The Size and Energy Loss of a Wave Packet

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Abstract: Combining formulas of quantum mechanics (QM) and the wave theory defines the diameter of a wave packet of circularly polarized light. This allows the calculation of the energy loss (red shift), if light traverses a thin plasma. In contrast to the Doppler effect, the total energy is conserved, also basic emission- and absorption-line physics is preserved.

Introduction

What is light? Depending on the experimental set-up visible light may be interpreted either as a wave phenomenon with interference phenomena or as a swarm of tiny photons that causes tiny hits on photosensitive material. Each of the two models can explain certain observations well, but fails in others. None can explain all observations consistent. Nevertheless, both describe the same physical phenomenon light. A combination of the formulas of QM and wave theory could be the key to a unified model.

Particularly interesting is the case, when low-energy photons are emitted from isolated, undisturbed atoms and can be received only as electromagnetic waves with dipole antennas. No photon detector responds to the [spectral line of hydrogen] at 1420 MHz. According to QM, the photon is circularly polarized and transports the energy 5.87433 µeV. The same must be true for the electromagnetic wave. The [half-width] of this HI line[1] is very short (only $\Delta f \approx 5\,kHz$) and from the formula $\Delta\lambda \cdot L \approx \lambda^2$ results a coherence length of about $L \approx 60\,km \approx 280 \cdot 10^3\,\lambda$. It is unclear, how a point-like photon is transformed into this remarkably long wave packet.

The receivers of radio astronomy are built on to the same principles as radios. The hardware is sensitive *only* to electromagnetic waves, nothing depends on QM. Where and how a photon converts into an electromagnetic wave?

Linking the formulas for momentum and energy density

Electromagnetic waves and photons have the (linear) pulse $\vec{p}$, whose direction coincides with the direction of propagation. Limiting to the amount of the pulse, in QM the formula

$$p = \frac{h}{\lambda} = \frac{h\,f}{c} \tag{1}$$

describes a single photon. This formula contains no indication for the volume of a photon, so far there are no corresponding measurement results. From the perspective of the electromagnetic waves is amazing that the QM makes no distinction between near and far field, ie ignoring the distance between the radiation source and observation. That may be the missing link.

For an [electromagnetic wave]

$$\vec{p} = \varepsilon_0 \int_V \vec{E} \times \vec{B}\, dV = \varepsilon_0 \mu_0 \int_V \vec{S}\, dV \tag{2}$$

holds, where $\vec{S}$ is the [Poynting vector]. In the near field, the relationship between $\vec{E}$ and $\vec{B}$ is

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location-dependent, difficult to describe, and completely different than in the far field. Only at large distances from the source, ie in the far field, $\vec{E}$ and $\vec{B}$ are proportional and always include a right angle, the momentum is pointing in the direction of propagation. Therefore, far away from the source, the formula can be simplified

$$p=\frac{\varepsilon_0}{c}\int_V E^2 dV \tag{3}$$

A wave packet carries exactly the amount of energy, which is calculated by QM

$$h f=\varepsilon_0\int_V E^2 dV \tag{4}$$

A calculation using the energy density $u=\varepsilon_0 E^2$ has the same result. A key element of this formula is the spatial extent and the volume of the wave packet. At this point it should be noted that the conventional representation of a wave is very problematic: In case of a linear polarized wave, the amplitude is usually described with a formula like

$$A=A_0\sin\left(2\pi f\left(\frac{x}{c}-t\right)+\varphi\right) \tag{5}$$

without limiting the allowable values of x and t. While this is mathematically convenient and simplifies the work, it describes an infinitely extended wave with infinite energy content. Thus, it can not describe the emission of a defined amount of energy in atomic processes. Only if the integration volume is limited, the electric field strength E can be measured and is not infinitesimally small. For this reason, a wave packet of *finite* extension is always assumed.

Linking the formulas for the angular momentum

Since the experiments of Beth[2] it has been known, that a circularly polarized plane wave of light does carry angular momentum. In QM, each photon has the spin

$$s=h/2\pi \tag{6}$$

that is oriented parallel to the direction of propagation. The angular momentum of the electromagnetic field is calculated by integration over the total volume of the wave packet

$$\vec{J}_{Classic}=\varepsilon_0\int_V \vec{r}\times(\vec{E}\times\vec{B})\,dV=\varepsilon_0\mu_0\int_V \vec{r}\times\vec{S}\,dV \tag{7}$$

The use of the Poynting vector $\vec{S}$ simplifies this formula a little bit. It should be noted that the spin of the photon and the classical angular momentum are related by the formula

$$J_{Classic}=\sqrt{2}\cdot s \tag{8}$$

In the far field, the wave packet of a circularly polarized electromagnetic wave is rotationally symmetric and a *finite* sequence of planar, circular wave fronts of *finite* extent transverse to the wave vector. The local area power density $S=\varepsilon_0 c E^2$ may have a different value at any point of the wave packet.

A cylindrical wave packet

Since a circularly polarized wave can not be a spherical wave, the wave packet must have a preferred direction of propagation. This is identical to the photons's direction of flight. Each wave packet has a limited length, but the course of the intervening envelope is unknown. Common but unproven assumptions are bell-shaped and exponential decay of the amplitude. Hereinafter, it is assumed the wave packet is a cylinder of radius *R* and length *L*, whose axis of symmetry coincides with the direction of propagation. The two integrals for momentum and total energy and angular momentum of the wave must have finite values. The intensity of the wave has the constant value *S* everywhere in the interior of the cylinder and is zero outside. The calculation with cylindrical coordinates is very simple and leads to

$$J=\varepsilon_0\mu_0 S\int_0^L\int_0^R\int_0^{2\pi} r^2\,dz\,dr\,d\varphi=\frac{2\pi}{3}\varepsilon_0\mu_0 S\,L\,R^3=\frac{2\pi\,\varepsilon_0 E^2 L\,R^3}{3c} \tag{9}$$

Since QM and electromagnetic waves describe the same physical phenomenon, one can equate the formulas for the angular momentum, which leads to

$$3\sqrt{2}\,h\,c=4\pi^2\varepsilon_0 E^2 L\,R^3 \tag{10}$$

From the formulas of the energy follows

$$h\,f=\varepsilon_0\int_V E^2\,dV=\varepsilon_0 E^2\pi R^2 L \tag{11}$$

The equations (10) and (11) are conserved quantities of a cylindrical wave packet, from which one can calculate the radius[3]

$$R=\frac{3\sqrt{2}\,c}{4\pi f}=\frac{3\sqrt{2}\,\lambda}{4\pi} \tag{12}$$

Usual receiving antennas confirm this value. λ/2 dipoles correspond approximately to the calculated diameter of a wave packet. Shortened dipoles cover only a fraction of the cross-sectional area of the cylinder and withdraw less energy from the wave packet. Extended dipoles hardly bring additional gain, because the parts far outside radiate more energy than they absorb. The diameter of the wave packet can also be an indication of the minimum diameter of photo diodes.

The formulas contain no indication of the length of the wave packet, which is why the coherence length *L* is used. Depending on the line width of the spectral line, *L* assumes values from the range $10^3\cdot\lambda<L<10^8\cdot\lambda$. Thus, the energy density

$$u=\frac{8\pi h f^3}{9c^2 L} \tag{13}$$

and the electric field strength

$$E^2=\frac{8\pi h\,\mu_0 f^3}{9L} \tag{14}$$

of the cylindrical wave packet are calculated. The value of the coherence length effects none of the subsequent results and may be selected as desired. The natural line width of optical spectral lines is very low when surrounding atoms do not interfere.

Examples: The natural line width of the sodium D-line is about 10 MHz. Each wave packet contains about 10^7 oscillation periods, is 6 m long and carries the energy *hf*. At a wavelength of 600 nm, the wave packet is a very thin cylinder with 400 nm diameter and 6 m in length. The electric field strength inside the wave packet is about 226 V / m.

The HI-line of hydrogen at 1420 MHz has a half width of about 5 kHz. The distance between the hydrogen atoms in the gas cloud is so large that independent wave packets are generated. The formula $\Delta\lambda \cdot L \approx \lambda^2$ gives a coherence length of 60 km and a volume of 960 m³. The energy density is very low and the electric field strength is only 10^{-8} V / m. Under optimal conditions, each wave packet induces a voltage of 10^{-9} V in a λ / 2 dipole. This is below the detection limit of radio telescopes, so a signal can only be registered when the amplitudes of several wave packets add up. This is obviously not a problem, given the large coherence length.

In the X-ray range the wavelength is at 1 pm, the coherence length shrinks to $L \approx 1000\,\lambda$ and the electric field strength increases to about 10^{16} V / m. This means that all atoms can be ionized.

The red shift at low energy loss

A right-circularly polarized wave has the two components

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = E_0 \begin{pmatrix} -\sin(\omega t) \\ \cos(\omega t) \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = B_0 \begin{pmatrix} -\cos(\omega t) \\ -\sin(\omega t) \\ 0 \end{pmatrix} \tag{15}$$

The Poynting vector

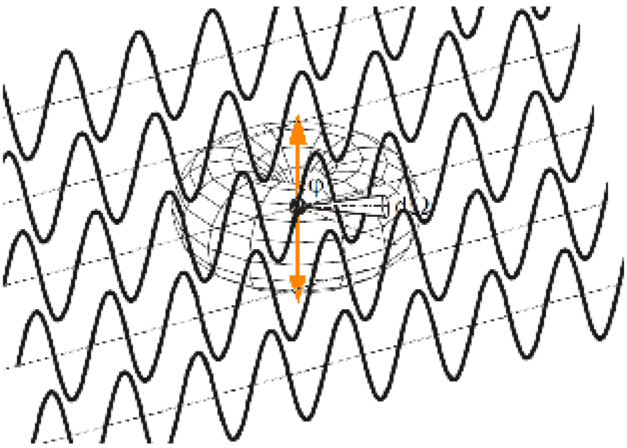
$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = \frac{E_0 B_0}{\mu_0} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{16}$$

is constant, pointing in the direction of wave propagation and has the value

$$S = \frac{8 \pi h f^3}{9 c L} \tag{17}$$

L is the length of the wave packet. Because the wave packet carries finite energy, the field strength and the magnetic field are zero outside the period $0 \le t \le L/c$.

On the way from a distant gas cloud to Earth, the light passes through the interstellar medium (ISM), a "warm ionized gas" temperature of 8000 K, about 20% to 50% of all atoms are ionized. With the help of radio astronomy, the mean density of free electrons of $4 \cdot 10^5$ per m³ was calculated, resulting in an average distance of electrons of d ≈ 14 mm[4] [5]. In the intergalactic medium (IGM), the average distance is estimated to be much longer.



The electric field of a vertical polarized wave packet accelerates an electron, which radiates energy like a dipole antenna.

Each unbound electron that is "touched" by the wave packet, absorbs a tiny amount of energy

$$e_1=\sigma_{Elektron}\int_0^{L/c} S\,dt=\sigma_{Elektron}\frac{8\pi h f^3}{9c^2} \tag{18}$$

where $\sigma_{Elektron}=6.65\cdot 10^{-29} m^2$ is the classical scattering cross section of the electron. An free electron is structureless and is not bound to a potential well. Therefore, it can not store energy and radiates symmetrically, as shown in the picture.

Below it is shown that the tiny energy loss reduces the frequency of the wave packet. At a frequency of 1420 MHz, no electron can absorb all the energy of the wave packet because the apparent cross-section much, much smaller than the cross-sectional area of the wave packet (0.016 m^2). For this reason, the electron can not scatter the wave packet and change its direction. Only in X-rays at much higher frequencies, the areas are comparable and Compton scattering may occur.

Each unbound electron takes away a tiny fraction e_1 from the the wave packets total energy *hf*. Planck's theory describes the smallest possible change of energy of a physical system that can oscillate harmoniously. Only then applies $W_{min}=hf$. An unbound electron experiences no restoring force into a rest position and is therefore not an oscillator. Hence an unbound electron can take away an arbitrarily small amount of energy from the wave packet and emit it immediately. If the wave packet comes from the distance *D*, traversing a region with the average electron density n_e, it affects

$$N=n_e\left(\frac{3\sqrt{2}c}{4\pi f}\right)^2\pi D \tag{19}$$

electrons. These electrons reduce the energy of the wave packet by the energy

$$A=N\,e_1=n_e\left(\frac{3\sqrt{2}c}{4\pi f}\right)^2\pi D\sigma_{Elektron}\frac{8\pi h f^3}{9c^2}=n_e\sigma_{Elektron}h\,f\,D \tag{20}$$

Because every electron radiates symmetrically, as shown in the picture for a vertically polarized electromagnetic wave, the wave packet undergoes no transverse momentum, its direction is not changed.

Reduced electric field strength or reduced frequency?

For small distances, the condition $A\ll hf$ is satisfied and it can be assumed that the diameter of the wave packet is constant. *Before* the wave packet touches the free electrons, the above derived formulas

$$3\sqrt{2}hc=4\pi^2\varepsilon_0 E_1^2 L_1 R_1^3 \quad\text{and}\quad h f_1=\varepsilon_0 E_1^2\pi L_1 R_1^2 \tag{21}$$

apply. *After* the wave packet has passed all the electrons and lost some energy, the formulas

$$3\sqrt{2}hc=4\pi^2\varepsilon_0 E_2^2 L_2 R_2^3 \quad\text{and}\quad h f_2=\varepsilon_0 E_2^2\pi L_2 R_2^2 \tag{22}$$

apply. These four equations must be satisfied to conserve energy and angular momentum of the wave packet.

- From the classical point of view, any energy loss is equivalent to a reduction of the amplitude and the electric field strength of the wave packet. The frequency should be constant.
- As QM does not deal with the electric field strength of the wave packet, any energy loss is equivalent to a reduction of the frequency.

QM and classical wave theory contradict each other. What is the solution, if the formulas of both areas are linked?

From the assumption of a constant frequency $f_1 = f_2$ follows $R_1 = R_2$ and $E_1^2 L_1 = E_2^2 L_2$. Then neither energy nor angular momentum of the wave packet would change, a contradiction to the previously calculated energy loss. That would mean that Thomson scattering does not exist - incompatible with the experimental experience.
If a frequency change is *not* excluded, the equations (21) and (22) have the solution

$$\frac{3\sqrt{2}\, c\, h(R_2 - R_1)}{4\pi R_2 R_1} = h(f_1 - f_2) \qquad (23)$$

The frequency decreases, the radius of the wave packet increases. With low energy loss, the relative frequency shift has the value

$$\frac{\Delta f}{f} = z = n_e \sigma_{Elektron} D \quad . \qquad (24)$$

This formula satisfies some astronomical observations:

- z does not depend on frequency or wavelength. Hence, the wavelength ratios in the spectra are conserved.
- z does not depend on the coherence length or intensity of the wave packet
- at small distances ($\sigma_{Elektron} n_e D \ll 1$), z is proportional to the distance *D*
- the decribed effect gives systematic redshifts but not blueshifts.

In contrast to the Doppler effect, the total energy is conserved. Basic emission- and absorption-line physics is preserved.

The comparison of the above approximation $z = n_e \sigma_{Elektron} D$ with the Hubble formula $c\,z = H_0 D$ yields

$$c\,\sigma_{Elektron} n_e = H_0 \qquad (25)$$

Using the currently accepted value of the Hubble constant $H_0 \approx 70 \frac{km}{s \cdot Mpc}$, the average density of unbound electrons in vicinity of our galaxy is

$$n_e = \frac{H_0}{c\,\sigma_{Elektron}} \approx \frac{114}{m^3} \qquad (26)$$

The mean density of free electrons in our own galaxy is $4 \cdot 10^5$ per m^3 and was derived from existing radio and optical observations of H II regions[4].

The calculation of the energy loss in the comparatively much larger Fresnel zones[6] leads to a similar result. Because of the lower energy density in this huge volume, the calculated factor

$w = \frac{3 q_e^4 \mu_0^2}{512 \pi m_e^2} = 2.33 \cdot 10^{-30} m^2$ is smaller than $\sigma_{Elektron} = 6.65 \cdot 10^{-29} m^2$ and may be neglected.

The redshift at large energy loss

If the distance or the density of free electrons is very large, the total energy loss increases and the condition $V \ll hf$ is not met, the radius of the wave packet increases markedly with decreasing frequency and the above assumption of a long cylinder with constant diameter is no longer acceptable. To calculate the total energy loss, the overall Distance *D* is divided into successive short cylinders of length $dx = \frac{\Delta f}{f\, n_e\, \sigma_{Elektron}}$. Integrating both sides and using the definition $f_{observed}(1+z) = f_{emit}$, one obtains

$$D = \frac{\ln(z+1)}{n_e\, \sigma_{Elektron}} \quad . \tag{27}$$

This non-linear equation is valid for very large values of the redshift. The assumed Hubble linear relationship between redshift and distance is only an approximation for $z \approx 0$.

[1] http://www.astro.washington.edu/users/jd/A541/Lectures/lec13_HI_lines.pdf
[2] Beth R A 1936 Mechanical detection and measurement of the angular momentum of light Phys. Rev. 50 115-25
[3] Dong-Lin Zu, The Classical Structure Model of Single Photon and Classical Point of View with Regard to Wave-Particle Duality of Photon, http://www.jpier.org/PIERL/pierl01/14.07111101.pdf
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