Posted on ArXiV September 13, 2014
Version:

Comments on recent papers by S. Marshall claiming proofs of several conjectures in number theory

Matthias Lesch

Abstract. In a recent series of preprints S. Marshall [Mar14] claims to give proofs of several famous conjectures in number theory, among them the twin prime conjecture and Goldbach's conjecture. A claimed proof of Beal's conjecture would even imply an elementary proof of Fermat's Last Theorem.

It is the purpose of this note to point out serious errors. It is the opinion of this author that it is safe to say that the claims of the above mentioned papers are lacking any basis.

1. The papers `vixra:1408.0169, 1408.0174, 1408.0201, 1408.0209,` and `1408.0212`

No less than five times the author reproduces exercise 161 of the book [DKMe07], where the solution on page 136 in loc. cit. is copied verbatim, with a few more details added. The author quotes [DKMe07], however he does not indicate clearly enough that his proof is copied more or less verbatim from there. At least the abstracts seem to suggest that he publishes the result as his own.

The statement itself is a quite nice consequence of Wilson's Theorem: given integers $p > 1$ and $d > 0$ consider the rational number

$$n(p,d) = (p-1)!\Big(\frac{1}{p} + \frac{(-1)^d d!}{p+d}\Big) + \frac{1}{p} + \frac{1}{p+d}. \tag{1.1}$$

Then p and $p+d$ are both primes if and only if $n(p,d)$ is an integer.

Multiplying Eq. (1.1) by $p \cdot (p+d)$ we find for any pair of integers $p > 1, d > 0$ that

$$n(p,d) \cdot p \cdot (p+d) = (p-1)!(p+d+(-1)^d d!p) + 2p + d. \tag{1.2}$$

In particular, the right hand side and hence both sides of this equation are integers. In the papers listed in the title to this section (e.g. `vixra:1408.0169`, p. 7, `vixra:1408.0174`, p. 7) it is erroneously concluded that if Eq. (1.2) holds for integers $p > 1, d > 0$ and a rational number n then n must be an integer. This is obviously not true as we know from Eq. (1.1). Take $p = 3, d = 3$ then $p+d$ is not prime and by Eq. (1.1) $n(p,d)$ is not an integer.

1.α. The paper `vixra:1408.0169` on Fibonacci primes

Let F_n be the sequence of *Fibonacci* numbers, that is

$$F_0 = F_1 = 1; \quad F_n = F_{n-1} + F_{n-2}, n \geq 2.$$

A Fibonacci prime is a Fibonacci number which is prime. It is unknown whether there are infinitely many Fibonacci primes.

`vixra:1408.0169` claims to prove this conjecture. Starting on page 6 a pair $p = F_{y-1}, d = F_{y-2}$ is considered. Using only the identities Eq. (1.1), (1.2) the author arrives at the conclusion that $p + d$ must be prime. The paper therefore proves a much stronger statement which is obviously wrong, namely that for given $p > 1, d > 0$ the sum $p + d$ is automatically prime.

Clearly, if a method of proof proves something false, the method cannot be correct, unless mathematics is contradictory. As a side remark I refer to Scott Aaronson's list [**Aar**] of indicators that a claimed mathematical breakthrough is wrong. Most of the items on that list are just indicators. However, item no. 3 (method proves something which is obviously wrong) is more than an indicator.

1.β. The paper `vixra:1408.0174` on Polignac's conjecture

Polignac's conjecture states that for any integer $k \geq 1$ there are infinitely many primes p such that $p + 2k$ is prime, too. The case $k = 1$ is the twin prime conjecture.

Starting on page 6 in `vixra:1408.0174` it is seemingly shown that for given $k \geq 1$ and a prime p then also $p + 2k$ is prime. There are many obvious counterexamples to this. The main error in the middle of page 7 is the one explained after Eq. (1.2).

At the end of the paper Goldbach's conjecture is derived from Polignac's conjecture. The proof is also incorrect but we do not have to discuss the details any more.

1.γ. The papers `vixra:1408.0201, 1408.0209`, and `1408.0212`

We won't discuss the three remaining papers on this Section's list in detail. The decisive error pointed out above is repeated three more times.

2. The paper `vixra:1408.0173` on Beal's conjecture

Beal's conjecture is relatively recent. It states that if

$$A^x + B^y = C^z \tag{2.1}$$

with positive integers A, B, C and positive integers $x, y, z > 2$ then A, B, C have a common prime factor.

This is really a far reaching conjecture which obviously implies Fermat's Last Theorem. Since no *elementary* proof of Fermat's Last Theorem is known yet, an elementary proof of Beal's conjecture would be a major breakthrough.

In top of page 3 of `vixra:1408.0173` a proof by contradiction is attempted. However, the author fails to formulate the negation to Beal's conjecture in a correct way. Namely, it is said that if we have a solution to Eq. (2.1) with positive integers A, B, C and positive integers $x, y, z > 2$ then A, B, C cannot have a common prime factor. Well, for this to disprove it suffices to give one counterexample, e.g. $3^3 + 6^3 = 3^5$.

Since the ability to negate statements in a correct way is so basic I have to admit that I stopped reading the paper at this point.

3. Conclusion

The 6 papers [**Mar14**] fall into two categories.

In the papers `vixra:1408.0169` and `vixra:1408.0174` on the infinitude of the Fibonacci primes and on Polignac's conjecture the approach proves something which is clearly wrong. The decisive error of these papers is repeated three more times in the papers `vixra:1408.0201, 1408.0209`, and `1408.0212`.

In `vixra:1408.0173` a proof by contradiction is attempted where the negation of the claim is stated incorrectly.

In light of this it is my opinion that the six papers are wrong and the conjectures are still open.

References

[Aar] S. Aaronson, *Ten signs a claimed mathematical breakthrough is wrong*, `http://www.scottaaronson.com/blog/?p=304`. 2

[DKMe07] J.-M. De Koninck and A. Mercier, *1001 problems in classical number theory*, American Mathematical Society, Providence, RI, 2007, Translated from the 2004 French original by De Koninck. MR 2302879 (2007m:11001) 1

[Mar14] S. Marshall, *Three papers vixra*, `viXra:1408.0169, 1408.0173, 1408.0174, 1408.0201, 1408.0209, 1408.0212 [Number Theory], Note: vixra.org, not arxiv.org`. 1, 3

E-mail address: `matthias@mlesch.de`