Bootstrapping generations

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Abstract

A supersymmetric version of Chew's "democratic bootstrap" argument predicts the existence of three generations of particles, with a quark, of type "up", more massive that the other five.

The origin of the now popular "dual models" can be traced to advocacy of Geoffrey Chew of the study of the S-matrix, eventually looking for a self consistent unique consistent solution, which "bootstraps" itself. A guiding principle was "democracy", where no particle could be claimed to be elementary, as in some dual it could be seen as composed of resonances or bound states of all the other particles.

On other hand, a dual model with fermions seems to require a special symmetry, supersymmetry, relating bosonic and fermionic degrees of freedom. Schwarz [] has proposed a "quark-gluon" dual model, and given the stringy character of these models, it seems reasonable to consider that the gluon part is constituted by a string terminated in a pair of quarks.

With this imaginery, we can try a weaker version of the democratic bootstrap. We keep agnostic about if quarks and leptons are composed of themselves or other particles, but we at least require that scalar leptons and scalar quarks are composed of pairs of quarks bound by some gluonic string.

Following an emergent tradition, we will call squarks and sleptons to the scalar particles, and we will refer to the requeriment of matching between the number of composite scalar particles and the number of degrees of freedom of standard model fermions as the "sBootstrap".

Lets first taste the sBootstrap condition at cask strength: N generations of quarks of types u, d should produce N^2 scalars of charge $+1/3$, but we have $2N$ fermionic degrees of freedom with such charge, so $N = 2$ for the matching in the "down antiquark" sector. But they also produce, from pairs of two down-type quark, a total of $N(N + 1)/2$ scalars of charge -2/3, again to be matched to $2N$ degrees of freedom in the "up antiquark" sector. So $N = 3$ for the matching in this sector.

We need thus to dilute out condition for it to be palatable. We do it in the following maner: requesting that not every quark can terminate the string. A way of seeing this is

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to imagine that some of the families have very high masses, perhaps of the order of the electroweak scale, so that on one hand they can not reach the relativistic speed at the extremes of the string, and on the other hand they would even disintegrate and decay before hadronization, not being able to form a durable bound state.

We postulate thus that only a subset of "light quarks" are in the terminations of the string and then able to form composites. Given that we could even have "mixed" generations, with one light and a heavy quark, we have now at our dispossal three parameters: the number of generations N, the number of light quarks of the down kind, say r, and the number of light quarks of the up kind, say s.

$$rs = 2N \tag{1}$$

$$\frac{r(r+1)}{2} = 2N \tag{2}$$

The integer solution for N is that it must be half of an hexagonal number

$$2N = \cancel{1}, 6, \cancel{15}, 28, \cancel{45}, 66, \cancel{91}, 120, ... \tag{3}$$

So the smallest admisible solution is $N = 3$. We could consider the requeriment of asymptotic freedom [] in the beta function of QCD to put an upper limit to the number of solutions, asking the number of flavours to be such that $2n_f < 33$. If this limitation applies to all the flavours, light or heavy, then the solution is unique.

We had N=3 generations of quarks with r=3 light down quarks but only s=2 light up quarks.

It could be argued perhaps that the beta function should be built only with the light quarks. In this case we have a short list of solutions

N	s	r	Total light flavours
3	2	3	5
14	4	7	11
33	6	11	17

Where we have included the third solution only because 17 flavours are very near of the theoretical bound of 16.5. Of course all the solutions beyond N=3 already contain a lot of heavy generations. Note that we always have $r = 2s - 1$.

To search for another source of uniqueness, lets look the slepton sector.

For the charged leptons, we have nothing new. Every charged scalar lepton is a composite of a quark and antiquark of different type, so $rs = 2N$. The question is what equation must be use for the match in the neutral sector; we have options for both sides of the matching.

On the composition side, the question is what group must be use to classify the color neutral composites. Given that they all similar to mesons (they can even be the mesons themselves) we will classify them with $SU(r + s)$. We could try $U(r + s)$, having an extra neutral state, but then we had a odd number of neutral states and no solution.

On the Standard Model side, we could consider the known degrees of freedom of neutrinos, only as left- handed particles, or add a right-handed neutrino in each generation. So we have finally two possible equations.

1) With both legt and right handed neutrinos:

$$r^2 + s^2 - 1 = 4N \tag{4}$$

2) With only left handed neutrinos

$$r^2 + s^2 - 1 = 2N \tag{5}$$

And again we can discard the second option: it has no integer solution. The sBootstrap can be extended to the lepton sector only if we use the extra degrees of freedom that come with the addition of right-handed neutrinos.

Furthermore, the extension can be done only in an unique case, N=3.

So, even if we do not want to use QCD beta-function as an argument, we also have an unique solution if we ask the the sBootstrap to produce all the scalars in the SSM.

To conclude, we have found that the sBootstrap requisites do predict that the number of fermion families must be three.

Looking at it group-theoretically, the scalars are produced from 3 "down quarks" and 2 "up quarks" according the decomposition of $SU(5)$ to $SU(3) \times SU(2)$

We had the sleptons extracted from a **24** of this flavour group

$$24 = (1,1) + (3,1) + (2,3) + (2,\bar{3}) + (1,8) \tag{6}$$

and the squarks from the two sextets that appear in the decomposition of **15**.

$$15 = (3,1) + (2,3) + (1,6) \tag{7}$$

And similarly the anti-squarks.

If a third generation is discovered with a massive "up type" quark, this should be a striking ~~prediction~~ postdiction from dual models, or string theory, how they call it nowadays.

References

(I am afraid that by 2014 all the references needed in this paper should be considered obsolete)