An Introduction to *m(GR)* Theory

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Abstract: *Usually, physicists propose the alternative theories of gravity to the General Relativity (GR). In this paper we propose something different – m(GR) theory. This theory is the modification of GR theory, where the metric tensor* $g_{\mu\nu}$ *has been replaced by the effective mass tensor* $m_{\mu\nu}$. *The m(GR) theory is a new theory of gravitation, which describes the gravitational phenomena not as geometric properties of the space-time but as the dynamical properties of the bodies which, under the influence of the external gravitational field, behave like bodies with the effective mass.*

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Introduction

General Relativity (GR) is a theory which describes the gravitational phenomena as geometric properties of the space-time. Although GR is widely accepted as a fundamental theory of gravitation, for the many physicists still this is not a perfect theory. There have been many different attempts to constructing an ideal theory of gravity [1, 2]. Most of them are based on assumptions that were the starting point for considerations in the GR with various modifications.

In this paper we would like propose *m(GR) theory*, which is a modification of GR theory, where *the metric tensor* $g_{\mu\nu}$ has been replaced by *the effective mass tensor* (EMT) $m_{\mu\nu}$ [3].

The *m(GR)* theory is the postulated a new theory of gravitation, which describes the gravitational phenomena not as geometric properties of the space-time but as the dynamical properties of the bodies which, under the influence of the external gravitational field, behave like bodies with the effective mass $m_{\mu\nu}$.

Motivation

The concept of the effective mass of the body plays important role in the contemporary physics. The effective mass is well-known in the solid-state physics [4]. When an electron is moving inside a solid

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material, the force between other atoms will affect its movement and it will not be described according to Newton's second law. Therefore introduced the concept of the effective mass to describe the movement of the electron inside a solid material, according to Newton's equation of motion. Generally, in the absence of an electric or magnetic field, the concept of effective mass does not apply. In the solid-state physics a *three-dimensional effective mass tensor* is define as:

$$\frac{1}{m_{ij}} = \frac{d^2 E}{dp_i dp_j} \tag{1}$$

where: E is the energy, p is the momentum of the electron, components i and j are the spatial components (i, j = 1, 2, 3). For the classical kinetic energy and in the isotropic medium the effective mass tensor $m_{ij} = m^{bare}$, where: m^{bare} denotes *the bare mass*.

The concept of effective mass is a very attractive because the effective mass of the body in the equations of the motion includes full information about all fields (for example electromagnetic *etc.*) surrounding the body without their exact analysis. Effective mass can be isotropic or anisotropic, positive or negative. For the free body his effective mass is equal to the bare mass.

Postulates

The *m(GR)* theory based on the following postulates:

1. In absence of the outer (gravitational, *etc.*) fields the mass of the body is described by *the bare mass tensor* $m_{\mu\nu}^{bare}$, where: $m_{\mu\nu}^{bare} = m^{bare} \cdot \eta_{\mu\nu} = \mathrm{diag}(-m^{bare}, m^{bare}, m^{bare}, m^{bare})$, $\eta_{\mu\nu}$ is *the Minkowski tensor*, the space-time components: μ, ν = 0, 1, 2, 3.
2. Under the influence of the outer (gravitational, *etc.*) fields $m_{\mu\nu}^{bare}$ becomes $m_{\mu\nu}$, ($m_{\mu\nu}^{bare} \to m_{\mu\nu}$).
3. All of the outer (gravitational, *etc.*) fields are described by the by *the stress-energy tensor* $T_{\mu\nu}$, which determines the distribution of the matter and any surrounding fields (in the Universe).
4. Effective mass of the body is a result of the mutual (gravitational, *etc.*) interactions between the body and all others surrounding bodies (*Mach's Principle*).
5. The source of the effective mass is the stress-energy tensor $T_{\mu\nu}$.
6. EMT $m_{\mu\nu}$ includes full information about all (gravitational, *etc.*) fields surrounding the body without their exact analysis.
7. In the weak (gravitational, *etc.*) fields we can decompose EMT $m_{\mu\nu}$ to the very simple form: $m_{\mu\nu} = m_{\mu\nu}^{bare} + \overset{*}{m}_{\mu\nu}$, where: $\overset{*}{m}_{\mu\nu} << m_{\mu\nu}^{bare}$ is *the small perturbation* of the effective mass.
8. There exists relation between the metric tensor $g_{\mu\nu}$ and EMT $m_{\mu\nu}$ in the form $g_{\mu\nu} = \frac{m_{\mu\nu}}{m^{bare}}$ and the metric $ds^2(g_{\mu\nu}) = ds^2(m_{\mu\nu})$, where: $ds^2(g_{\mu\nu}) = g_{\mu\nu} dx^\mu dx^\nu$, $ds^2(m_{\mu\nu}) = \frac{m_{\mu\nu}}{m^{bare}} dx^\mu dx^\nu$.
9. In the absence of the all (gravitational, *etc.*) fields $m_{\mu\nu} \to m_{\mu\nu}^{bare}$ and the metric $ds^2(m_{\mu\nu}) = \frac{m_{\mu\nu}}{m^{bare}} dx^\mu dx^\nu$ becomes the Minkowski metric $ds^2(m_{\mu\nu}^{bare}) = \eta_{\mu\nu} dx^\mu dx^\nu$.

Because at the moment there is no field equation for EMT, all consideration are realized in GR framework, where the metric tensor $g_{\mu\nu}$ has been replaced by EMT $m_{\mu\nu}$.

What we expect?

A. Einstein was postulated that the gravitational forces have be expressed by the curvature of a metric tensor $ds^2(g_{\mu\nu}) = g_{\mu\nu}dx^\mu dx^\nu$ on a four-dimension space-time manifold with signature (-, +, +, +). He also postulated that space-time is curved and that its curvature is locally determined by the matter stress-energy tensor, which determines the distribution of the matter.

The relationship in the form $ds^2(g_{\mu\nu}) = ds^2(m_{\mu\nu})$, postulated in this paper, opens up a new possibilities for the study of the gravitational phenomena not as geometric properties of the space-time, but as the dynamical properties of the bodies which, under the influence of the external gravitational fields, behave like bodies with the effective mass.

Under the influence of the outer (gravitational, *etc.*) fields $m_{\mu\nu}^{bare}$ becomes the EMT $m_{\mu\nu}$. All of the outer (gravitational, *etc.*) fields are described by the by *the stress-energy tensor* $T_{\mu\nu}$, which is the source of the effective mass. The metric $ds^2(m_{\mu\nu}) = \frac{m_{\mu\nu}}{m^{bare}} dx^\mu dx^\nu$ determines the causal structure of the distribution of the matter in the space-time as well as its metric relations (see to section **Clocks and rods**).

We expect that the *m(GR)* theory should satisfy the classical tests of GR also, but their physical interpretation and consequences should be different than in GR.

The equation of motion

Let us consider the Lagrangian function

$$L = \frac{1}{2} m_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} \tag{2}$$

The equation of motion have the form

$$\frac{dp_\beta}{d\tau} + \Gamma^*_{\beta\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0 \tag{3}$$

where: $p_\beta = m_{\beta\nu} \frac{dx^\nu}{d\tau}$ is *the four-momentum*, $\Gamma^*_{\beta\mu\nu} = \frac{1}{2}\left(\frac{\partial m_{\beta\nu}}{\partial x^\mu} + \frac{\partial m_{\beta\mu}}{\partial x^\nu} - \frac{\partial m_{\mu\nu}}{\partial x^\beta} \right)$. Please note that $\Gamma^*_{\beta\mu\nu}$ has the dimension [kg/m]. The same dimension has *the linear density*.

The Riemann tensor, Ricci tensor and the Ricci scalar expressed by EMT

According to the Postulate 8, *the Christoffel symbols of the first kind* we can now express by the EMT $m_{\mu\nu}$

$$\Gamma_{\beta\mu\nu} = \frac{1}{2m^{bare}} \left(\frac{\partial m_{\beta\nu}}{\partial x^\mu} + \frac{\partial m_{\beta\mu}}{\partial x^\nu} - \frac{\partial m_{\mu\nu}}{\partial x^\beta} \right) \tag{4}$$

The Riemann tensor has the form

$$R_{\kappa\lambda\mu\nu} = \frac{\partial\Gamma_{\mu\nu\lambda}}{\partial x^{\kappa}} - \frac{\partial\Gamma_{\mu\nu\kappa}}{\partial x^{\lambda}} + \Gamma^{\pi}_{\mu\lambda}\Gamma_{\pi\nu\kappa} - \Gamma^{\pi}_{\mu\kappa}\Gamma_{\pi\nu\lambda} \quad (5)$$

where: $\Gamma^{\pi}_{\mu\nu}$ is *the Christoffel symbol of the second kind* and $\Gamma^{\pi}_{\mu\nu} = m^{bare} m^{\pi\lambda}\Gamma_{\lambda\mu\nu}$. Please note that the Riemann tensor has the dimension [1/m^2] and is expressed by the second derivatives of the EMT $m_{\mu\nu}$. In the absence of the all (gravitational, *etc.*) fields $m_{\mu\nu} \to m^{bare}_{\mu\nu}$ and $R_{\kappa\lambda\mu\nu} = 0$, which means that the distribution of mass as a homogeneous, isotropic and independent of the time.

From the equation (5) we can calculate *the Ricci tensor* according to the formula

$$R_{\kappa\mu} = m^{bare} m^{\lambda\nu} R_{\kappa\lambda\mu\nu} \quad (6)$$

The Ricci scalar we can calculate according to the formula

$$R = m^{bare} m^{\kappa\mu} R_{\kappa\mu} \quad (7)$$

Modified Einstein field equations

Because at the moment there is no field equation for EMT, we use *the Einstein field equation* in the altered form

$$\tau^{eff}_{\mu\nu} - \frac{1}{2}\frac{m_{\mu\nu}}{m^{bare}}\tau = 8\pi \cdot T_{\mu\nu} \quad (8)$$

where: $\tau^{eff}_{\mu\nu} = \frac{c^4}{G} R_{\mu\nu}$ we will call *the effective energy tensor*, which should include full information regarding the existence of any surrounding fields, G is the Newton's gravitational constant, c is the speed of light in vacuum, $\frac{c^4}{G}$ is *the Planck force*, $\tau = \frac{c^4}{G} R$ is *the bare energy*.

How to interpreted the modified Einstein field equation?

Under the influence of the outer (gravitational, *etc.*) fields $m^{bare}_{\mu\nu} \to m_{\mu\nu}$. The stress-energy tensor $T_{\mu\nu}$, which determines the distribution of the matter (in the Universe), acts as the source of this conversion *i.e.* $m^{bare}_{\mu\nu} \xrightarrow{T_{\mu\nu}} m_{\mu\nu}$. When $T_{\mu\nu} \to 0$ then $\tau^{eff}_{\mu\nu} \to \tau^{bare}_{\mu\nu}$ and $m_{\mu\nu} \to m^{bare}_{\mu\nu}$.

Solution for the Schwarzschild vacuum

In *the Schwarzschild vacuum* the modified field equation take the form

$$\tau^{eff}_{\mu\nu} - \frac{1}{2}\frac{m_{\mu\nu}}{m^{bare}}\tau = 0 \quad (9)$$

The modified Schwarzschild's solution for the empty space-time around a spherical body with the effective mass m_{00}^{*} is

$$c^2 d\tau^2 = -\left(1 - \frac{m_{00}^{*}}{m^{bare}}\right) c^2 dt^2 + \left(1 - \frac{m_{00}^{*}}{m^{bare}}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2 \tag{10}$$

Clocks and rods

In the static and the spherically symmetric gravitational field all clocks and rods have the effective mass m_{00}^{*}. When there is no gravitational field all clocks and rods have the bare masses m^{bare}. All clocks with the effective mass m_{00}^{*} runs different than the clocks with the bare mass m^{bare}

$$d\tau = dt\sqrt{1 - \frac{m_{00}^{*}}{m^{bare}}} \tag{11}$$

where: τ is the proper time, t is the time coordinate. If we assume that $m_{00}^{*} \uparrow$ then clocks with the effective mass will measure different time than the clocks with the bare mass and $d\tau < dt$. But if we assume that $m_{00}^{*} \to 0$ then clocks with the effective mass and the clocks with the bare mass will measure the same time and $d\tau = dt$.

Rods with effective mass m_{00}^{*} will show a radial distance different than the rods with the bare mass

$$dR = \frac{dr}{\sqrt{1 - \frac{m_{00}^{*}}{m^{bare}}}} \tag{12}$$

where r is the radial distance. If we assume that $m_{00}^{*} \uparrow$ then rods with the effective mass will measure different radial distance than the rods with the bare mass and $dR > dr$. But if we assume that $m_{00}^{*} \to 0$ then rods with the effective mass and the rods with the bare mass will measure the same radial distance $dR = dr$. This is a consequence of the apply the metric $ds^2(m_{\mu\nu}) = \frac{m_{\mu\nu}}{m} dx^{\mu} dx^{\nu}$ to describe the gravity, where clocks and rods have (the effective) mass.

Solution for the non-interacting dust in Newtonian limit

For the non-interacting dust the stress–energy tensor $T_{\mu\nu}$ takes a form

$$T_{\mu\nu} = \rho u_{\mu} u_{\nu} \tag{13}$$

where: ρ is *the mass-energy density*, u_{μ} is the fluid's *four velocity*. In the weak gravitational field and for the small velocity the field equation (8) takes a form

$$\tau_{00}^{eff} = 8\pi T_{00} \tag{14}$$

where: $\tau_{00}^{eff} = \frac{c^4}{G}\nabla^2\left(\frac{m_{00}^*}{m^{bare}}\right)$ and $T_{00} = \rho c^2$. The field equation (14) we can express in a slightly different form and we get *the modified Poisson equation* for the static gravitational field

$$\nabla^2\left(\frac{m_{00}^*}{m^{bare}}\right) = \frac{8\pi G}{c^2}\rho \tag{15}$$

Equation (15) satisfies Mach's Principle (Postulate No. 4) in the Newtonian limit. If (we assume that there) exists relation between m_{00}^* and *the gravitational potential* V in the form

$$\frac{m_{00}^*}{m^{bare}} = \frac{2V}{c^2} \tag{16}$$

then we get well-known *the Poisson equation* for the weak gravitational field

$$\nabla^2 V = 4\pi G\rho \tag{17}$$

For the relation (16) the modified Schwarzschild's solution (10) becomes the well-known *Schwarzschild's solution.*

Is there a mass anisotropy in *the Solar System*?

To test whether the concept of the EMT $m_{\mu\nu}$ correctly describes the gravity phenomena, we offer search the mass anisotropy in the Solar System.

The Schwarzschild EMT $m_{\mu\nu}^{Schwarzschild}$ in the matrix representation has the form

$$m_{\mu\nu}^{Schwarzschild} = m^{bare}\begin{bmatrix} -\left(1-\frac{2GM}{c^2r}\right) & 0 & 0 & 0 \\ 0 & \left(1-\frac{2GM}{c^2r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 sin^2\theta \end{bmatrix} \tag{18}$$

The time component of the effective Schwarzschild mass

$$m_{tt}^{Schwarzschild} = -m^{bare}\left(1-\frac{2GM}{c^2r}\right) \tag{19}$$

and *the radial component of the effective Schwarzschild mass*

$$m_{rr}^{Schwarzschild} = m^{bare}\left(1-\frac{2GM}{c^2r}\right)^{-1} \tag{20}$$

depends on the distance r between the planet and the star.

As we known in the Solar System the distance r between the Earth and the Sun is changed (the orbit is an ellipse). The changes in the mass anisotropy for the components m_{tt} and m_{rr} we should observe during a whole year, measuring from perihelion (or aphelion) to perihelion (or aphelion). We expect that the estimated values for the annual change for $\frac{|\Delta m_{tt}|}{m^{bare}}$ and for $\frac{|\Delta m_{rr}|}{m^{bare}}$ are equal to

$$\frac{|\Delta m_{tt}|}{m^{bare}} = \frac{1}{m^{bare}}\left|(m_{tt})_{perih} - (m_{tt})_{aphel}\right| = \frac{2GM}{c^2}\left|\frac{1}{r_{perih}} - \frac{1}{r_{aphel}}\right| \approx 6{,}6\cdot 10^{-10} \tag{21}$$

and

$$\frac{|\Delta m_{rr}|}{m^{bare}} = \frac{1}{m^{bare}}\left|(m_{rr})_{perih} - (m_{rr})_{aphel}\right| = \left|\frac{1}{1-\frac{2GM}{c^2 r_{perih}}} - \frac{1}{1-\frac{2GM}{c^2 r_{aphel}}}\right| \approx 6{,}6\cdot 10^{-10} \tag{22}$$

The change in mass during the annual motion of the Earth around the Sun can be measured by means of the ground-based atomic clocks [5].

In the section **Clocks and rods** we have shown that the clocks with the effective mass in a gravitational field measure time according to the formula (11). If we place one atomic clock in the perihelion and the second in aphelion then the estimated relative change of time is

$$\left|\frac{\Delta\tau_{perih} - \Delta\tau_{aphel}}{\Delta\tau_{aphel}}\right| \approx \frac{GM}{c^2}\left|\frac{1}{r_{perih}} - \frac{1}{r_{aphel}}\right| \approx 3{,}3\cdot 10^{-10} \tag{23}$$

where $\Delta\tau$ describes the proper time for the atomic clock in the perihelion (or aphelion).

Conclusion

In this paper we presented the *m(GR)* theory, which is the modification of GR theory, where the metric tensor $g_{\mu\nu}$ has been replaced by EMT $m_{\mu\nu}$. This theory describes the gravitational phenomena not as geometric properties of the space-time but as **the dynamical properties of the bodies** which, under the influence of the external gravitational field, behave like bodies with the effective mass $m_{\mu\nu}$.

Both theories GR and *m(GR)* satisfies the classical tests of GR – for example: *the perihelion shift*, *the deflection of light by the Sun* and *the gravitational redshift*, but the physical interpretation and consequences of the those phenomena in *m(GR)* theory is different than in GR.

The *m(GR)* theory predicts additionally the mass anisotropy, which we can measure using the atomic clocks. The estimated value for the relative change of the time for clocks located in the perihelion and aphelion amounts 3,3 x 10^{-10}.

Experimentally confirmed the concept of the effective mass in GR opens up a new possibilities for the study of the gravitational phenomena and may throw a new light on the mysteries of the our Universe.

The table below illustrates the difference in the physical interpretation for the both theories.

GR theory	***m(GR)* theory**
Geometry of space-time = Matter content	*Effective energy = Matter content*

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