Conjectured Primality and Compositeness Tests for Numbers of Special Forms

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Abstract: Conjectured polynomial time primality and compositeness tests for numbers of special forms are introduced.

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1 Introduction

In number theory the Riesel primality test [1], is the fastest deterministic primality test for numbers of the form $k \cdot 2^n - 1$ with $k$ odd and $k < 2^n$. The test was developed by Hans Riesel and it is based on Lucas-Lehmer test [2]. In 1960 Kusta Inkeri provided unconditional, deterministic, lucasian type primality test for Fermat numbers [3]. In 2008 Ray Melham provided unconditional, probabilistic, lucasian type primality test for generalized Mersenne numbers [4]. In 2010 Pedro Berrizbeitia, Florian Luca and Ray Melham provided polynomial time compositeness test for numbers of the form $(2^n + 1)/3$, see Theorem 2 in [5]. In this note I present lucasian type primality and compositeness tests for numbers of special forms.

2 The Main Result

Definition 2.1. Let $P_m(x) = 2^{-m} \cdot \left( (x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where $m$ and $x$ are positive integers.

Conjecture 2.1. Let $N = k \cdot 2^n - 1$ such that $n > 2$, $3 \mid k$, $k < 2^n$ and

\[
\begin{align*}
\begin{cases}
  k \equiv 1 \pmod{10} & \text{with } n \equiv 2, 3 \pmod{4} \\
  k \equiv 3 \pmod{10} & \text{with } n \equiv 0, 3 \pmod{4} \\
  k \equiv 7 \pmod{10} & \text{with } n \equiv 1, 2 \pmod{4} \\
  k \equiv 9 \pmod{10} & \text{with } n \equiv 0, 1 \pmod{4}
\end{cases}
\end{align*}
\]
Let \( S_i = P_2(S_{i-1}) \) with \( S_0 = P_k(3) \), thus

\( N \) is prime iff \( S_{n-2} \equiv 0 \pmod{N} \)

**Conjecture 2.2.** Let \( N = k \cdot 2^n - 1 \) such that \( n > 2, 3 \mid k, k < 2^n \) and

\[
\begin{align*}
    &k \equiv 3 \pmod{42} \text{ with } n \equiv 0, 2 \pmod{3} \\
    &k \equiv 9 \pmod{42} \text{ with } n \equiv 0 \pmod{3} \\
    &k \equiv 15 \pmod{42} \text{ with } n \equiv 1 \pmod{3} \\
    &k \equiv 27 \pmod{42} \text{ with } n \equiv 1, 2 \pmod{3} \\
    &k \equiv 33 \pmod{42} \text{ with } n \equiv 0, 1 \pmod{3} \\
    &k \equiv 39 \pmod{42} \text{ with } n \equiv 2 \pmod{3}
\end{align*}
\]

Let \( S_i = P_2(S_{i-1}) \) with \( S_0 = P_k(5) \), thus

\( N \) is prime iff \( S_{n-2} \equiv 0 \pmod{N} \)

**Conjecture 2.3.** Let \( N = k \cdot 2^n + 1 \) such that \( n > 2, k < 2^n \) and

\[
\begin{align*}
    &k \equiv 5, 19 \pmod{42} \text{ with } n \equiv 0 \pmod{3} \\
    &k \equiv 13, 41 \pmod{42} \text{ with } n \equiv 1 \pmod{3} \\
    &k \equiv 17, 31 \pmod{42} \text{ with } n \equiv 2 \pmod{3} \\
    &k \equiv 23, 37 \pmod{42} \text{ with } n \equiv 0, 1 \pmod{3} \\
    &k \equiv 11, 25 \pmod{42} \text{ with } n \equiv 0, 2 \pmod{3} \\
    &k \equiv 1, 29 \pmod{42} \text{ with } n \equiv 1, 2 \pmod{3}
\end{align*}
\]

Let \( S_i = P_2(S_{i-1}) \) with \( S_0 = P_k(5) \), thus

\( N \) is prime iff \( S_{n-2} \equiv 0 \pmod{N} \)

**Conjecture 2.4.** Let \( N = k \cdot 2^n + 1 \) such that \( n > 2, k < 2^n \) and

\[
\begin{align*}
    &k \equiv 1 \pmod{6} \text{ and } k \equiv 1, 7 \pmod{10} \text{ with } n \equiv 0 \pmod{4} \\
    &k \equiv 5 \pmod{6} \text{ and } k \equiv 1, 3 \pmod{10} \text{ with } n \equiv 1 \pmod{4} \\
    &k \equiv 1 \pmod{6} \text{ and } k \equiv 3, 9 \pmod{10} \text{ with } n \equiv 2 \pmod{4} \\
    &k \equiv 5 \pmod{6} \text{ and } k \equiv 7, 9 \pmod{10} \text{ with } n \equiv 3 \pmod{4}
\end{align*}
\]

Let \( S_i = P_2(S_{i-1}) \) with \( S_0 = P_k(8) \), thus

\( N \) is prime iff \( S_{n-2} \equiv 0 \pmod{N} \)

**Conjecture 2.5.** Let \( N = 3 \cdot 2^n + 1 \) such that \( n > 2 \) and \( n \equiv 1, 2 \pmod{4} \)

\[
\begin{align*}
    &\text{Let } S_i = P_2(S_{i-1}) \text{ with } \\
    &S_0 = \begin{cases} 
        P_3(32), & \text{if } n \equiv 1 \pmod{4} \\
        P_3(28), & \text{if } n \equiv 2 \pmod{4}
    \end{cases}
\end{align*}
\]

thus

\( N \) is prime iff \( S_{n-2} \equiv 0 \pmod{N} \)
Conjecture 2.6. Let $N = 5 \cdot 2^n + 1$ such that $n > 2$ and $n \equiv 1, 3 \pmod{4}$

Let $S_i = P_2(S_{i-1})$ with

\[
S_0 = \begin{cases} 
  P_5(28), & \text{if } n \equiv 1 \pmod{4} \\
  P_5(32), & \text{if } n \equiv 3 \pmod{4}
\end{cases}
\]

thus

$N$ is prime iff $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 2.7. Let $N = 7 \cdot 2^n + 1$ such that $n > 2$ and $n \equiv 0, 2 \pmod{4}$

Let $S_i = P_2(S_{i-1})$ with

\[
S_0 = \begin{cases} 
  P_7(8), & \text{if } n \equiv 0 \pmod{4} \\
  P_7(32), & \text{if } n \equiv 2 \pmod{4}
\end{cases}
\]

thus

$N$ is prime iff $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 2.8. Let $N = 9 \cdot 2^n + 1$ such that $n > 2$ and $n \equiv 2, 3 \pmod{4}$

Let $S_i = P_2(S_{i-1})$ with

\[
S_0 = \begin{cases} 
  P_9(28), & \text{if } n \equiv 2 \pmod{4} \\
  P_9(32), & \text{if } n \equiv 3 \pmod{4}
\end{cases}
\]

thus

$N$ is prime iff $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 2.9. Let $N = 11 \cdot 2^n + 1$ such that $n > 2$ and $n \equiv 1, 3 \pmod{4}$

Let $S_i = P_2(S_{i-1})$ with

\[
S_0 = \begin{cases} 
  P_{11}(8), & \text{if } n \equiv 1 \pmod{4} \\
  P_{11}(28), & \text{if } n \equiv 3 \pmod{4}
\end{cases}
\]

thus

$N$ is prime iff $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 2.10. Let $N = 13 \cdot 2^n + 1$ such that $n > 2$ and $n \equiv 0, 2 \pmod{4}$

Let $S_i = P_2(S_{i-1})$ with

\[
S_0 = \begin{cases} 
  P_{13}(32), & \text{if } n \equiv 0 \pmod{4} \\
  P_{13}(8), & \text{if } n \equiv 2 \pmod{4}
\end{cases}
\]

thus

$N$ is prime iff $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 2.11. Let $F = 2^{2^n} + 1$ such that $n \geq 2$. Let $S_i = P_4(S_{i-1})$ with $S_0 = 8$, thus

$F$ is prime iff $S_{2^{n-1}-1} \equiv 0 \pmod{F}$
Conjecture 2.12. Let \( N = k \cdot 6^n - 1 \) such that \( n > 2, k > 0, k \equiv 3, 9 \pmod{10} \) and \( k < 6^n \)

Let \( S_i = P_6(S_{i-1}) \) with \( S_0 = P_{3k}(P_3(3)) \), thus

\( N \) is prime iff \( S_{n-2} \equiv 0 \pmod{N} \)

Conjecture 2.13. Let \( N = k \cdot 6^n - 1 \) such that \( n > 2, k > 0, k \equiv 5 \pmod{42} \) and \( k < 6^n \)

Let \( S_i = P_6(S_{i-1}) \) with \( S_0 = P_{3k}(P_3(5)) \), thus

\( N \) is prime iff \( S_{n-2} \equiv 0 \pmod{N} \)

Conjecture 2.14. Let \( N = k \cdot b^n - 1 \) such that \( n > 2, k \text{ is odd}, 3 \nmid k, b \text{ is even}, 3 \nmid b, 5 \nmid b \), \( k < b^n \).

Let \( S_i = P_b(S_{i-1}) \) with \( S_0 = P_{bk/2}(P_{b/2}(4)) \), thus

\( N \) is prime iff \( S_{n-2} \equiv 0 \pmod{N} \)

Conjecture 2.15. Let \( N = k \cdot b^n - 1 \) such that \( n > 2, k < b^n \) and

\[
\begin{align*}
&k \equiv 3 \pmod{30} \text{ with } b \equiv 2 \pmod{10} \text{ and } n \equiv 0, 3 \pmod{4} \\
&k \equiv 3 \pmod{30} \text{ with } b \equiv 4 \pmod{10} \text{ and } n \equiv 0, 2 \pmod{4} \\
&k \equiv 3 \pmod{30} \text{ with } b \equiv 6 \pmod{10} \text{ and } n \equiv 0, 1, 2, 3 \pmod{4} \\
&k \equiv 3 \pmod{30} \text{ with } b \equiv 8 \pmod{10} \text{ and } n \equiv 0, 1 \pmod{4}
\end{align*}
\]

Let \( S_i = P_b(S_{i-1}) \) with \( S_0 = P_{bk/2}(P_{b/2}(5778)) \), thus

\( N \) is prime iff \( S_{n-2} \equiv 0 \pmod{N} \)

Conjecture 2.16. Let \( N = k \cdot b^n - 1 \) such that \( n > 2, k < b^n \) and

\[
\begin{align*}
&k \equiv 9 \pmod{30} \text{ with } b \equiv 2 \pmod{10} \text{ and } n \equiv 0, 1 \pmod{4} \\
&k \equiv 9 \pmod{30} \text{ with } b \equiv 4 \pmod{10} \text{ and } n \equiv 0, 2 \pmod{4} \\
&k \equiv 9 \pmod{30} \text{ with } b \equiv 6 \pmod{10} \text{ and } n \equiv 0, 1, 2, 3 \pmod{4} \\
&k \equiv 9 \pmod{30} \text{ with } b \equiv 8 \pmod{10} \text{ and } n \equiv 0, 3 \pmod{4}
\end{align*}
\]

Let \( S_i = P_b(S_{i-1}) \) with \( S_0 = P_{bk/2}(P_{b/2}(5778)) \), thus

\( N \) is prime iff \( S_{n-2} \equiv 0 \pmod{N} \)

Conjecture 2.17. Let \( N = k \cdot b^n - 1 \) such that \( n > 2, k < b^n \) and

\[
\begin{align*}
&k \equiv 21 \pmod{30} \text{ with } b \equiv 2 \pmod{10} \text{ and } n \equiv 2, 3 \pmod{4} \\
&k \equiv 21 \pmod{30} \text{ with } b \equiv 4 \pmod{10} \text{ and } n \equiv 1, 3 \pmod{4} \\
&k \equiv 21 \pmod{30} \text{ with } b \equiv 8 \pmod{10} \text{ and } n \equiv 1, 2 \pmod{4}
\end{align*}
\]

Let \( S_i = P_b(S_{i-1}) \) with \( S_0 = P_{bk/2}(P_{b/2}(3)) \), thus

\( N \) is prime iff \( S_{n-2} \equiv 0 \pmod{N} \)

Conjecture 2.18. Let \( F_n(b) = b^{2^n} + 1 \) such that \( n > 1, b \text{ is even}, 3 \nmid b \) and \( 5 \nmid b \).
Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(P_{b/2}(8))$, thus $F_n(b)$ is prime iff $S_{2n-2} \equiv 0 \pmod{F_n(b)}$

**Conjecture 2.19.** Let $N = k \cdot 3^n - 2$ such that $n \equiv 0 \pmod{2}$, $n > 2$, $k \equiv 1 \pmod{4}$ and $k < 3^n$.

Let $S_i = P_3(S_{i-1})$ with $S_0 = P_{3k}(4)$, thus

If $N$ is prime then $S_{n-1} \equiv P_1(4) \pmod{N}$

**Conjecture 2.20.** Let $N = k \cdot 3^n - 2$ such that $n \equiv 1 \pmod{2}$, $n > 2$, $k \equiv 1 \pmod{4}$ and $k < 3^n$.

Let $S_i = P_3(S_{i-1})$ with $S_0 = P_{3k}(4)$, thus

If $N$ is prime then $S_{n-1} \equiv P_3(4) \pmod{N}$

**Conjecture 2.21.** Let $N = k \cdot 3^n + 2$ such that $n > 2$, $k \equiv 1, 3 \pmod{8}$ and $k < 3^n$.

Let $S_i = P_3(S_{i-1})$ with $S_0 = P_{3k}(6)$, thus

If $N$ is prime then $S_{n-1} \equiv P_3(6) \pmod{N}$

**Conjecture 2.22.** Let $N = k \cdot 3^n + 2$ such that $n > 2$, $k \equiv 5, 7 \pmod{8}$ and $k < 3^n$.

Let $S_i = P_3(S_{i-1})$ with $S_0 = P_{3k}(6)$, thus

If $N$ is prime then $S_{n-1} \equiv P_1(6) \pmod{N}$

**Conjecture 2.23.** Let $N = k \cdot 2^n - c$ such that $n > 2c$, $k > 0$, $c > 0$ and $c \equiv 3, 5 \pmod{8}$

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(6)$, thus

If $N$ is prime then $S_{n-1} \equiv -P_{1/c/2}(6) \pmod{N}$

**Conjecture 2.24.** Let $N = k \cdot 2^n + c$ such that $n > 2c$, $k > 0$, $c > 0$ and $c \equiv 3, 5 \pmod{8}$

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(6)$, thus

If $N$ is prime then $S_{n-1} \equiv -P_{1/c/2}(6) \pmod{N}$

**Conjecture 2.25.** Let $N = k \cdot 2^n - c$ such that $n > 2c$, $k > 0$, $c > 0$ and $c \equiv 1, 7 \pmod{8}$

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(6)$, thus

If $N$ is prime then $S_{n-1} \equiv P_{1/c/2}(6) \pmod{N}$

**Conjecture 2.26.** Let $N = k \cdot 2^n + c$ such that $n > 2c$, $k > 0$, $c > 0$ and $c \equiv 1, 7 \pmod{8}$

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(6)$, thus

If $N$ is prime then $S_{n-1} \equiv P_{1/c/2}(6) \pmod{N}$

**Conjecture 2.27.** Let $N = k \cdot 10^n - c$ such that $n > 2c$, $k > 0$, $c > 0$ and $c \equiv 3, 5 \pmod{8}$

Let $S_i = P_{10}(S_{i-1})$ with $S_0 = P_{5k}(P_5(6))$, thus

If $N$ is prime then $S_{n-1} \equiv -P_{5[c/2]}(6) \pmod{N}$
Conjecture 2.28. Let $N = k \cdot 10^n + c$ such that $n > 2c$, $k > 0$, $c > 0$ and $c \equiv 3, 5 \pmod{8}$.

Let $S_i = P_{10}(S_{i-1})$ with $S_0 = P_{5k}(P_5(6))$, thus

If $N$ is prime then $S_{n-1} \equiv -P_{5[c/2]}(6) \pmod{N}$

Conjecture 2.29. Let $N = k \cdot 10^n - c$ such that $n > 2c$, $k > 0$, $c > 0$ and $c \equiv 1, 7 \pmod{8}$.

Let $S_i = P_{10}(S_{i-1})$ with $S_0 = P_{5k}(P_5(6))$, thus

If $N$ is prime then $S_{n-1} \equiv P_{5[c/2]}(6) \pmod{N}$

Conjecture 2.30. Let $N = k \cdot 10^n + c$ such that $n > 2c$, $k > 0$, $c > 0$ and $c \equiv 1, 7 \pmod{8}$.

Let $S_i = P_{10}(S_{i-1})$ with $S_0 = P_{5k}(P_5(6))$, thus

If $N$ is prime then $S_{n-1} \equiv P_{5[c/2]}(6) \pmod{N}$

Conjecture 2.31. Let $R = (3^p - 1)/2$ such that $p > 3$ and $p$ is an odd prime.

Let $S_i = P_3(S_{i-1})$ with $S_0 = P_3(4)$, thus

If $R$ is prime then $S_{p-1} \equiv P_3(4) \pmod{R}$

Conjecture 2.32. Let $R = (10^p - 1)/9$ such that $p$ is an odd prime.

Let $S_i = P_{10}(S_{i-1})$ with $S_0 = P_3(6)$, thus

If $R$ is prime then $S_{p-1} \equiv P_3(6) \pmod{R}$

Conjecture 2.33. Let $N = b^n - b - 1$ such that $n > 2$, $b \equiv 0, 6 \pmod{8}$.

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, thus

If $N$ is prime then $S_{n-1} \equiv P_{(b+2)/2}(6) \pmod{N}$

Conjecture 2.34. Let $N = b^n - b - 1$ such that $n > 2$, $b \equiv 2, 4 \pmod{8}$.

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, thus

If $N$ is prime then $S_{n-1} \equiv -P_{b/2}(6) \pmod{N}$

Conjecture 2.35. Let $N = b^n + b + 1$ such that $n > 2$, $b \equiv 0, 6 \pmod{8}$.

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, thus

If $N$ is prime then $S_{n-1} \equiv P_{b/2}(6) \pmod{N}$

Conjecture 2.36. Let $N = b^n + b + 1$ such that $n > 2$, $b \equiv 2, 4 \pmod{8}$.

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, thus

If $N$ is prime then $S_{n-1} \equiv -P_{(b+2)/2}(6) \pmod{N}$

Conjecture 2.37. Let $N = b^n - b + 1$ such that $n > 3$, $b \equiv 0, 2 \pmod{8}$.

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, thus

If $N$ is prime then $S_{n-1} \equiv P_{b/2}(6) \pmod{N}$

Conjecture 2.38. Let $N = b^n - b + 1$ such that $n > 3$, $b \equiv 4, 6 \pmod{8}$.
Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, thus
If $N$ is prime then $S_{n-1} \equiv -P_{(b-2)/2}(6) \pmod{N}$

**Conjecture 2.39.** Let $N = b^n + b - 1$ such that $n > 3$, $b \equiv 0, 2 \pmod{8}$.

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, thus
If $N$ is prime then $S_{n-1} \equiv P_{(b-2)/2}(6) \pmod{N}$

**Conjecture 2.40.** Let $N = b^n + b - 1$ such that $n > 3$, $b \equiv 4, 6 \pmod{8}$.

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, thus
If $N$ is prime then $S_{n-1} \equiv -P_{b/2}(6) \pmod{N}$

**References**


