Quantum F-matrix Theory

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ABSTRACT

Yang–Mills Theory coupled to Gravitation is found, using the 6d spaces [1], f-matrices which are doublets of gamma matrices and spin-vectors. Lagrangians for the fermion sector with internal symmetry G=SU(4) coupled to Gravitation, and the scalar sector with quadratic potentials are derived. Calculations given for Higgs bare mass 125.3GeV, Dark matter scalar particle mass 41.8GeV, the cosmological constant 1.15e-52 m^{-2}, Top quark mass 172.7GeV, Higgs VEV 246.2GeV. Inflaton field is a negative complex potential. Electric charges of quarks and leptons are determined. Matter-antimatter asymmetry is resolved. Symmetry breaking of G results in QCD SU(3), QED U(1), and Chiral Electroweak group SU(2)xU(1) is an emergent interaction are the only other gauge groups acting on 3 generations.

Introduction

It is well known that the Standard model (SM) of particle physics is valid only in Minkowski Space-Time. A possible clue to the search for a theory which includes Quantum Gravity is the derivative of tensors and spinors:

- Tensors: the derivative requires the affine connection
- Spinors: the derivative requires the spin connection

Thus if spinors are coupled to a vector (to be designated as spin-vector), then it follows that the spinors will also be coupled to the affine connection.

From the paper [1] it was shown that extending pseudo-Riemann geometry to include an asymmetric metric resulted in a determination of the dimension of these spaces to be 6. The 7 real 6d vector spaces are { (0,6), (6.0), (3,3), (2,4), (4,2), (1,5), (5,1) } to be designated as f-spaces.

f-spinors are the 2, 4d irreducible Weyl complex spinors on the f-spaces, and are local gauge invariant under the Lie gauge group G=SU(4).

1 The ensemble of *f*-states

The f-spaces are related by Wick type rotation of basis vectors (space-like to time-like and vice versa) which are to be interpreted as a physical phenomenon. In general for a 6 dimensional space define f-matrices f_σ as a doublet of 2 gamma matrices over a 2d vector space with basis (e_1, e_2) that satisfies the Clifford Algebra.

The f-matrices f_σ are

$$f_\sigma = \begin{pmatrix} a_1\gamma_1 \\ a_2\gamma_2 \end{pmatrix}_\sigma \qquad \textbf{1.1}$$
$$\gamma_{\sigma 1}^2 = 1, \gamma_{\sigma 2}^2 = -1$$

The coefficients a_i are complex. The probability that a basis vector is $\gamma_{\sigma i}$ is

$$p_i = \langle f_{\sigma i} | f_\sigma \rangle \qquad \textbf{1.2}$$

The integral is over a 1d real space. Form the 6d state (designated as an f-space) $\beta_\sigma f_\sigma$ and then the density matrix of an ensemble F of f-spaces is

$$F = \beta_{n\sigma} f_{n\sigma} \qquad \textbf{1.3}$$

F is a function of 6 real dimensions with co-ordinates x_μ and $\beta_\sigma \in \mathbb{R}$

The f-spaces have a pure basis when each element of the set $\{f_\sigma\}$ is in a pure state.

2 Emergent Space-Time

The partial direct sums of pure f-spaces are similar to the Principle of Superposition of Hilbert state vectors.

The general case is given by **2.1** with the particular case of the partial direct sums of the f-spaces (3,3) and (1,5) given by **2.2**

$$(p,q) \bigoplus (r,s) = \{(p+r,0).(p,s),(q,r),(0,q+s),(p+r,s),(p+r,q),(r,q+s),(p,q+s)\} \qquad \textbf{2.1}$$
$$(1,5) \bigoplus (3,3) = \{(4,0).(1,3)\} \qquad \textbf{2.2}$$
$$(1,3) \bigoplus (3,3) = \{(4,0).(1,3),(3,3),(0,6)\} \qquad \textbf{2.3}$$

Only spaces of dimension ≤ 6 are allowed.
Space-Time (1,3) is emergent from the partial direct sums of the f-spaces (1,5) and (3,3) given by **2.2**. A further partial direct sum of Space-Time with the f-space (3,3) is given by **2.3**.

3 Scalar Sector

From [1] write the 6d vector $B_\sigma = g_\sigma \varphi^\dagger \varphi$ then the 6d complex scalar equation on the f-spaces is:

$$\mathrm{D}_\sigma(g^\sigma \varphi^\dagger \varphi) - g^\sigma \Gamma^\mu_{\mu\sigma}(\varphi^\dagger\varphi) + g_\sigma g^\sigma (\varphi^\dagger\varphi)^2 = 0 \qquad \textbf{3.1}$$

The complex scalars φ

$$\varphi = \omega \int dx^\mu a_i \qquad \textbf{3.2}$$

Energy of the complex scalar field $E = \varphi^\dagger\varphi$, it follows using **3.2** that $E \le \omega^2$. The complex scalar field has an upper energy ω^2. When $E = \omega^2$ the f-matrices norm to ± 1 and therefore at the upper energy bound Space-Time is Minkowski and hence no curvature. Since all particle momentum depends on the energy of the complex scalars and using the Heisenberg Uncertainty principle it follows that the uncertainty in $\Delta x \ge L$, where L is a minimum length.

The presence of the g-vector g_σ and the symmetric connection $\Gamma^{\mu}_{\mu\sigma}$ implies that for the special case (1,3) vector space, gravitation is coupled to the complex scalar fields φ.

From **2.3** the (4,0) is a real 4d vector space, identify as the 4d real representation of a 2d complex vector ie Higgs field.

The Higgs field acquires kinetic energy during inflation so the scalar Lagrangian in (1,3) space takes the form

$$\mathcal{L}_\varphi = \left(D_\mu\varphi\right)^\dagger (D^\mu\varphi) - \frac{1}{L} V(\varphi^\dagger\varphi) \qquad \textbf{3.3}$$

$$V(\varphi^\dagger\varphi) = -g^\sigma \Gamma^{\mu}_{\mu\sigma}(\varphi^\dagger\varphi) + g_\sigma g^\sigma (\varphi^\dagger\varphi)^2 \qquad \textbf{3.4}$$

The components of the dimensionless coupling strength vector are assumed to be the gravitational coupling strengths.

4 Inflation & Dark Energy

When the connection vanishes the potential $V(\varphi^\dagger\varphi)$ reduces to

$$V(\varphi^\dagger\varphi) = g_\sigma g^\sigma (\varphi^\dagger\varphi)^2 \qquad \textbf{4.1}$$

When $g^2 = g_\sigma g^\sigma < 0$ the potential $V(\varphi^\dagger\varphi)$ acts like the inflaton field. Summing over the 3 particle-antiparticle f-spaces (section 9) the inflaton field is

$$\Lambda_{eff} = \frac{3}{2} g^2 (\varphi^\dagger\varphi)^2 \qquad \textbf{4.2}$$

$g_\sigma g^\sigma < 0$ is possible only for vector spaces with indefinite signature. Thus inflation can only arise if the space has an indefinite signature. The scalar field $\varphi^\dagger\varphi$ rolls down to the minimum $\langle 0|0\rangle$ with the inflaton field approaching the cosmological constant Λ_0

$$\Lambda_0 \sim \frac{3}{2} g^2 \left(\frac{\langle 0|0\rangle}{\hbar c}\right)^2 \qquad \textbf{4.3}$$

Evaluating **4.3** at the threshold of electron-positron pair production:

$$\Lambda_0 \sim \frac{3}{2}\left(\frac{2m_e}{m_p}\right)^4 \left(\frac{\langle 0|0\rangle}{\hbar c}\right)^2 \qquad \textbf{4.4}$$

Where the Planck mass is $m_p \sim 1.22 \times 10^{19}\mathrm{GeV}$, the Higgs VEV is $\langle 0|0\rangle \sim 246\mathrm{GeV}$ and the electron mass is $m_e \sim 0.511\mathrm{MeV}$. It follows that the cosmological constant in free space is $\Lambda_0 \sim 1.15 \times 10^{-52}\mathrm{m}^{-2}$. This is in agreement with the Planck data [2]

The production of gravitational waves during inflation implies that the symmetric connection is non-zero therefore the complex potential **3.4** is restored. The resulting Space-Time (1,3) is one of many outcomes dependent on which f-space(s) has the condition $g_\sigma g^\sigma < 0$.

5 Higgs Vacuum

It is assumed that the Higgs Vacuum state is a quantum field, H. A quantum field has a lower energy bound E_l and an upper energy bound E_u and has f fermionic and b bosonic degrees of freedom [1]. From [1] the Higgs VEV $\langle 0|0\rangle$ was given by:

$$\langle 0|0\rangle = \sqrt{\frac{E_l E_u}{2\pi(7f+8b)^{3/4}}} = \left[\frac{27000\hbar^5 c^{17}\Lambda_0}{\pi^{13}G^3(7f+8b)^3}\right]^{1/8} \qquad \textbf{5.1}$$

The Higgs vacuum consists of fermion-antifermions, f=6*8*2, SU(4) gauge bosons b= 6*15*2 and 7 complex scalars. It follows from **5.1** that the Higgs VEV is 246.2GeV which is in agreement with the measured value [3].

6 The Higgs Boson

Identify the g_σ (1,3) potential as the Higgs sector field with a bare mass given by:

$$\frac{1}{\lambda_H^2} = \frac{1}{L} g^\sigma \Gamma^{\mu}_{\mu\sigma} \qquad \textbf{6.1}$$

For Higgs boson at rest, the spatial derivatives vanish. Eq **6.1** reduces to

$$\frac{1}{\lambda_H^2} = \frac{1}{L\lambda}\mathrm{g}^0 \qquad \textbf{6.2}$$

$$E_H = \sqrt{E_L E_\lambda \mathrm{g}^0} \qquad \textbf{6.3}$$

Let $\sqrt{E_L E_\lambda} = \langle 0|0\rangle$ and $\mathrm{g}^0 = \sum \alpha_G(f,b)$

From [1] the gravitational coupling at the upper energy scale is

$$\alpha_G(f,b) = \frac{60}{\pi^3(7f+8b)} \qquad \textbf{6.4}$$

Where f and b are the degeneracy's of the fermions and bosons

The Higgs boson is assumed to be a condensate of the complex scalar with a virtual quantum field of spin 2 quanta.

The Higgs boson has a self coupling and so b=1 for spin 0. The Higgs boson also has a field of virtual bosons due to the asymmetric metric fields. There are 7 *f*-spaces hence the asymmetric metric fields contribute 7x2=14 degrees of freedom, hence b=14.

$$\mathrm{g}^0 = \alpha_G(0,1) + \alpha_G(0,14) \qquad \mathbf{6.5}$$

$$E_H = \sqrt{\frac{225}{28\pi^3}}\langle 0|0\rangle \qquad \mathbf{6.6}$$

The prediction for the Higgs bare mass 125.3GeV is in agreement with the latest measurement of 125GeV [4]

7 Dark Matter Field

Summing the potentials of the 7 *f*-spaces, the mass term has the co-efficient

$$\frac{1}{\lambda_D^2} = \frac{1}{L}\sum_{\mathrm{i}}\left(g^\sigma\Gamma^{\mu}_{\mu\sigma}\right)_{\mathrm{i}} \qquad \mathbf{7.1}$$

For Dark matter particle at rest, the spatial derivatives vanish.

Consider an ensemble of potentials in equilibrium so the coupling $g^\sigma = \mathrm{g}^0$ for all potentials. Write $\frac{1}{\lambda} = \sum_{\mathrm{i}}\left(\Gamma^{\mu}_{\mu\sigma}\right)_{\mathrm{i}}$, Eq **7.1** reduces to

$$\frac{1}{\lambda_D^2} = \frac{1}{L\lambda}\mathrm{g}^0 \qquad \mathbf{7.2}$$

There are 21 time-like derivates, giving rise to 21 scalars:

$$\mathrm{g}^0 = \alpha_G(0,21) + \alpha_G(0,14) \qquad \mathbf{7.3}$$

The prediction for dark matter mass is 41.8GeV.

8 Standard Model Gauge Groups

The chiralities of the 2 4d *f*-spinors on each of the 6d spaces (p,q) are +1 and -1. The (1,3) 4d Dirac spinor without the U(1) labels are spinons, with chirality +1 and -1. The *f*-spinors acquire spin by coupling to the spinons. The 2 *f*-spinors can form a doublet with the same 6d chirality only by coupling to a spinon with chirality -1.

It follows that the Lie gauge group SU(2)xU(1), the Electroweak gauge group acts on chiral -1 fermion doublets as is observed [5].

The number of Higgs bosons in the BEH mechanism is [6]

$$H = 2n - N + M \qquad \mathbf{8.1}$$

Where N is the dimension the gauge group SU(n), M is the dimension of the subgroup, 2n is the real dimension of a complex scalar field.

Setting H=1, the SU(4) has subgroup SU(3), the QCD gauge group **[7]**. Thus a 4d complex scalar field breaks the symmetry of SU(4) to SU(3). Symmetry breaking of SU(4) also results in 7 massive bosons. When the energy scale drops below the 4d complex field ground state, these massive bosons become massless. One of the bosons is the T_{15} generator, which is proposed to be the photon of QED.

The group SU(4) cannot be spontaneously broken to the subgroups SU(2) or U(1) with H=1 as the dimension of the scalar field required is not a positive integer.

In summary, the only Lie gauge groups are SU(4), SU(3), U(1) and chiral SU(2)xU(1).

9 Matter-Antimatter asymmetry

The *f*-spaces (p,q) and (q,p) are related by a Wick rotation of i to each basis vector.

The direct sum of these *f*-spaces is

$$(p,q) \bigoplus (q,p) = \{(p+q,0).(p,p),(q,q),(0,q+p)\} \quad \textbf{9.1}$$

Since $p+q=6$, **9.1** reduces to

$$(p,q) \bigoplus (q,p) = \{(6,0).(p,p),(q,q),(0,6)\} \quad \textbf{9.2}$$

The generators of SU(4) on (6,0) are constructed using the Lie bracket of the gamma matrices

$$\sigma_{\mu\nu} = \frac{i}{4}[\gamma_\mu, \gamma_\nu] \quad \textbf{9.3}$$

The SU(4) generators on (0,6) are obtained by a Wick rotation of the basis vectors of (6,0), resulting in the generators

$$\sigma_{\mu\nu} = -\frac{i}{4}[\gamma_\mu, \gamma_\nu] \quad \textbf{9.4}$$

Thus the SU(4) generators are opposite sign, and hence the *f*-spinors on the *f*-spaces (p,q) and (q,p) are SU(4) charge-anti-charge pairs.

The (3,3) *f*-spinors are coupled to the gauge space (0.6) (**2.3**). The absence of the coupling to the gauge space (6,0) implies that the (3.3) *f*-spinors when coupled to Space-Time (1,3) are charge chiral, ie no anti-charge states thus forming the matter field.

For each f-spinor, the electric charge quantum numbers Q_i are assumed proportional to the diagonal matrix T_{15} of $SU(4)$ after symmetry breaking,

$$Q_i = c_i diag(-1, -1, -1, 3) \tag{9.5}$$

Where i=1,2 denotes the 2 f-spinors and c_i are constants

The sum of the electric charges equal to zero is equivalent to the condition:

$$Tr(Q_1) + Tr(Q_2) = 0 \tag{9.6}$$

The electric charges on the sub-group SU(3) are

$$Q_i = c_i diag(-1, -1, 2) \tag{9.7}$$

Since $Tr(Q_i) = 0$ set $c_i = 1$
Electric charge quantum number is in the range $Q \in [-1,1]$
Dividing these 2 matrices by 3 results in the 2 electric charges -1/3 and 2/3

$$\begin{gathered} Q_1 = diag(-1/3, -1/3, -1/3, q_1) \\ Q_2 = diag(2/3, 2/3, 2/3, q_2) \end{gathered} \tag{9.8}$$

Using the conditions:

$$\begin{gathered} Tr(Q_1) + Tr(Q_2) = 0 \\ Q_1 - Q_2 = -1 \end{gathered} \tag{9.9}$$

It follows that:

$$\begin{gathered} q_1 - q_2 = -1 \\ q_1 + q_2 = -1 \end{gathered} \tag{9.10}$$

Hence $q_1 = -1$ and $q_2 = 0$
Thus the electric charge quantum numbers are $q \in [-1, -1/3, 0, 2/3]$
The 2 f-spinors have electric quantum numbers:

$$Q_1 = \left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, -1\right), \qquad Q_2 = \left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, 0\right) \tag{9.11}$$

Thus Quarks are the triplet states of electric charge -1/3 and 2/3. Leptons are the -1 and 0 electric charge states.

10 Fermion Sector

The (1,3) Dirac spinor ζ is the source of spin (to be designated as a spinon) forms a 4d vector current – spin-vector j_σ:

$$j_\sigma = \zeta^\dagger \gamma_\sigma \zeta \tag{10.1}$$

The f-spinors ψ couple to the 4d spin-vector to form fermions.

The gauge group G=SU(4) (section 8). The 4d f-spinors come in 3 pairs (p,q) and (q,p) coupled to the spin-vector forming fermion-anti-fermion pairs (section 9). There are 2 4d irreducible spinors, resulting in 8 states for each of the (p,q) spaces hence there are 3 generations of fermions. The (3,3) f-spinors are chiral with respect to the G gauge group charge, thus forming matter (section 9)

The (1,3) Yang-Mills Lagrangian [8] for massless s=1/2 fermions is

$$\mathcal{L} = (\psi j^{\sigma})^{\dagger}\gamma^{\mu}\nabla_{\mu}\psi j_{\sigma} + F_{\mu\nu}F^{\mu\nu} + R \qquad \textbf{10.2}$$

∇_{μ} is the gauged spinor/tensor derivative. The Lagrangian is local gauge invariant under G. Thus gravitation is coupled to f-spinors via the vector index on the spin-vector.

The Hilbert energy-stress tensor is a 2nd rank tensor hence the matter Lagrangian is supplemented with

$$F_{\mu\nu}F^{\mu\nu} + R + \cdots \qquad \textbf{10.3}$$

The scalars $R_{\mu\nu}R^{\mu\nu}, R_{\sigma\rho\mu\nu}R^{\sigma\rho\mu\nu}$ lead to higher rank Hilbert energy-stress tensors.

Gravitation is a consequence of the spinons, i.e. the (1,3) Dirac spinor and is an integral part of the Fermion Lagrangian.

11 Top Quark Mass

Fermions are coupled to the Higgs field via a Yukawa dimensionless coupling g:

$$E = g\langle 0|0\rangle \qquad \textbf{11.1}$$

For scalar field, $g = \sqrt{\alpha_G(f,b)}$, where $\alpha_G(f,b)$ is the gravitational coupling **6.4.**
For fermions, let $g = \sqrt[4]{\alpha_G(f,b)}$. Evaluating **11.1** with $\alpha_G(f,b) = \alpha_G(0,1)$ gives

$$E = \left(\frac{15}{2\pi^3}\right)^{1/4} \langle 0|0\rangle \qquad \textbf{11.2}$$

Evaluating **11.2** gives $E = 172{,}67\mathrm{GeV}$, in agreement with the top quark mass [9].

Conclusion

Fermions & Gauge Interactions

The introduction of f-matrices as a mixed state of gamma matrices results in the 6d spaces (p,q) and the sub-spaces being in a mixed state. Space-Time and the Higgs field emerge as partial superposition's of the (1,5) and (3,3) f-spaces. A fermion state is determined by the charge (4d f-spinors), spin (coupling to spinons), and mass (coupling to the Higgs field). The phenomenon of charge–spin separation seen in condensed matter experiments may support the fermion state description given here. The (3,3) f-spinors are charge chiral forming fermions i.e. no anti-fermion states, while the f-spinors on the 6 other f-spaces form fermion-anti-fermion pairs, thus resolving the matter-antimatter asymmetry.

The gauge group SU(4) spontaneously breaks to QCD SU(3) and QED U(1) via the BEH mechanism while the chiral Electroweak group SU(2)xU(1) is emergent. 3 generations of quarks and leptons with the correct electric charges are predicted with an upper fermion mass of 172.7GeV – the top quark.

Scalar sector

The Higgs boson bare mass 125GeV agrees well with the measurements from CERN. It is proposed that dark matter is a scalar with mass 41GeV. The inflaton field arises naturally as a complex potential on a vector space with an indefinite signature only.

Gravitation

Spinons are the Dirac 4d spinors on the vector space (1,3) have spin only. Spinons form a 4d current vector, the spin-vector to which *f*-spinors couple. The Lagrangian for *f*-spinors coupled to the spin-vector forming fermions automatically requires a tensor derivative. Gravitational field is the gauge field consisting of the symmetric affine connection.

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