Three formulas that generate easily certain types of triplets of primes

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Abstract. In this paper I present three formulas, each of them with the following property: starting from a given prime \( p \), are obtained in many cases two other primes, \( q \) and \( r \). I met the triplets of primes \( \{p, q, r\} \) obtained with these formulas in the study of Carmichael numbers; the three primes mentioned are often the three prime factors of a 3-Carmichael number.

Note:

To refer to the three formulas easily I will name them the formula alpha, beta or gama and the triplets obtained the triplet alpha, beta or gama.

Formula alpha:

The formula alpha is \( 30a*n - (ap + a - 1) \). The first prime of a triplet alpha is \( p \) and the other two ones are obtained giving to \( n \) values of integers, under the condition that \( ap + a - 1 \) is prime.

Examples:

: For \( p = 11 \) and \( a = 2 \) the condition that \( ap + a - 1 \) is prime is met because \( 2*11 + 2 - 1 = 23 \) which is prime; the formula alpha becomes \( 60n - 23 \); it can be seen that for \( n = 1 \) is obtained 47 (prime) and for \( n = 2 \) is obtained 97 (prime) so we have the triplet alpha \([11, 47, 97]\); also for \( n = 3 \) is obtained 157 (prime) so other two triplets alpha are \([11, 47, 157]\) and \([11, 97, 157]\);

: For \( p = 7 \) and \( a = 3 \) the condition that \( ap + a - 1 \) is prime is met because \( 3*7 + 3 - 1 = 23 \) which is prime; the formula alpha becomes \( 90n - 23 \); it can be seen that for \( n = 1 \) is obtained 67 (prime) and for \( n = 2 \) is obtained 157 (prime) so we have the triplet alpha \([7, 67, 157]\); also for \( n = 4 \) is obtained 337 (prime) so other two triplets alpha are \([7, 67, 337]\) and \([7, 157, 337]\).

Note: see the sequence A182416 in OEIS for the connection between Carmichael numbers and formula alpha.
**Formula beta:**

The formula beta is $30an + (ap + a - 1)$. The first prime of a triplet beta is $p$ and the other two ones are obtained giving to $n$ values of integers, under the condition that $ap + a - 1$ is prime.

**Examples:**

For $p = 11$ and $a = 2$ the condition that $ap + a - 1$ is prime is met because $2*11 + 2 - 1 = 23$ which is prime; the formula beta becomes $60n + 23$; it can be seen that for $n = 1$ is obtained 83 (prime) and for $n = 4$ is obtained 263 (prime) so we have the triplet beta $[11, 83, 263]$; also for $n = 6$ is obtained 383 (prime) so other two triplets beta are $[11, 83, 383]$ and $[11, 263, 383]$.

For $p = 19$ and $a = 3$ the condition that $ap + a - 1$ is prime is met because $3*19 + 3 - 1 = 59$ which is prime; the formula beta becomes $90n + 59$; it can be seen that for $n = 1$ is obtained 149 (prime) and for $n = 2$ is obtained 239 (prime) so we have the triplet beta $[59, 149, 239]$; also for $n = 4$ is obtained 419 (prime) so other two triplets beta are $[59, 149, 419]$ and $[59, 239, 419]$.

Note: see the sequence A182416 in OEIS for the connection between Carmichael numbers and formula beta.

**Formula gama:**

The formula gama is $2pn - 2n + p$. The first prime of a triplet gama is $p$ and the other two ones are obtained giving to $n$ values of integers, under the condition that $2p - 1$ is prime.

**Example:**

For $p = 7$ the condition that $2p - 1$ is prime is met; the formula gama becomes $12n + 7$; for $n = 1$ is obtained 19 (prime) and for $n = 2$ is obtained 31 so we have the triplet gama $[7, 19, 31]$; also for $n = 3$ is obtained 43 so other two triplets gama are $[7, 19, 43]$ and $[7, 31, 43]$.

Note: see the sequence A182207 in OEIS for the connection between Carmichael numbers and formula gama.