 1$ ,  $12^k + 1$  and  $18^k + 1$  are all three primes.

**Conjecture 6:**

There exist an infinity of positive integers  $k$  such that  $6^k + 1$ ,  $12^k + 1$  and  $18^k + 13$  are all three primes.

**Conjecture 7:**

There exist an infinity of positive integers  $k$  such that  $k$ ,  $2^k - 1$  and  $5^k - 4$  are all three primes.

**Conjecture 8:**

There exist an infinity of positive integers  $k$  such that  $k$ ,  $2^k - 1$  and  $3^k - 2$  are all three primes.

**Conjecture 9:**

There exist an infinity of positive integers  $k$  such that  $k$ ,  $3^k - 2$  and  $4^k - 3$  are all three primes.



**Conjecture 10:**

There exist an infinity of positive integers  $k$  such that  $40*k + 1$ ,  $60*k + 1$  and  $100*k + 1$  are all three primes.

**Conjecture 11:**

There exist an infinity of positive integers  $k$  such that  $k$ ,  $2*k - 1$ ,  $7*k - 6$  and  $14*k - 13$  are all four primes.

**Conjecture 12:**

There exist an infinity of positive integers  $k$  such that  $k$ ,  $2*k - 1$ ,  $6*k - 5$  and  $12*k - 11$  are all four primes.

**Conjecture 13:**

There exist an infinity of pairs of distinct non-null positive integers  $m$ ,  $n$  such that  $60*m*n - 29$  and  $60*m*n - (60*m + 29)$  are both primes.

**Conjecture 14:**

There exist an infinity of pairs of distinct non-null positive integers  $[m, n]$  such that  $40*m - 10*n - 29$  and  $40*m - 10*n - 129$  are both primes.

**Conjecture 15:**

For any pair of twin primes  $[q, r]$  there exist an infinity of primes  $p$  of the form  $p = 7200*q*r^n + 1$ , where  $n$  is positive integer.

Examples:

: for  $[q, r] = [5, 7]$ ,  $p$  is prime for  $n = 1, 2, 4, 7 (\dots)$

: for  $[q, r] = [11, 13]$ ,  $p$  is prime for  $n = 1, 3, 8 (\dots)$

**Conjecture 16:**

There exist an infinity of primes  $p$  of the form  $p = n*s(p) - n + 1$ , where  $n$  is positive integer and  $s(p)$  is the sum of the digits of  $p$ .

**Conjecture 17:**

There exist an infinity of primes  $p$  of the form  $p = n*s(p) + n - 1$ , where  $n$  is positive integer and  $s(p)$  is the sum of the digits of  $p$ .

**Conjecture 18:**

There exist an infinity of primes  $p$  of the form  $p = n*s(p) + n - 1$ , where  $n$  is positive integer and  $s(p)$  is the sum of the digits of  $p$ .

**Conjecture 19:**

There exist an infinity of primes  $p$  of the form  $p = m*n + m - n$ , where  $m$  and  $n$  are distinct odd primes.

**Conjecture 20:**

There exist an infinity of primes  $p$  of the form  $p = m^2 - m*n + n$ , where  $m$  and  $n$  are distinct odd primes.

**Conjecture 21:**

There exist an infinity of primes  $p$  of the form  $p = (q + 5^k)/10$ , where  $q$  is prime and  $k$  positive integer.

**Conjecture 22:**

There exist an infinity of primes  $p$  of the form  $p = (q + 5^k)/30$ , where  $q$  is prime and  $k$  positive integer.

**Conjecture 23:**

There exist an infinity of primes  $p$  of the form  $p = q^3 + 60$ , where  $q$  is prime.

**Conjecture 24:**

There exist an infinity of primes  $p$ , for  $n$  positive integer, of the following forms:

- :  $20*n^2 + 12*n + 1$ ;
- :  $1800*n^2 + 840*n + 1$ ;
- :  $3*n^2 + 6*n + 4$ ;
- :  $4*n^2 + 172*n + 529$ ;
- :  $20*n^2 + 364*n + 177$ ;
- :  $n^2 + 81*n + 39$ ;
- :  $n^2 + 10*n + 10$ .

### 36. An interesting relation between the squares of primes and the number 96 and two conjectures

**Abstract.** In this paper I make two conjectures based on the observation of an interesting relation between the squares of primes and the number 96.

#### Conjecture 1:

If  $p$  is a prime greater than or equal to 5, then the sequence  $q = p^2 + 96 \cdot k$ , where  $k$  is positive integer, contains an infinity of numbers which are primes or squares of primes.

#### Example:

: for  $p = 5$  are obtained the primes  $q = 313, 409, 601 (\dots)$  for  $k = 3, 4, 6 (\dots)$  and the squares of primes  $q = 11^2, 37^2 (\dots)$  for  $k = 1, 14 (\dots)$ .

#### Conjecture 2:

If  $p$  is a prime greater than or equal to 5, then the sequence  $q = p^2 + 96 \cdot k$ , where  $k$  is positive integer, contains an infinity of semiprimes  $q = m \cdot n$ , where  $m < n$ , with the following property: the number  $n - m + 1$  is a prime or a square of a prime.

#### Example:

: for  $p = 5$  are obtained the semiprimes  $q = 217 = 7 \cdot 31$  (and  $31 - 7 + 1 = 5^2$ ) for  $k = 2$ ,  $q = 505 = 5 \cdot 101$  (and  $101 - 5 + 1 = 97$ , prime) for  $k = 5$ ,  $q = 697 = 17 \cdot 41$  (and  $41 - 17 + 1 = 5^2$ ) for  $k = 7$ ,  $q = 793 = 13 \cdot 61$  (and  $61 - 13 + 1 = 7^2$ ) for  $k = 8$ ,  $q = 889 = 7 \cdot 127$  (and  $127 - 7 + 1 = 11^2$ ) for  $k = 9$ ,  $q = 985 = 5 \cdot 197$  (and  $197 - 5 + 1 = 193$ , prime) for  $k = 10$ ,  $q = 1081 = 23 \cdot 47$  (and  $47 - 23 + 1 = 5^2$ ) for  $k = 11$ ,  $q = 1177 = 11 \cdot 107$  (and  $107 - 11 + 1 = 97$ , prime) for  $k = 12$ ,  $q = 1273 = 19 \cdot 67$  (and  $67 - 19 + 1 = 7^2$ ) for  $k = 13$ ,  $q = 1465 = 5 \cdot 293$  (and  $293 - 5 + 1 = 17^2$ ) for  $k = 15$ .

Note that, for  $p = 5$ , were obtained for  $1 \leq k \leq 15$  only primes, squares of primes and semiprimes with the property mention above.

Taking randomly a prime, id est 233, is obtained:

: for  $k = 1$ , the semiprime  $q = 329 = 7 \cdot 47$  ( $47 - 7 + 1 = 41$ );  
: for  $k = 3$ , the prime  $q = 521$ ;  
: for  $k = 4$ , the prime  $q = 617$ ;  
: for  $k = 5$ , the semiprime  $q = 713 = 23 \cdot 31$  ( $31 - 23 + 1 = 3^2$ );  
: for  $k = 6$ , the prime  $q = 809$ .

Taking randomly another prime, id est 769, is obtained:

: for  $k = 1$ , the semiprime  $q = 865 = 5 \cdot 173$  ( $173 - 5 + 1 = 13^2$ );  
: for  $k = 2$ , the square of prime  $q = 31^2$ ;  
: for  $k = 4$ , the prime  $q = 1153$ ;

- : for  $k = 5$ , the prime  $q = 1249$ ;
- : for  $k = 7$ , the semiprime  $q = 1441 = 11 \cdot 131$  ( $131 - 11 + 1 = 11^2$ ).

**Conclusion:**

It is clear from these examples that the formula  $p^2 + 96 \cdot k$ , where  $p$  is prime and  $k$  is positive integer, has the property to generate primes, squares of primes and semiprimes with the property shown.

### 37. A formula that seems to generate easily big numbers that are primes or products of very few primes

**Abstract.** The formula  $N = (p^4 - 2p^2 + m)/(m - 1)$ , where  $p$  is an odd prime and  $m$  is a positive integer greater than 1, seems to generate easily primes or products of very few primes.

#### Observation:

The formula  $N = (p^4 - 2p^2 + m)/(m - 1)$ , where  $p$  is an odd prime and  $m$  is a positive integer greater than 1, seems to generate easily primes or products of very few primes.

#### Examples:

For  $m = 2$  the formula becomes  $N = p^4 - 2p^2 + 2$  and were obtained the following results for the sequence of the first five consecutive primes of the form  $10k + 1$ :

- : for  $p = 11$ ,  $N = 14401$  prime;
- : for  $p = 31$ ,  $N = 921601$  prime;
- : for  $p = 41$ ,  $N = 2822401 = 113 \cdot 24977$ ;
- : for  $p = 61$ ,  $N = 13838401 = 3313 \cdot 4177$ ;
- : for  $p = 71$ ,  $N = 25401601 = 101 \cdot 251501$ .

For a larger prime of the same form,  $p = 961752931$ , is obtained  $N = 855855567096510789934200845104477377601$ , a semiprime with 39 digits.

For  $m = 3$  the formula becomes  $N = (p^4 - 2p^2 + 3)/2$  and were obtained the following results for the sequence of the first five consecutive primes of the form  $20k + 9$ :

- : for  $p = 29$ ,  $N = 352801 = 17 \cdot 20753$ ;
- : for  $p = 89$ ,  $N = 31363201$  prime;
- : for  $p = 109$ ,  $N = 70567201 = 2659 \cdot 26539$ ;
- : for  $p = 149$ ,  $N = 246420001$  prime;
- : for  $p = 229$ ,  $N = 1374976801 = 11 \cdot 124997891$ .

For  $m = 4$  the formula becomes  $N = (p^4 - 2p^2 + 4)/3$  and were obtained the following results for the sequence of the first eight consecutive primes of the form  $30k + 13$ :

- : for  $p = 13$ ,  $N = 9409 = 97^2$ ;
- : for  $p = 43$ ,  $N = 1138369$  prime;
- : for  $p = 73$ ,  $N = 9462529 = 1609 \cdot 5881$ ;
- : for  $p = 103$ ,  $N = 37509889 = 43 \cdot 872323$ ;
- : for  $p = 163$ ,  $N = 235286209$  prime;
- : for  $p = 193$ ,  $N = 462471169$  prime;
- : for  $p = 223$ ,  $N = 824291329$  prime;
- : for  $p = 283$ ,  $N = 2138029249$  prime.

For two larger primes of the same form is obtained:

- : for  $p = 1299763$ ,  $N = 951339271160353903881409$  prime;
- : for  $p = 1299853$ ,  $N = 951602794365121103901889$  prime.

Taking randomly a prime, id est  $p = 29$ , are obtained the following results:

- : for  $m = 2$ ,  $N = 705601$ , a semiprime;
- : for  $m = 3$ ,  $N = 352801$ , a semiprime;
- : for  $m = 4$ ,  $N = 235201$ , a semiprime;
- : for  $m = 5$ ,  $N = 176401$ , a prime;
- : for  $m = 6$ ,  $N = 141121$ , a prime;
- : for  $m = 7$ ,  $N = 117601$ , a semiprime;
- : for  $m = 8$ ,  $N = 100801$ , a prime;
- : for  $m = 9$ ,  $N = 88201$ , a semiprime;
- : for  $m = 10$ ,  $N = 78401$ , a prime;
- : for  $m = 11$ ,  $N = 70561$ , a semiprime;
- : for  $m = 12$ ,  $N$  is not integer;
- : for  $m = 13$ ,  $N = 58801$ , a semiprime;
- : for  $m = 14$ ,  $N$  is not integer;
- : for  $m = 15$ ,  $N = 50401$ , a semiprime;
- : for  $m = 16$ ,  $N = 47041$ , a prime;
- : for  $m = 17$ ,  $N = 44101$ , a prime (...)

### 38. Four conjectures based on the observation of a type of recurrent sequences involving semiprimes

**Abstract.** In this paper I make four conjectures starting from the observation of the following recurrent relations:  $((p^*q - p)^2 - p)^2 - p \dots$ ), respectively  $((p^*q - q)^2 - q)^2 - q \dots$ ), where  $p, q$  are distinct odd primes.

#### Observation:

Let  $a(i)$  be the general term of the sequence formed in the following way:  $a(i) = (((p^*q - p)^2 - p)^2 - p \dots)$  and  $b(i)$  be the general term of the sequence formed in the following way:  $b(i) = (((p^*q - q)^2 - q)^2 - q \dots)$ , where  $p, q$  are distinct odd primes,  $p < q$ . Very interesting patterns can be observed between  $a(i)$  and  $b(i)$  in the case of the same semiprime  $p^*q$  or between the terms of this recurrence relation for different semiprimes:

Let  $p^*q = 7^*13 = 91$ ; then:

$$\begin{aligned} &: a(1) = 2^*91 - 7 = 175; \\ &: a(2) = 2^*175 - 7 = 343; \\ &: a(3) = 2^*343 - 7 = 679 = 7^*97; \\ &: a(4) = 2^*679 - 7 = 1351 = 7^*193; \\ &: a(5) = 2^*1351 - 7 = 2695; \\ &: a(6) = 2^*2695 - 7 = 5383 = 7^*769 \\ &(\dots) \\ &: b(1) = 2^*91 - 13 = 169; \\ &: b(2) = 2^*169 - 13 = 325; \\ &: b(3) = 2^*325 - 13 = 637; \\ &: b(4) = 2^*637 - 13 = 1261 = 13^*97; \\ &: b(5) = 2^*1261 - 13 = 2509 = 13^*193; \\ &: b(6) = 2^*2509 - 13 = 5005; \\ &: b(7) = 2^*5005 - 13 = 9997 = 13^*769 \\ &(\dots) \end{aligned}$$

Note that  $a(3)/p = b(4)/q = 97$ ,  $a(4)/p = b(5)/q = 193$  and  $a(6)/p = a(7)/q = 769$ .

Let  $p^*q = 11^*13 = 143$ ; then:

$$\begin{aligned} &: a(1) = 2^*143 - 11 = 275; \\ &: a(2) = 2^*275 - 11 = 539; \\ &: a(3) = 2^*539 - 11 = 1067 = 11^*97; \\ &: a(4) = 2^*1067 - 11 = 2123 = 11^*193; \\ &: a(5) = 2^*2123 - 11 = 4235; \\ &: a(6) = 2^*4235 - 11 = 8459 = 11^*769; \\ &: a(7) = 2^*8459 - 11 = 16907; \\ &: a(8) = 2^*16907 - 11 = 33803; \\ &: a(9) = 2^*33803 - 11 = 67595; \\ &: a(10) = 2^*67595 - 11 = 135179 = 11^*12289 \\ &(\dots) \\ &: b(1) = 2^*143 - 13 = 273; \\ &: b(2) = 2^*273 - 13 = 533 = 13^*41; \end{aligned}$$

:  $b(3) = 2 \cdot 533 - 13 = 1053$ ;  
 :  $b(4) = 2 \cdot 1053 - 13 = 2093$ ;  
 :  $b(5) = 2 \cdot 2093 - 13 = 4173$ ;  
 :  $b(6) = 2 \cdot 4173 - 13 = 8333 = 13 \cdot 641$ ;  
 :  $b(7) = 2 \cdot 8333 - 13 = 16653$ ;  
 :  $b(8) = 2 \cdot 16653 - 13 = 33293$ ;  
 :  $b(9) = 2 \cdot 33293 - 13 = 66573$ ;  
 :  $b(10) = 2 \cdot 66573 - 13 = 133133$ ;  
 :  $b(11) = 2 \cdot 66573 - 13 = 266253$ ;  
 :  $b(12) = 2 \cdot 266253 - 13 = 532493 = 13 \cdot 40961$   
 (...)

Note that, in the case of this semiprime, were obtained for  $a(i)/p$  the primes obtained for the first semiprime, id est 97, 193, 769, 12289 (which are primes of the form  $6 \cdot 2^n + 1$ , see the sequence A039687 in OEIS) but for  $b(i)/q$  other primes, id est 41, 641, 40961 ((which are primes of the form  $5 \cdot 2^n + 1$ , see the sequence A050526 in OEIS).

Let  $p \cdot q = 7 \cdot 11 = 77$ ; then:

:  $a(1) = 2 \cdot 77 - 7 = 147$ ;  
 :  $a(2) = 2 \cdot 147 - 7 = 287 = 7 \cdot 41$ ;  
 :  $a(3) = 2 \cdot 287 - 7 = 567$ ;  
 :  $a(4) = 2 \cdot 567 - 7 = 1127$ ;  
 :  $a(5) = 2 \cdot 1127 - 7 = 2247$ ;  
 :  $a(6) = 2 \cdot 2247 - 7 = 4487 = 7 \cdot 641$ .  
 (...)

:  $b(1) = 2 \cdot 77 - 11 = 143$  then for the following terms see  $a(i)$  in the first example of  $p \cdot q = 11 \cdot 13$ .

Let  $p \cdot q = 193 \cdot 199$ ; then we obtain, as  $b(i)/q$ , the primes 769, 12289 (which are primes of the form  $6 \cdot 2^n + 1$ , obtained above) but for  $a(i)/p$  other set of primes not met before: 397, 3169, 6337 (...). To make things even more complicated, for  $p \cdot q = 197 \cdot 199$  we obtain, for  $a(i)/p$ , the set of primes 397, 3169, 6337 mentioned above but for  $b(i)/q$  other set of primes not met before: 3137, 50177 (...), which are primes of the form  $49 \cdot 2^n + 1$  (see the sequence A077498 in OEIS). Note also the interesting thing that 397, 3169 and 6337 are all three primes of the form  $99 \cdot 2^n + 1$ .

Let  $p \cdot q = 13 \cdot 233$ ; then we obtain, as  $a(i)/p$ , the primes 929, 59393, which are primes of the form  $29 \cdot 2^n + 1$ . Seems amazing how many possible infinite sequences of primes can be obtained starting from a simple recurrence relation and a randomly chosen pair of distinct odd primes.

### Conjecture 1:

Let  $a(i)$  be the general term of the sequence formed in the following way:  $a(i) = (((p \cdot q - p) \cdot 2 - p) \cdot 2 - p) \dots$  and  $b(i)$  be the general term of the sequence formed in the following way:  $b(i) = (((p \cdot q - q) \cdot 2 - q) \cdot 2 - q) \dots$ , where  $p, q$  are distinct odd primes. Then there exist an infinity of primes of the form  $a(i)/p$  as well as an infinity of primes of the form  $b(i)/q$  for any pair  $[p, q]$ .



**Conjecture 2:**

Let  $a(i)$  be the general term of the sequence formed in the following way:  $a(i) = (((p \cdot q - p)^2 - p)^2 - p) \dots$  and  $b(i)$  be the general term of the sequence formed in the following way:  $b(i) = (((p \cdot q - q)^2 - q)^2 - q) \dots$ , where  $p, q$  are distinct odd primes. Then there exist an infinity of pairs  $[p, q]$  such that the sequence of primes  $a(i)/p$  is the same with the sequence of primes  $b(i)/q$ .

**Conjecture 3:**

There exist an infinity of primes, for  $k$  positive integer, of the form  $n \cdot 2^k + 1$ , for  $n$  equal to 5, 6, 29, 49 or 99 (note that this conjecture is a consequence of Conjecture 1 and the examples observed above).

**Conjecture 4:**

There exist an infinity of positive integers  $n$  such that the sequence  $n \cdot 2^k + 1$ , where  $k$  is positive integer, contains an infinity of primes.

### 39. Conjecture that states that a Mersenne number with odd exponent is either prime either divisible by a 2-Poulet number

**Abstract.** In this paper I make a conjecture which states that any Mersenne number (number of the form  $2^n - 1$ , where  $n$  is natural) with odd exponent  $n$ , where  $n$  is greater than or equal to 3, also  $n$  is not a power of 3, is either prime either divisible by a 2-Poulet number. I also generalize this conjecture stating that any number of the form  $P = ((2^m)^n - 1)/3^k$ , where  $m$  is non-null positive integer,  $n$  is odd, greater than or equal to 5, also  $n$  is not a power of 3, and  $k$  is equal to 0 or is equal to the greatest positive integer such that  $P$  is integer, is either a prime either divisible by at least a 2-Poulet number (I will name this latter numbers Mersenne-Coman numbers) and I finally enunciate yet another related conjecture.

#### Note:

For a list of 2-Poulet numbers see the sequence A214305 which I posted on OEIS. For a list of Mersenne numbers see the sequence A000225 in OEIS.

#### Conjecture 1:

Any Mersenne number  $2^n - 1$  with odd exponent  $n$ , where  $n$  is greater than or equal to 3, also  $n$  is not a power of 3, is either prime either divisible by a 2-Poulet number.

#### Verifying the conjecture:

(For the first thirteen such  $n$ )

- :  $2^3 - 1 = 7$ , prime;
- :  $2^5 - 1 = 31$ , prime;
- :  $2^7 - 1 = 127$ , prime;
- :  $2^{11} - 1 = 2047 = 23 \cdot 89$ , a 2-Poulet number;
- :  $2^{13} - 1 = 8191$ , prime;
- :  $2^{15} - 1 = 32767 = 7 \cdot 31 \cdot 151$ , which is divisible by  $4681 = 31 \cdot 151$ , a 2-Poulet number;
- :  $2^{17} - 1 = 131071$ , prime;
- :  $2^{19} - 1 = 524287$ , prime;
- :  $2^{21} - 1 = 2097151 = 7^2 \cdot 127 \cdot 337$ , which is divisible by  $42799 = 127 \cdot 337$ , a 2-Poulet number;
- :  $2^{23} - 1 = 8388607 = 47 \cdot 178481$ , a 2-Poulet number;
- :  $2^{25} - 1 = 33554431 = 31 \cdot 601 \cdot 1801$ , which is divisible by  $1082401 = 601 \cdot 1801$ , a 2-Poulet number;
- :  $2^{29} - 1 = 536870911 = 233 \cdot 1103 \cdot 2089$ , which is divisible by  $256999 = 233 \cdot 1103$ , a 2-Poulet number;
- :  $2^{31} - 1 = 2147483647$ , prime.

#### Conjecture 2:

Any Mersenne-Coman number of the form  $P = ((2^m)^n - 1)/3^k$ , where  $m$  is non-null positive integer,  $n$  is odd, greater than or equal to 5, also  $n$  is not a power of 3, and  $k$  is

equal to 0 or is equal to the greatest positive integer such that P is integer, is either a prime either divisible by at least a 2-Poulet number.

**Verifying the conjecture:**

(For  $m = 2$  and the first twelve such  $n$ )

- :  $(4^5 - 1)/3 = 341 = 11*31$ , a 2-Poulet number;
- :  $(4^7 - 1)/3 = 5461 = 43*127$ , a 2-Poulet number;
- :  $(4^{11} - 1)/3 = 1398101 = 23*89*683$ , which is divisibe by:
  - :  $2047 = 23*89$ , a 2-Poulet number;
  - :  $15709 = 23*683$ , a 2-Poulet number;
  - :  $60787 = 89*683$ , a 2-Poulet number.
- :  $(4^{13} - 1)/3 = 22369621 = 2731*8191$ , a 2-Poulet number;
- :  $(4^{15} - 1)/3^2$  is divisibe by:
  - :  $341 = 11*31$ , a 2-Poulet number;
  - :  $4681 = 31*151$ , a 2-Poulet number;
  - :  $10261 = 31*331$ , a 2-Poulet number;
  - :  $49981 = 151*331$ , a 2-Poulet number.
- :  $(4^{17} - 1)/3 = 5726623061 = 43691*131071$ , a 2-Poulet number;
- :  $(4^{19} - 1)/3 = 91625968981 = 174763*52487$ , a 2-Poulet number;
- :  $(4^{21} - 1)/3^2$  divides 5461, 14491, 233017, 42799, 688213 and 1826203, all of them 2-Poulet numbers;
- :  $(4^{23} - 1)/3 = 23456248059221 = 47*178481*2796203$ , which is divisibe by:
  - :  $8388607 = 47*178481$ , a 2-Poulet number;
  - :  $131421541 = 47*2796203$ , a 2-Poulet number;
  - :  $499069107643 = 178481*2796203$ , a 2-Poulet number.
- :  $(4^{29} - 1)/3 = 96076792050570581 = 59*233*1103*2089*3033169$ , which is divisibe by:
  - :  $13747 = 59*233$ , a 2-Poulet number;
  - :  $65077 = 59*1103$ , a 2-Poulet number;
  - :  $123251 = 59*2089$ , a 2-Poulet number;
  - :  $178956971 = 59*3033169$ , a 2-Poulet number;
  - :  $256999 = 233*1103$ , a 2-Poulet number;
  - :  $486737 = 233*2089$ , a 2-Poulet number;
  - :  $706728377 = 233*3033169$ , a 2-Poulet number;
  - :  $2304167 = 1103*2089$ , a 2-Poulet number;
  - :  $3345585407 = 1103*3033169$ , a 2-Poulet number;
  - :  $6336290041 = 2089*3033169$ , a 2-Poulet number.
- :  $(4^{31} - 1)/3 = 1537228672809129301 = 715827883*2147483647$ , a 2-Poulet number;
- :  $(4^{37} - 1)/3 = 6296488643826193618261 = 223*1777*25781083*616318177$ , which is divisibe by  $396271 = 223*1777$  and other 2-Poulet numbers.

**Verifying the conjecture:**

(For  $m = 3$  and the first four such  $n$ )

- :  $8^5 - 1 = 32767 = 7*31*151$ , which is divisibe by  $4681 = 31*151$ , a 2-Poulet number;
- :  $8^7 - 1 = 2097151 = 7^2*127*337$ , which is divisibe by  $42799 = 127*337$ , a 2-Poulet number;

- :  $8^{11} - 1 = 8589934591 = 7 \cdot 23 \cdot 89 \cdot 599479$ , which is divisible by  $2047 = 23 \cdot 89$ , a 2-Poulet number;
- :  $8^{13} - 1 = 549755813887 = 7 \cdot 79 \cdot 8191 \cdot 121369$ , which is divisible by  $647089 = 79 \cdot 8191$ , a 2-Poulet number.

**Verifying the conjecture:**

(For  $m = 4$  and the first four such  $n$ )

- :  $(16^5 - 1)/3$  divides  $341 = 11 \cdot 31$ , a 2-Poulet number;
- :  $(16^7 - 1)/3$  divides  $5461 = 43 \cdot 127$ , a 2-Poulet number;
- :  $(16^{11} - 1)/3$  divides  $2047 = 23 \cdot 89$ , a 2-Poulet number;
- :  $(16^{13} - 1)/3$  divides  $8321 = 53 \cdot 157$ , a 2-Poulet number.

**Note:**

The Mersenne-Coman primes (Mersenne-Coman numbers which are primes) seems to be very rare. For  $m = 2$  (*i.e.*  $4^n - 1$ , where  $n$  is odd,  $n \geq 5$ ) there is no such a prime up to  $n = 107$ .

**Conjecture 3:**

For any prime  $p$  greater than or equal to 5 the number  $(4^p - 1)/3$  is either prime either a product of primes  $p_1 \cdot p_2 \cdot \dots \cdot p_n$  such that all the numbers  $p_i \cdot p_j$  are 2-Poulet numbers for  $1 \leq i < j \leq n$ .

**Note:**

This Conjecture is verified for  $p$  up to 31 (see the Conjecture 2 above).

#### 40. Conjecture that states that a Fermat number is either prime either divisible by a 2-Poulet number

**Abstract.** In this paper I make a conjecture which states that any Fermat number (number of the form  $2^{(2^n)} + 1$ , where  $n$  is natural) is either prime either divisible by a 2-Poulet number. I also generalize this conjecture stating that any number of the form  $N = ((2^m)^p + 1)/3^k$ , where  $m$  is non-null positive integer,  $p$  is prime, greater than or equal to 7, and  $k$  is equal to 0 or is equal to the greatest positive integer such that  $N$  is integer, is either a prime either divisible by at least a 2-Poulet number (I will name this latter numbers Fermat-Coman numbers) and I finally enunciate yet another related conjecture.

##### Note:

For a list of 2-Poulet numbers see the sequence A214305 which I posted on OEIS. For a list of Fermat numbers see the sequence A000215 in OEIS.

##### Conjecture 1:

Any Fermat number  $F = 2^{(2^n)} + 1$  is either prime either divisible by a 2-Poulet number.

##### Note:

It is known that the first 5 Fermat numbers (3, 5, 17, 257, 65537) are primes. Also, for  $n = 5$  is obtained  $F = 4294967297 = 641 \cdot 6700417$ , which is, indeed, a 2-Poulet number (for the next two (composite) Fermat numbers, 18446744073709551617 340282366920938463463374607431768211457, semiprimes, I couldn't verify if they are 2-Poulet numbers).

##### Conjecture 2:

Any Fermat-Coman number of the form  $N = ((2^m)^p + 1)/3^k$ , where  $m$  is non-null positive integer,  $p$  is prime, greater than or equal to 7, and  $k$  is equal to 0 or is equal to the greatest positive integer such that  $N$  is integer, is either a prime either divisible by at least a 2-Poulet number.

##### Verifying the conjecture:

(For  $m = 1$  and the first eight such  $p$ )

- :  $(2^7 + 1)/3 = 43$ , prime;
- :  $(2^{11} + 1)/3 = 683$ , prime;
- :  $(2^{13} + 1)/3 = 2731$ , prime;
- :  $(2^{17} + 1)/3 = 43691$ , prime;
- :  $(2^{19} + 1)/3 = 174763$ , prime;
- :  $(2^{23} + 1)/3 = 2796203$ , prime;
- :  $(2^{29} + 1)/3 = 178956971 = 59 \cdot 3033169$ , a 2-Poulet number;
- :  $(2^{31} + 1)/3 = 715827883$ , prime;

**Verifying the conjecture:**

(For  $m = 2$  and the first four such  $p$ )

- :  $4^7 + 1 = 16385 = 5 \cdot 29 \cdot 113$ , which is divisible by  $3277 = 29 \cdot 113$ , a 2-Poulet number;
- :  $4^{11} + 1 = 4194305 = 5 \cdot 397 \cdot 2113$ , which is divisible by  $838861 = 397 \cdot 2113$ , a 2-Poulet number;
- :  $4^{13} + 1 = 67108865 = 5 \cdot 53 \cdot 157 \cdot 1613$ , which is divisible by:
  - :  $8321 = 53 \cdot 157$ , a 2-Poulet number;
  - :  $85489 = 53 \cdot 1613$ , a 2-Poulet number;
  - :  $253241 = 157 \cdot 1613$ , a 2-Poulet number;
- :  $4^{17} + 1 = 17179869185 = 5 \cdot 137 \cdot 953 \cdot 26317$ , which is divisible by:
  - :  $130561 = 137 \cdot 953$ , a 2-Poulet number;
  - :  $3605429 = 137 \cdot 26317$ , a 2-Poulet number;
  - :  $25080101 = 953 \cdot 26317$ , a 2-Poulet number.

**Verifying the conjecture:**

(For  $m = 3$  and the first two such  $p$ )

- :  $(8^7 + 1)/3^2 = 233017 = 43 \cdot 5419$ , a 2-Poulet number;
- :  $(8^{11} + 1)/3^2 = 954437177 = 67 \cdot 683 \cdot 20857$ , which is divisible by  $1397419 = 67 \cdot 20857$ , a 2-Poulet number.

**Note:**

The Fermat-Coman primes (Fermat-Coman numbers which are primes) seems to be very rare.

**Conjecture 3:**

For any prime  $p$  greater than or equal to 7 the number  $(4^p + 1)/5$  is either prime either a product of primes  $p_1 \cdot p_2 \cdot \dots \cdot p_n$  such that all the numbers  $p_i \cdot p_j$  are 2-Poulet numbers for  $1 \leq i < j \leq n$  (this Conjecture is verified for  $p$  up to 17 (see the Conjecture 2 above).

## 41. Two exciting classes of odd composites defined by a relation between their prime factors

**Abstract.** In this paper I will define two interesting classes of odd composites often met (by the author of this paper) in the study of Fermat pseudoprimes, which might also have applications in the study of big semiprimes or in other fields. This two classes of composites  $n = p(1)*...*p(k)$ , where  $p(1), \dots, p(k)$  are the prime factors of  $n$  are defined in the following way:  $p(j) - p(i) + 1$  is a prime or a power of a prime, respectively  $p(i) + p(j) - 1$  is a prime or a power of prime for any  $p(i), p(j)$  prime factors of  $n$  such that  $p(1) \leq p(i) < p(j) \leq p(k)$ .

### Definition 1:

We name the odd composites  $n = p(1)*...*p(k)$ , where  $p(1), \dots, p(k)$  are the prime factors of  $n$ , with the property that  $p(j) - p(i) + 1$  is a prime or a power of a prime for any  $p(i), p(j)$  prime factors of  $n$  such that  $p(1) \leq p(i) < p(j) \leq p(k)$ , Coman composites of the first kind. If  $n = p*q$  is a squarefree semiprime,  $p < q$ , with the property that  $q - p + 1$  is a prime or a power of a prime, then  $n$  it will be a Coman semiprime of the first kind.

Examples:

- :  $2047 = 23*89$  is a Coman semiprime of the first kind because  $89 - 23 + 1 = 67$ , a prime;
- :  $4681 = 31*151$  is a Coman semiprime of the first kind because  $151 - 31 + 1 = 121$ , a power of a prime;
- :  $1729 = 7*13*19$  is a Coman composite of the first kind because  $19 - 7 + 1 = 13$ , a prime,  $19 - 13 + 1 = 7$ , a prime, and  $13 - 7 + 1 = 7$ , a prime.
- :  $2821 = 7*13*31$  is a Coman composite of the first kind because  $13 - 7 + 1 = 7$ , a prime,  $31 - 13 + 1 = 19$ , a prime, and  $31 - 7 + 1 = 25$ , a power of a prime.

Note that not incidentally I chose Fermat pseudoprimes to base two with two prime factors (2-Poulet numbers) and absolute Fermat pseudoprimes as examples: they are often Coman composites.

### Definition 2:

We name the odd semiprimes  $n_1 = p_1*q_1$ ,  $p_1 < q_1$ , with the property that  $n_2 = q_1 - p_1 + 1 = p_2*q_2$ ,  $p_2 < q_2$ , is a Coman semiprime of the first kind, a Coman semiprime of the first kind of the second degree, also the odd semiprimes  $n_2$  with the property that  $n_3 = q_2 - p_2 + 1$  is a Coman semiprime of the first kind of the second degree, a Coman semiprime of the first kind of the third degree and so on.

Examples:

- :  $679 = 7*97$  is a Coman semiprime of the first kind of the second degree because  $97 - 7 + 1 = 91$ , a Coman semiprime of the first kind because  $91 = 7*13$  and  $13 - 7 + 1 = 7$ , a prime;

- :  $8983 = 13*691$  is a Coman semiprime of the first kind of the third degree because  $691 - 13 + 1 = 679$ , which is a Coman semiprime of the first kind of the second degree.

**Definition 3:**

We name the odd composites  $n = p(1)*...*p(k)$ , where  $p(1), \dots, p(k)$  are the prime factors of  $n$ , with the property that  $p(j) + p(i) - 1$  is a prime or a power of a prime for any  $p(i), p(j)$  prime factors of  $n$  such that  $p(1) \leq p(i) < p(j) \leq p(k)$ , Coman composites of the second kind. If  $n = p*q$  is a squarefree semiprime,  $p < q$ , with the property that  $q + p - 1$  is a prime or a power of a prime, then  $n$  it will be a Coman semiprime of the second kind.

Examples:

- :  $341 = 11*31$  is a Coman semiprime of the second kind because  $11 + 31 - 1 = 41$ , a prime;
- :  $1729 = 7*13*19$  is a Coman composite of the second kind because  $7 + 13 - 1 = 19$ , a prime,  $13 + 19 - 1 = 31$ , a prime, and  $7 + 19 - 1 = 25$ , a power of a prime.

**Definition 4:**

We name the odd semiprimes  $n_1 = p_1*q_1$ ,  $p_1 < q_1$ , with the property that  $n_2 = q_1 + p_1 - 1 = p_2*q_2$ ,  $p_1 < q_1$ , is a Coman semiprime of the second kind, a Coman semiprime of the second kind of the second degree, also the odd semiprimes  $n_2$  with the property that  $n_3 = q_2 + p_2 - 1$  is a Coman semiprime of the second kind of second degree, a Coman semiprime of the second kind of the third degree and so on.

**Notes:**

- : The odd semiprimes of the type  $n = p*q$ ,  $p < q$ , where  $\text{abs}\{p - q + 1\}$  or  $q^2 - p + 1$  or  $\text{abs}\{q - p^2 + 1\}$  or  $q^2 - p^2 + 1$  or  $\text{abs}\{p^2 - q^2 + 1\}$  or, respectively,  $p^2 + q - 1$  or  $p + q^2 - 1$  or  $p^2 + q^2 - 1$  is also prime, seems also to be interesting to be studied;
- : The numbers of the type  $n = p^2 + q^2 - 1$  respectively  $n = q^2 - p^2 + 1$ , where  $p, q$  primes,  $p < q$ , are often, if not primes, Coman composites;
- : The seventh Fermat number,  $18446744073709551617 = 274177*67280421310721$  is a Coman semiprime of the second kind because the number  $67280421584897 = 67280421310721 + 274177 - 1$  is a prime;
- : Many Mersenne numbers are Coman composites:  $2047 = 23*89$  is a Coman semiprime of the first kind because  $89 - 23 + 1 = 67$ , a prime;  $32767 = 7*31*151$  is a Coman composite of the second kind because  $7 + 31 - 1 = 37$ , a prime,  $7 + 151 - 1 = 157$ , a prime, and  $31 + 151 - 1 = 181$ , a prime;  $33554431 = 31*601*1801$  because  $31 + 601 - 1 = 631$ , a prime,  $31 + 1801 - 1 = 1831$ , a prime, and  $601 + 1801 - 1 = 2401 = 7^4$ , a power of a prime;
- : In the papers from the references given below there are few conjectures about Coman composites.



**References:**

- : A formula which conducts to primes or to a type of composites that could form a class themselves, Marius Coman;
- : An elementary formula which seems to conduct often to primes, Marius Coman;
- : Seven conjectures on a certain way to write primes including two generalizations of the twin primes conjecture, Marius Coman;
- : Ten conjectures about certain types of pairs of primes arising in the study of 2-Poulet numbers, Marius Coman;
- : The notion of chameleonic numbers, a set of composites that “hide” in their inner structure an easy way to obtain primes, Marius Coman;
- : Twenty-four conjectures about “the eight essential subsets of primes”, Marius Coman;
- : Two types of pairs of primes that could be associated to Poulet numbers, Marius Coman.

## 42. A formula for generating a certain kind of semiprimes based on the two known Wieferich primes

**Abstract.** In one of my previous papers, “A possible infinite subset of Poulet numbers generated by a formula based on Wieferich primes” I pointed an interesting relation between Poulet numbers and the two known Wieferich primes (not the known fact that the squares of these two primes are Poulet numbers themselves but a way to relate an entire set of Poulet numbers by a Wieferich prime). Exploring further that formula I found a way to generate primes, respectively semiprimes of the form  $q_1 * q_2$ , where  $q_2 - q_1$  is equal to a multiple of 30.

### Note:

In the paper I was talking about in Abstract I conjectured that there exist, for every Wieferich prime  $W$ , an infinity of Poulet numbers which are equal to  $n * W - n + 1$ , where  $n$  is integer,  $n > 1$ . Examples of such Poulet numbers are  $3277 = 1093 * 3 - 2$ ,  $4369 = 1093 * 4 - 3$ ,  $5461 = 1093 * 5 - 4$ , respectively  $49141 = 1093 * 14 - 13$ . In other words, I conjectured that there exist an infinity of pairs of Poulet numbers  $(P_1, P_2)$  such that  $P_2 - P_1 + 1 = 1093$ , respectively an infinity of pairs of Poulet numbers  $(P_1, P_2)$  such that  $P_2 - P_1 + 1 = 3511$ . Examples of such pairs of Poulet numbers are  $(1729, 2821)$ ,  $(3277, 4369)$ ,  $(4369, 5461)$ . Playing with this formula I noted that in many cases the number  $P + W - 1$ , where  $P$  is a Poulet number and  $W$  a Wieferich prime, is equal to a semiprime  $q_1 * q_2$ , where  $q_2 - q_1 = 30$  (examples of such semiprimes are  $37 * 67 = 1387 + 1093 - 1$  and  $43 * 73 = 2047 + 1093 - 1$ ). But, more than that, I noticed that often the numbers of the type  $q_1 * q_2 - W + 1$  (and implicitly, as we will see further, of the type  $q_1 * q_2 + W - 1$ ), where  $q_1$  and  $q_2$  are primes such that  $q_2 - q_1 = 30 * k$ , where  $k$  positive integer, are often equal to  $q_3 * q_4$ , where  $q_3$  and  $q_4$  are primes such that  $q_4 - q_3 = 30 * h$ , where  $h$  positive integer.

### Conjecture 1:

For every prime  $p$ ,  $p > 5$ , there exist an infinity of primes  $q$ ,  $q = p + 30 * n$ , where  $n$  positive integer, such that the number  $p * q + 1092$  is equal to a semiprime  $p_i * q_i$ , where  $q_i - p_i = 30 * m$ , where  $m$  positive integer.

### Conjecture 2:

For every prime  $p$ ,  $p > 5$ , there exist an infinity of primes  $q$ ,  $q = p + 30 * n$ , where  $n$  positive integer, such that the number  $p * q + 1092$  is equal to a prime.

### The first three such semiprimes corresponding to $p = 17$ :

:  $17 * 47 + 1092 = 31 * 61$ ;  
:  $17 * 107 + 1092 = 41 * 71$ ;  
:  $17 * 137 + 1092 = 11 * 311$ .

### The first three such primes corresponding to $p = 17$ :

- :  $17 \cdot 167 + 1092 = 3931$ , prime;
- :  $17 \cdot 197 + 1092 = 4441$ , prime;
- :  $17 \cdot 137 + 1092 = 4951$ , prime.

**The first three such semiprimes corresponding to  $p = 23$ :**

- :  $23 \cdot 173 + 1092 = 11 \cdot 461$ ;
- :  $23 \cdot 353 + 1092 = 61 \cdot 151$ ;
- :  $23 \cdot 443 + 1092 = 29 \cdot 389$ .

**The first three such primes corresponding to  $p = 23$ :**

- :  $23 \cdot 53 + 1092 = 2311$ , prime;
- :  $23 \cdot 83 + 1092 = 3001$ , prime;
- :  $23 \cdot 113 + 1092 = 3691$ , prime.

**Conjecture 3:**

For every prime  $p$ ,  $p > 5$ , there exist an infinity of primes  $q$ ,  $q = p + 30 \cdot n$ , where  $n$  positive integer, such that the number  $p \cdot q + 3510$  is equal to a semiprime  $p_i \cdot q_i$ , where  $q_i - p_i = 30 \cdot m$ , where  $m$  positive integer.

**Conjecture 4:**

For every prime  $p$ ,  $p > 5$ , there exist an infinity of primes  $q$ ,  $q = p + 30 \cdot n$ , where  $n$  positive integer, such that the number  $p \cdot q + 3510$  is equal to a prime.

**The first three such semiprimes corresponding to  $p = 17$ :**

- :  $17 \cdot 107 + 3510 = 73 \cdot 73$ ;
- :  $17 \cdot 167 + 3510 = 7 \cdot 907$ ;
- :  $17 \cdot 347 + 3510 = 97 \cdot 97$ .

**The first three such primes corresponding to  $p = 17$ :**

- :  $17 \cdot 137 + 3510 = 5839$ , prime;
- :  $17 \cdot 227 + 3510 = 7369$ , prime;
- :  $17 \cdot 257 + 3510 = 7879$ , prime.

**The first three such semiprimes corresponding to  $p = 23$ :**

- :  $23 \cdot 293 + 3510 = 37 \cdot 277$ ;
- :  $23 \cdot 383 + 3510 = 97 \cdot 127$ ;
- :  $23 \cdot 503 + 3510 = 17 \cdot 887$ .

**The first three such primes corresponding to  $p = 23$ :**

- :  $23 \cdot 53 + 3510 = 4729$ , prime;
- :  $23 \cdot 83 + 3510 = 5419$ , prime;
- :  $23 \cdot 173 + 3510 = 7489$ , prime.

### 43. Formula that uses primes as input values for obtaining larger primes as output, based on the numbers 7 and 186

**Abstract.** In this paper I present a formula, based on the numbers 7 and 186, that, using primes as input values, often leads, as output values, to larger primes, also to squares of primes and semiprimes. I found this formula by chance, playing with two of my favourite numbers, 13 and 31, and observing that  $7*13^2 + 6*31 = 37^2$  (to be noted, without necessarily connection with this paper, that the difference between the two known Wieferich primes, 1093 and 3511, is equal to  $6*13*31$ ).

#### Observation:

The formula  $7*p^2 + 186$ , where  $p$  is prime, often conducts to primes, squares of primes and semiprimes.

#### Exemplification:

(taking as input values  $p$  the first 27 primes; note that were obtained 13 primes, 6 squares of primes and 8 semiprimes)

:	$7*3^2$	$+ 186 = 3*83$ , semiprime;
:	$7*5^2$	$+ 186 = 19^2$ , square of prime;
:	$7*7^2$	$+ 186 = 23^2$ , square of prime;
:	$7*11^2$	$+ 186 = 1033$ , prime;
:	$7*13^2$	$+ 186 = 37^2$ , square of prime;
:	$7*17^2$	$+ 186 = 47^2$ , square of prime;
:	$7*19^2$	$+ 186 = 2713$ , prime;
:	$7*23^2$	$+ 186 = 3889$ , prime;
:	$7*29^2$	$+ 186 = 6073$ , prime;
:	$7*31^2$	$+ 186 = 31*223$ , semiprime;
:	$7*37^2$	$+ 186 = 9769$ , prime;
:	$7*41^2$	$+ 186 = 11953$ , prime;
:	$7*43^2$	$+ 186 = 19*691$ , semiprime;
:	$7*47^2$	$+ 186 = 15649$ , prime;
:	$7*53^2$	$+ 186 = 23*863$ , semiprime;
:	$7*59^2$	$+ 186 = 43*571$ , semiprime;
:	$7*61^2$	$+ 186 = 37*709$ , semiprime;
:	$7*67^2$	$+ 186 = 73*433$ , semiprime;
:	$7*71^2$	$+ 186 = 43^2$ , square of prime;
:	$7*73^2$	$+ 186 = 37489$ , prime;
:	$7*79^2$	$+ 186 = 73*601$ , semiprime
:	$7*83^2$	$+ 186 = 48409$ , prime;
:	$7*89^2$	$+ 186 = 55633$ , prime;
:	$7*97^2$	$+ 186 = 257^2$ , square of prime;
:	$7*101^2$	$+ 186 = 71593$ , prime;
:	$7*103^2$	$+ 186 = 74449$ , prime;
:	$7*107^2$	$+ 186 = 80329$ , prime.

#### Exemplification:



## 44. Conjectures on Smarandache generalized Fermat numbers

**Abstract.** In this paper I make few conjectures on few classes of generalized Fermat numbers, i.e. the numbers of the form  $F(k) = 2^{(2^k)} + n$ , where  $k$  is positive integer and  $n$  is an odd number, the numbers of the form  $F(k) = 4^{(4^k)} + 3$  and the numbers of the form  $F(k) = m^{(m^k)} + n$ , where  $m + n = p$ , where  $p$  is prime, all subclasses of Smarandache generalized Fermat numbers, i.e. the numbers of the form  $F(k) = a^{(b^k)} + c$ , where  $a, b$  are integers greater than or equal to 2 and  $c$  is integer such that  $(a, c) = 1$ .

### Conjecture 1:

Let be  $F(k) = 2^{(2^k)} + n$ , where  $k$  is positive integer and  $n$  is an odd number. Then, there exist an infinity of numbers  $n$  such that  $F(k)$  is prime for  $k = 0, k = 1$  and  $k = 2$ .

#### Note:

For  $n = 1$ , the numbers  $F(k)$  are the Fermat numbers and it is known that the first five such numbers are primes (it is conjectured that there are not Fermat numbers that are primes for  $k \geq 5$ ). For  $n = 3$  the three primes obtained are [5, 17, 19]. For  $n \geq 3$  obviously  $n$  can be only of the forms  $30*m + 15$  or  $30*m + 27$ , otherwise one from the three numbers  $F(0), F(1)$  and  $F(3)$  would be divisible with 3 or 5.

#### Examples:

(Of such numbers  $n$ )

- : For  $n = 15$  are obtained the primes [17, 19, 31];
- : For  $n = 27$  are obtained the primes [29, 31, 43];
- : For  $n = 57$  are obtained the primes [59, 61, 73];
- : For  $n = 135$  are obtained the primes [137, 139, 151];
- : For  $n = 147$  are obtained the primes [149, 151, 163].

#### Note:

Obviously this conjecture implies the conjecture that there exist an infinity of primes of the forms  $30*k + 1, 30*k + 13, 30*k + 17, 30*k + 19$  respectively  $30*k + 29$ , but I already conjectured in a previous paper (namely "Twenty-four conjectures about the eight essential subsets of primes") that there exist an infinity of primes of the form  $30*k + n$ , for any  $n$  equal to 1, 7, 11, 13, 17, 19, 23 or 29.

### Conjecture 2:

There exist an infinity of quadruplets of primes of the form  $[30*k + 17, 30*k + 19, 30*k + 31, 30*k + 43]$ . Such primes are, as can be seen above, for instance, [17, 19, 31, 43] or [137, 139, 151, 163].

### Comment:

I obviously could put the Conjecture 1 in a simpler form (i.e. Conjecture: there exist an infinity of odd numbers  $n$  such that the numbers  $n + 2, n + 4$  and  $n + 16$  are all three primes, but in this case it would appear like an arbitrary statement, which is not, but one

from the many possible interesting related conjectures on Smarandache generalized Fermat numbers like the following ones:

**Conjecture 3:**

Let be  $F(k) = 4^{(4^k)} + 3$ , where  $k$  is positive integer. Then, there exist an infinity of numbers  $k$  such that  $F(k)$  is equal to  $7 \cdot p$ , where  $p$  is prime.

**Examples:**

(Of such numbers  $F(k)$ )

- :  $F(1) = 259 = 7 \cdot 37$ ;
- :  $F(2) = 4294967299 = 7 \cdot 613566757$ ;
- :  $F(3) = 340282366920938463463374607431768211459 = 7 \cdot 48611766702991209066196372490252601637$

**Conjecture 4:**

Let be  $F(k) = m^{(m^k)} + n$ , where  $m$  is even and  $n$  is odd, such that  $m + n = p$ , where  $p$  is prime. Then, there exist at least a  $k$ , beside of course  $k = 0$ , for which  $F(k)$  has as a prime factor the number  $p$ .

**Conjecture 5:**

Let be  $F(k) = m^{(m^k)} + n$ , where  $m$  is odd and  $n$  is even, such that  $m + n = p$ , where  $p$  is prime. Then, there exist at least a  $k$ , beside of course  $k = 0$ , for which  $F(k)$  has as a prime factor the number  $p$ .

**Reference:**

Florentin Smarandache, *Conjecture (General Fermat numbers)*, in Collected Papers, vol. II, Kishinev University Press, Kishinev, 1997.

## 45. An interesting class of Smarandache generalized Fermat numbers

**Abstract.** In this paper I make few observations on a class of Smarandache generalized Fermat numbers, which are the numbers of the form  $F(k) = a^{(b^k)} + c$ , where  $a, b$  are integers greater than or equal to 2 and  $c$  is integer such that  $(a, c) = 1$ . The class that is observed in this paper includes the numbers of the form  $F(k) = m^{(n^k)} + n$ , where  $k$  is positive integer and  $m$  and  $n$  are coprime positive integers, not both of them odd or both of them even.

### Observation:

The numbers of the form  $F(k) = m^{(n^k)} + n$ , where  $k$  is positive integer and  $m$  and  $n$  are coprime positive integers, not both of them odd or both of them even are very difficult to be factorized, not just because their obviously large size but because seem to have very few prime factors. This is a known characteristic of Fermat numbers (for instance, the 8-th such number, having 78 digits, is a semiprime) and seems that this class of Smarandache generalized Fermat numbers share this feature also.

### Examples:

For  $F(k) = 4^{(3^k)} + 3$  are obtained the following values:

:  $F(1) = 67$  prime;  
 :  $F(2) = 262147$  prime;  
 :  $F(3) = 18014398509481987 * 1422061 * 12667809967$  semiprime;  
 :  $F(4) = 5846006549323611672814739330865132078623730171907 = 103 * 56757345139064190998201352726845942510910001669$  semiprime;  
 (the number  $F(5)$  has 147 digits)

For  $F(k) = 2^{(7^k)} + 7$  are obtained the following values:

:  $F(1) = 16387 = 7 * 2341$  semiprime;  
 :  $F(2) = 316912650057057350374175801351$  prime;  
 (the number  $F(3)$  has 207 digits)

For  $F(k) = 2^{(9^k)} + 9$  are obtained the following values:

:  $F(1) = 521$  prime;  
 :  $F(2) = 2417851639229258349412361 = 11 * 219804694475387122673851$  semiprime;  
 (the number  $F(3)$  has 220 digits)

For  $F(k) = 2^{(11^k)} + 11$  are obtained the following values:

:  $F(1) = 2059 = 29 * 71$  semiprime;  
 :  $F(2) = 2658455991569831745807614120560689163 = 13 * 131 * 1.561042860581228271172997134797821$ ;  
 (the number  $F(3)$  has 401 digits)



**Comment:**

Of course, not for every pair of  $(m, n)$  that satisfies the conditions described above are obtained relevant results, but I haven't find yet a convincing pattern for a special relation between  $m$  and  $n$ .

**Reference:**

Florentin Smarandache, *Conjecture (General Fermat numbers)*, in *Collected Papers*, vol. II, Kishinev University Press, Kishinev, 1997.

#### 46. Conjecture which states that an iterative operation of any pair of two odd primes conducts to a larger prime

**Abstract.** In this paper I make a conjecture which states that from any two odd primes  $p_1$  and  $p_2$  can be obtained, through an iterative and very simple operation, a prime  $p_3$  larger than  $p_1$  and also larger than  $p_2$ .

##### Conjecture:

Let  $p$  and  $q$  be two odd primes; then, through the iterative operation  $((((p + 1)*q + 1) + 1)*q + 1\dots)$  finally it will be obtained a prime.

##### Verifying the conjecture:

(For  $(p, q) = (1299709, 5)$ )

:  $1299709 + 1 = 1299710$ ;  
:  $1299710*5 + 1 = 6498551$ ;  
:  $6498551 + 1 = 6498552$ ;  
:  $6498552*5 + 1 = 32492761$ ;  
:  $32492761 + 1 = 32492762$ ;  
:  $32492762*5 + 1 = 162463811$ ;  
:  $162463811 + 1 = 162463812$ ;  
:  $162463812*5 + 1 = 812319061$ ;  
:  $812319061 + 1 = 812319062$ ;  
:  $812319062*5 + 1 = 4061595311$  which is prime.

##### Verifying the conjecture:

(For  $(p, q) = (1299721, 7)$ )

:  $1299721 + 1 = 1299722$ ;  
:  $1299722*7 + 1 = 9098055$ ;  
:  $9098055 + 1 = 9098056$ ;  
:  $9098056*7 + 1 = 63686393$ ;  
:  $63686393 + 1 = 63686394$ ;  
:  $63686394*7 + 1 = 445804759$ ;  
:  $445804759 + 1 = 445804760$ ;  
:  $445804760*7 + 1 = 3120633321$ ;  
:  $3120633321 + 1 = 3120633322$ ;  
:  $3120633322*7 + 1 = 21844433255$ ;  
:  $21844433255 + 1 = 21844433256$ ;  
:  $21844433256*7 + 1 = 152911032793$ ;  
:  $152911032793 + 1 = 152911032794$ ;  
:  $152911032794*7 + 1 = 1070377229559$ ;  
:  $1070377229559 + 1 = 1070377229560$ ;  
:  $1070377229560*7 + 1 = 7492640606921$ ;  
:  $7492640606921 + 1 = 7492640606922$ ;  
:  $7492640606922*7 + 1 = 52448484248455$ ;  
:  $52448484248455 + 1 = 52448484248456$ ;

:  $52448484248456 \cdot 7 + 1 = 367139389739193$ ;  
:  $367139389739194 + 1 = 367139389739194$ ;  
:  $367139389739194 \cdot 7 + 1 = 2569975728174359$ ;  
:  $2569975728174359 + 1 = 2569975728174360$ ;  
:  $2569975728174360 \cdot 7 + 1 = 17989830097220521$ ;  
:  $17989830097220521 + 1 = 17989830097220522$ ;  
:  $17989830097220522 \cdot 7 + 1 = 125928810680543655$ ;  
:  $125928810680543655 + 1 = 125928810680543656$ ;  
:  $125928810680543656 \cdot 7 + 1 = 881501674763805593$ ;  
:  $881501674763805593 + 1 = 881501674763805594$ ;  
:  $881501674763805594 \cdot 7 + 1 = 6170511723346639159$ , which is prime.

## 47. A probably infinite sequence of primes formed using Carmichael numbers, the number 584 and concatenation

**Abstract.** In this paper I make a conjecture which states that there exist an infinity of primes of the form  $N/3^m$ , where  $m$  is positive integer and  $N$  is the number formed concatenating to the left a Carmichael number with the number 584. Such primes are  $649081 = 5841729/3^2$ ,  $1947607 = 5842821/3$ ,  $1948867 = 5846601/3$  etc. I also make few comments about a certain kind of semiprimes.

### Conjecture 1:

Let  $p$  be an odd prime; then the formula  $n^*p + n - 1$ , where  $n$  integer,  $n > 1$ , conducts to an infinity of prime numbers (note that, for  $n = 2$ , the conjecture states the infinity of Sophie Germain primes).

### Examples:

(Of such primes)

:  $649081 = 5841729/3^2$ ;  
:  $1947607 = 5842821/3^1$ ;  
:  $1948867 = 5846601/3^1$ ;  
:  $649879 = 5848911/3^2$ ;  
:  $2164483 = 58441041/3^3$ ;  
:  $6495997 = 58463973/3^2$ ;  
:  $64909829 = 584188461/3^2$ ;  
:  $194799667 = 584399001/3^1$ ;  
:  $64972073 = 584748657/3^2$ ;  
:  $194946067 = 584838201/3^1$ .

### Conjecture 2:

There exist an infinity of semiprimes  $p^*q = N/3^m$ , where  $p < q$ ,  $m$  is positive integer and  $N$  is the number formed concatenating to the left a Carmichael number with the number 584, with the property that  $q - p + 1$  is a prime number.

### Examples:

(Of such semiprimes)

:  $58429341/3^2 = 109*59561$  and  $59561 - 109 + 1 = 59453$ , which is prime;  
:  $58452633/3^2 = 37*88969$  and  $88969 - 73 + 1 = 88897$ , which is prime;  
:  $584162401/3^0 = 37*15788173$  and  $15788173 - 37 + 1 = 15788137$ , which is prime.

Note:

For more about this type of semiprimes see my previous paper "Two exciting classes of odd composites defined by a relation between their prime factors". In the paper mentioned I observed the semiprimes with the property from above ( $p^*q$ ,  $p < q$ , such that  $q - p + 1$  is prime) but also the sequences of semiprimes  $p1^*q1$ , such that  $q1 - p1 =$

$p^2 \cdot q^2$ , such that  $q^2 - p^2 = p^3 \cdot q^3$  and so on; taking for instance the Carmichael number 1713045574801 concatenated to the left with 584 we have a chain of such consecutive semiprimes, finally the iterative operation ending with a prime:

- :  $5841713045574801/9 = 73 \cdot 8891496264193$  semiprime;
- :  $8891496264193 - 73 + 1 = 239 \cdot 37202913239$  semiprime;
- :  $37202913239 - 239 + 1 = 29 \cdot 1282859069$  semiprime;
- :  $1282859069 - 29 + 1 = 109 \cdot 11769349$  semiprime;
- :  $11769349 - 109 + 1 = 11 \cdot 1069931$  semiprime;
- :  $1069931 - 11 + 1 = 1069921$  prime.

**Comment:**

There exist yet another interesting relation between Carmichael numbers and the number 584; the numbers 561 and 1729, the first and the third Carmichael numbers, are both of the form  $584 \cdot n - 23$  (they are obtained for  $n = 1$ , respectively  $n = 3$ ). So we have the relation  $(1729 + 23)/(561 + 23) = 3$ , an integer, an interesting relation between the numbers 561, 1729 and 23, all three “cult numbers”. For other interesting relations between the number 561 and the Hardy-Ramanujan number, 1729, see my previous paper “Special properties of the first absolute Fermat pseudoprime, the number 561”.

#### 48. Recurrent formulas which conduct to probably infinite sequences of primes and a generalization of a Cunningham chain

**Abstract.** In this paper I define few formulas which conduct from any odd prime respectively from any pair of distinct odd primes to an infinity of probably infinite sequences of primes, also to such sequences of a certain kind of semiprimes, and I also make a generalization of a Cunningham chain of primes of the first kind, respectively of the second kind.

##### Conjecture 1:

Let  $p$  be an odd prime; then the formula  $(((((n * p + n - 1) * p + n - 1)) * p + n - 1) \dots)$  conducts to an infinity of prime numbers for any  $n$  integer,  $n > 1$ .

##### Examples:

First such prime for  $[p, n] = [104729, 2]$

:  $104729 * 2 + 1 = 209459$  prime.

First such prime for  $[p, n] = [104729, 3]$

:  $104729 * 3 + 2 = 314189$  prime.

First such prime for  $[p, n] = [104729, 4]$

:  $104729 * 4 + 3 = 418925$  prime.

First such prime for  $[p, n] = [104729, 5]$

:  $104729 * 5 + 4 = 523649$ ;

:  $523649 * 5 + 4 = 2618249$  prime.

##### Conjecture 2:

Let  $p$  and  $q$  be distinct odd primes; then the formula  $(((((n * p + (n - 1) * q) * p + (n - 1) * q) * p + (n - 1) * q) \dots))$  conducts to an infinity of prime numbers for any  $n$  integer,  $n > 1$ .

##### Examples:

First such prime for  $[p, q, n] = [104729, 3, 2]$

:  $104729 * 2 + 3 = 209461$ ;

:  $209461 * 2 + 3 = 418925$ ;

:  $418925 * 2 + 3 = 837853$  prime.

First such prime for  $[p, q, n] = [104729, 7, 3]$

:  $104729*3 + 7*2 = 314201$ ;  
 :  $314201*3 + 7*2 = 942617$ ;  
 :  $942617*3 + 7*2 = 2827865$ ;  
 :  $2827865*3 + 7*2 = 8483609$  prime.

First such prime for  $[p, q, n] = [104729, 13, 4]$

:  $104729*4 + 13*3 = 418955$ ;  
 :  $418955*4 + 13*3 = 1675859$  prime.

First such prime for  $[p, q, n] = [104729, 17, 7]$

:  $104729*5 + 17*4 = 523713$ ;  
 :  $523713*5 + 17*4 = 2618633$  prime.

**Definition:**

(generalization of a Cunningham chain of the first kind)

We name a Cunningham-Coman chain of primes of the first kind the primes  $p_1, p_2, \dots, p_k$  obtained through the recurrent formula:  $p_2 = 2*p_1 + q, p_3 = 4*p_1 + 3*q, p_4 = 8*p_1 + 7*q, \dots, p_i = 2^{(i-1)}*p_1 + 2^{(i-1)} - 1$ , where  $q$  is an odd prime.

**Example:**

For  $q = 5$ , we have the following Cunningham-Coman chain of length 6 starting with  $p_1 = 13$ :  $13, 31 (= 2*13 + 5), 67 (= 4*13 + 3*5), 139 (= 8*13 + 7*5), 283 (= 16*13 + 15*5), 571 (= 32*13 + 31*5)$ .

**Conjecture 3:**

Let  $p$  be an odd prime; then the formula  $(((((n*p + n - 1)*p + n - 1))*p + n - 1)...) conducts, for any  $n$  integer,  $n > 1$ , to an infinity of Coman semiprimes.$

Note: In a previous paper, “Two exciting classes of odd composites defined by a relation between their prime factors” I defined Coman semiprimes of the first kind the semiprimes  $p*q$  with the property that  $q_1 - p_1 + 1 = p_2*q_2$ , where the semiprime  $p_2*q_2$  has also the property that  $q_2 - p_2 + 1 = p_3*q_3$ , also a semiprime, and the operation is iterate until eventually  $p_k - q_k + 1$  is a prime. I also defined Coman semiprimes of the second kind the semiprimes  $p*q$  with the property that  $q_1 + p_1 - 1 = p_2*q_2$ , where the semiprime  $p_2*q_2$  has also the property that  $q_2 + p_2 - 1 = p_3*q_3$ , also a semiprime, and the operation is iterate until eventually  $p_k + q_k - 1$  is a prime.

**Examples:**

First such semiprime for  $[p, n] = [104729, 2]$

:  $104729*2 + 1 = 209459$ ;  
 :  $209459*2 + 1 = 418919$ ;  
 :  $418919*2 + 1 = 837839$ ;  
 :  $837839*2 + 1 = 1675679$ ;

:  $1675679*2 + 1 = 3351359$ ;  
 :  $3351359*2 + 1 = 6702719 = 139*48221$ , which is a Coman semiprime because  $48221 - 139 + 1 = 7*6869$  and  $6869 - 7 + 1 = 6863$  which is prime.

**Conjecture 4:**

Let  $p$  and  $q$  be distinct odd primes; then the formula  $(((((n*p + (n - 1)*q)*p + (n - 1)*q)*p + (n - 1)*q)...) conducts, for any  $n$  integer,  $n > 1$ , to an infinity of Coman semiprimes.$

**Examples:**

First such semiprime for  $[p, q, n] = [104729, 7, 3]$

:  $104729*3 + 7*2 = 43*7307$ , which is a Coman semiprime of the second kind because  $7307 + 43 - 1 = 7349$ , which is prime.

First such semiprime for  $[p, q, n] = [104729, 11, 5]$

:  $104729*5 + 11*4 = 523689$ ;  
 :  $523689*5 + 11*4 = 2618489 = 547*4787$ , which is at the same time a Coman semiprime of the first kind because  $4787 - 547 + 1 = 4241$ , which is prime, and a Coman semiprime of the second kind, because  $4787 + 547 - 1 = 5333$ , which is also prime.

**Definition:**

We name *an absolute Coman semiprime* a semiprime which is at the same time Coman semiprime of the first kind and Coman semiprime of the second kind.

**Conjecture 5:**

Let  $p$  be an odd prime; then the formula  $(((((n*p - n + 1)*p - n + 1))*p - n + 1)...) conducts to an infinity of prime numbers for any  $n$  integer,  $n > 1$ .$

**Conjecture 6:**

Let  $p$  and  $q$  be distinct odd primes; then the formula  $(((((n*p - (n + 1)*q)*p - (n + 1)*q)*p - (n + 1)*q)...) conducts to an infinity of prime numbers for any  $n$  integer,  $n > 1$ .$

**Definition:**

(generalization of a Cunningham chain of the second kind)

We name a Cunningham-Coman chain of primes of the first kind the primes  $p_1, p_2, \dots, p_k$  obtained through the recurrent formula:  $p_2 = 2*p_1 - q, p_3 = 4*p_1 - 3*q, p_4 = 8*p_1 - 7*q, \dots, p_i = 2^{(i - 1)}*p_1 - 2^{(i - 1)} + 1$ , where  $q$  is an odd prime.

**Conjecture 7:**

Let  $p$  be an odd prime; then the formula  $(((((n*p - n + 1)*p - n + 1))*p - n + 1)...) conducts, for any  $n$  integer,  $n > 1$ , to an infinity of Coman semiprimes.$



**Conjecture 8:**

Let  $p$  and  $q$  be distinct odd primes; then the formula  $((((n^*p - (n + 1)^*q)^*p - (n + 1)^*q)^*p - (n + 1)^*q) \dots)$  conducts, for any  $n$  integer,  $n > 1$ , to an infinity of Coman semiprimes.