Reflection of Plane Electromagnetic Wave from Conducting Plane

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Abstract

The phenomenon of reflection from conducting surface is considered in terms of exact solutions of Maxwell equations. Matching of waves and current density at the plane is completed. Amplitudes of reflected and transmitted waves are found as functions of incident wave and conductivity of the plane. This work is completed also for conducting plane lying between two distinct media. It is shown that in case of conducting interface waves with some certain parameters (polarization, incidence angle and frequency) and transform completely into waves of current density whereas amplitude of the reflected wave is equal to zero that is equivalent to total absorption.

1 Introduction

Conducting bodies are known to reflect electromagnetic waves. A limiting case of this phenomenon with the body being infinitesimally thin, or at least, having negligibly small thickness, which can be represented as a surface, is much simpler because in this case matching procedure is needed only on the surface itself. A theory of reflection from conducting surfaces would be an interesting and useful part of classical electrodynamics and mathematical physics. In this work we consider the simplest case of this phenomenon in which plane wave propagates towards a uniform conducting plane. The task is to derive amplitudes of reflected and transmitted waves as functions of polarization of the wave, angle of incidence and conductivity of the plane which is assumed to be constant everywhere on it. The problem is considered in its general form which admits that the plane lies between two distinct media. So, if their dielectric parameters are different, under some conditions transmitted wave is evanescent, therefore waves of this kind are also to be in the scope and we present our description of these waves at the very beginning.

In this work we propose an exact solution of the problem of reflection and transmission of plane electromagnetic waves from a uniform conducting plane. The work is organized as follows. We start with our representation of Maxwell equations in term of exterior calculus and plane waves as their particular solution. Then we show how to match the field to surface current density on the plane. Finally, we link this density to electric strength of the wave and conductivity of the plane and finally derive the values of amplitudes of incident, reflected and transmitted waves for given angle of incidence and conductivity of the plane.

2 Surface densities of the source of the field on a plane

When considering interaction of electromagnetic wave with a conducting plane, some sources of the field appear in the form of charge and current densities on it, therefore, the complete version of Maxwell equations are also to be considered. In this case the 1-form A is strictly real and the field equation for it reads

$$\mathrm{d}\,{}^*\mathrm{d}A = -4\pi I. \tag{1}$$

The right-hand side of this equation stands for the 3-form of 4-dimensional current density

$$I = \rho \mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}z + j_x \mathrm{d}t \wedge \mathrm{d}y \wedge \mathrm{d}z + j_y \mathrm{d}t \wedge \mathrm{d}z \wedge \mathrm{d}x + j_z \mathrm{d}t \wedge \mathrm{d}x \wedge \mathrm{d}y \tag{2}$$

where ρ stands for the volume charge density and j_a's do for components of the spatial current density. Surface densities of charge and current are limiting cases of these ones which have δ-function singularity on the support. For example, the 2-form of surface charge density $\sigma \mathrm{d}x \wedge \mathrm{d}y$ on the plane $z = 0$ can also be represented as an ordinary volume density $\sigma\delta(z)\mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}z$ and so for the current. Since all the rest considerations are being carried in three dimensions, below I employ only 3-dimensional exterior calculus, therefore the exterior factor $\mathrm{d}t$ will not appear in the current density. Nevertheless, one important difference between 3-dimensional and (1+3)-dimensional version of exterior calculus must be mentioned. The asterisk conjugations in these dimensions sometimes differs in sign. That is why the sign "minus" appeared in the field equation (1). This sign agreement provides that the time component of A called electrostatic potential Φ,

$$A = \Phi \mathrm{d}t + f\mathrm{d}x + g\mathrm{d}y + h\mathrm{d}z$$

satisfies Laplace equation with correct sign, however, one of Maxwell equations takes the form

$$\mathrm{d}H = -4\pi I \tag{3}$$

(note the sign "minus" at the current density). Current density I appears under electric induction Δ of a electromagnetic wave incident at a conducting plane. Its value is proportional to induction due to the relation

$$I = \kappa\Delta \tag{4}$$

where κ stands for conductivity of the plane.

3 Plane and evanescent electromagnetic waves

Plane and evanescent waves have been discussed in our recent work [3]. The corresponding strengths and inductions as they are derived there, have the form

$$\begin{aligned} E =& \quad -\omega \sin(\omega t - px - qz)\, \mathrm{d}y, \\ H =& \quad -\frac{q}{\mu} \sin(\omega t - px - qz)\, \mathrm{d}x + \frac{p}{\mu} \sin(\omega t - px - qz)\, \mathrm{d}z, \\ B =& \quad p \sin(\omega t - px - qz)\, \mathrm{d}x \wedge \mathrm{d}y - q \sin(\omega t - px - qz)\, \mathrm{d}y \wedge \mathrm{d}z, \\ \Delta =& \quad -\omega\epsilon \sin(\omega t - px - qz)\, \mathrm{d}z \wedge \mathrm{d}x \end{aligned} \tag{5}$$

and

$$\begin{aligned} E =& \quad \frac{q}{\epsilon} \sin(\omega t - px - qz)\, \mathrm{d}x - \frac{p}{\epsilon} \sin(\omega t - px - qz)\, \mathrm{d}z, \\ H =& \quad -\omega \sin(\omega t - px - qz)\, \mathrm{d}y, \\ B =& \quad -\omega\mu \sin(\omega t - px - qz)\, \mathrm{d}z \wedge \mathrm{d}x, \\ \Delta =& \quad q \sin(\omega t - px - qz)\, \mathrm{d}y \wedge \mathrm{d}z - p \sin(\omega t - px - qz)\, \mathrm{d}x \wedge \mathrm{d}y \end{aligned} \tag{6}$$

for two orthogonal polarizations. Components p and q of the wave vector satisfy the well-known dispersion equation

$$\epsilon\mu\omega^2 = p^2 + q^2. \tag{7}$$

If the conducting plane lies between two distinct media, it may happen that $p > \omega n$. In this case this equation is broken and it is convenient to introduce another constant q by

$$q^2 = p^2 - \epsilon\mu\omega^2. \tag{8}$$

Then the wave in the less optically dense medium is evanescent and its fields has strengths and inductions of the form

$$\begin{aligned} E =& \quad a\omega e^{-q|z|} \sin(\omega t - px)\, \mathrm{d}y, \\ H =& \quad -\frac{aq}{\mu} e^{-q|z|} \cos(\omega t - px)\, \mathrm{d}x - \frac{ap}{\mu} e^{-q|z|} \sin(\omega t - px)\, \mathrm{d}z, \\ B =& \quad -ape^{-q|z|} \sin(\omega t - px)\, \mathrm{d}x \wedge \mathrm{d}y - aqe^{-q|z|} \cos(\omega t - px)\, \mathrm{d}y \wedge \mathrm{d}z, \\ \Delta =& \quad a\omega\epsilon e^{-q|z|} \sin(\omega t - px)\, \mathrm{d}z \wedge \mathrm{d}x \end{aligned} \tag{9}$$

for one polarization and

$$\begin{aligned} E =& \quad -\frac{aq}{\mu} e^{-q|z|} \sin(\omega t - px)\, \mathrm{d}x + \frac{ap}{\mu} e^{-q|z|} \cos(\omega t - px)\, \mathrm{d}z, \\ H =& \quad \omega e^{-q|z|} \cos(\omega t - px) \\ B =& \quad a\omega\mu e^{-q|z|} \cos(\omega t - px)\, \mathrm{d}z \wedge \mathrm{d}x \\ \Delta =& \quad ape^{-q|z|} \cos(\omega t - px)\, \mathrm{d}x \wedge \mathrm{d}y - aqe^{-q|z|} \sin(\omega t - px)\, \mathrm{d}y \wedge \mathrm{d}z, \end{aligned} \tag{10}$$

for another.

4 Plane waves from a conducting plane. The case of divergence-free current density

Let a plane, put it as the coordinate plane $z = 0$, be conducting and carry a surface current density which depends on t and x as $\cos(\omega t - px)$, for example, $I = j\cos(\omega t - px)\,\mathrm{d}z \wedge \mathrm{d}x$. In the simpler case of current orthogonal to the vector of propagation, electromagnetic field produced by such a current density is a wave like those considered above and on the plane it has a jump of the tangent component of the magnetic strength which is equal to the surface current density on the plane $z = 0$:

$$I = j\cos(\omega t - px)\,\mathrm{d}x = (H_+ - H_-)_x \mathrm{d}x, \quad z = 0. \tag{11}$$

Therefore the entire field can be represented in the form of plane wave as before, but in this case the wave propagates outwards from the plane, thus, in positive direction of z in the $z > 0$ half-space and in negative direction otherwise. The corresponding strengths and inductions are as in the equations (5):

$$\begin{aligned}
E =& \quad a\omega \sin(\omega t - px - q|z|)\,\mathrm{d}y, \\
H =& \quad -\varepsilon(z)\frac{aq}{\mu}\sin(\omega t - px - q|z|)\,\mathrm{d}x - \varepsilon(z)\frac{ap}{\mu}\sin(\omega t - px - q|z|)\,\mathrm{d}z, \\
B =& \quad -ap\sin(\omega t - px - q|z|)\,\mathrm{d}x \wedge \mathrm{d}y - aq\sin(\omega t - px - q|z|)\,\mathrm{d}y \wedge \mathrm{d}z, \\
\Delta =& \quad a\omega\epsilon \sin(\omega t - px - q|z|)\,\mathrm{d}z \wedge \mathrm{d}x
\end{aligned}$$

where ε is the well-known step function which stands for the sign of z. In this case the boundary conditions are (11) and

$$(E_+ - E_-)_y = (B_+ - B_-)_{xy} = (\Delta_+ - \Delta_-)_{xy} = 0, \quad z = 0. \tag{12}$$

These three conditions are completed trivially and the previous one, (11) equalizes current density and discontinuity of the x-component of magnetic strength:

$$j = -\frac{2aq}{\mu}.$$

So, as soon as the surface current density, or actually, the coefficient j is specified, the waves amplitude can be found from this equation. If the constant q (8) in the form of the current density is real-valued, the wave is evanescent at both sides of the plane, hence, in this case we have bilateral evanescent wave. The corresponding strengths and inductions have the form (9). It is easy to verify that these expressions satisfy both the field equations and boundary conditions (11,12).

5 Plane waves from a conducting plane. The case of non-zero charge density

Consider a current density on a plane which has the same direction as the the vector of propagation, $I = j\cos(\omega t - px)\,\mathrm{d}y$. Since this current density accumulates electric charge, hence such a current density is obligatory accompanied with the corresponding 2-form of surface charge density $\sigma \mathrm{d}x \wedge \mathrm{d}y$ linked to the 2-form I by the continuity equation

$$\frac{\partial \sigma}{\partial t} + \mathrm{d}I = 0. \tag{13}$$

Then, the surface charge density depends on t and spatial coordinates the same way and I have them in the form

$$\sigma = s\cos(\omega t - px)\,\mathrm{d}x \wedge \mathrm{d}y, \quad I = j\cos(\omega t - px)\,\mathrm{d}y \tag{14}$$

Substituting these forms into the equation (13) yields simple relation

$$jp = s\omega. \tag{15}$$

The next task is to match plane waves using the surface densities in the boundary conditions. These conditions read that discontinuity of the 2-form Δ on the plane is equal to the surface density and that the equation (11) is now to be used for the x-component of the magnetic strength:

$$(E_+ - E_-)_y = (B_+ - B_-)_{xy} = 0, \quad (\Delta_+ - \Delta_-)_{xy} = \sigma, \quad z = 0. \tag{16}$$

It is seen that in this case the wave has another polarization than that considered above and is given by the equations (6):

$$\begin{aligned} E =&\ \frac{aq}{\epsilon}\sin(\omega t - px - q|z|)\,\mathrm{d}x + \varepsilon(z)\frac{ap}{\epsilon}\sin(\omega t - px - q|z|)\,\mathrm{d}z, \\ H =&\ \varepsilon(z)a\omega\sin(\omega t - px - q|z|)\,\mathrm{d}y, \\ B =&\ \varepsilon(z)a\omega\mu\sin(\omega t - px - q|z|)\,\mathrm{d}z \wedge \mathrm{d}x, \\ \Delta =&\ \varepsilon(z)ap\sin(\omega t - px - q|z|)\,\mathrm{d}x \wedge \mathrm{d}y + aq\sin(\omega t - px - q|z|)\,\mathrm{d}y \wedge \mathrm{d}z. \end{aligned} \tag{17}$$

Substituting this into the boundary conditions (16) yields only two equalities,

$$2a\omega = j, \quad 2ap = \sigma \tag{18}$$

which are in full agreement with the consequence (15) of the continuity equation (13). Thus, if surface current density on the plane has the form (14), then charge and current densities satisfy the equation (15) and the current produces electromagnetic wave given by the equation (17) with amplitude found from the equations (18) and the component p of the wave vector as in the equation (7). Again, if the constant q (8) in the form of the current density is real-valued, the wave is evanescent at both sides of the plane, hence, in this case we have bilateral evanescent wave. The corresponding strengths and inductions are given by the equations (10). It is easy to verify that these expressions satisfy both the field equations and boundary conditions (11,12).

6 Reflection and transmission at a conducting plane. E tangent to the plane

In this and the next sections I consider propagation of a plane wave from a dielectric onto a conducting plane which again is specified by the coordinate plane $z = 0$. The total wave in the $z > 0$ half-space is to be represented as the sum of incident and reflected waves and that in the $z < 0$ is what the surface of the current density on the plane emits. In fact, the reflected wave also is of this origin. Due to special role of the electric strength, which produces the surface current density, reflection and refraction of waves with two different polarizations are completely different and will be considered separately, so, in this section I consider the case of E (co)-tangent to the plane.

Evidently, incident, transmitted and reflected waves can be represented by 1-forms ($q_i = q_t$)

$$\begin{aligned} A_i = &\quad a_i\left(\frac{\imath q}{\mu\omega}\cos\phi_+\mathrm{d}x - \cos\phi_+\mathrm{d}y + \frac{\imath p}{\mu\omega}\cos\phi_+\mathrm{d}z\right)\\ A_r = &\quad a_r\left(\frac{\imath q}{\mu\omega}\cos\phi_r\mathrm{d}x + \cos\phi_r\mathrm{d}y - \frac{\imath p}{\mu\omega}\cos\phi_r\mathrm{d}z\right)\\ A_t = &\quad a_t\left(\frac{\imath q}{\mu\omega}\cos\phi_-\mathrm{d}x - \cos\phi_-\mathrm{d}y + \frac{\imath p}{\mu\omega}\cos\phi_-\mathrm{d}z\right). \end{aligned}$$

Again, as a result, I have strengths and inductions of the three waves in the form

$$\begin{aligned} E_i = &\quad a_i\omega\sin(\omega t - px + qz)\,\mathrm{d}y,\\ H_i = &\quad -\frac{aq}{\mu}\sin(\omega t - px + qz)\,\mathrm{d}x - \frac{a_i p}{\mu}\sin(\omega t - px + qz)\,\mathrm{d}z,\\ B_i = &\quad -a_i p\sin(\omega t - px + qz)\,\mathrm{d}x\wedge\mathrm{d}y - a_i q\sin(\omega t - px + qz)\,\mathrm{d}y\wedge\mathrm{d}z,\\ \Delta_i = &\quad a_i\omega\epsilon\sin(\omega t - px + qz)\,\mathrm{d}z\wedge\mathrm{d}x \end{aligned}$$

for the incident wave,

$$\begin{aligned} E_r = &\quad -a_r\omega\sin(\omega t - px - qz)\,\mathrm{d}y,\\ H_r = &\quad -\frac{a_r q}{\mu}\sin(\omega t - px - qz)\,\mathrm{d}x + \frac{a_r p}{\mu}\sin(\omega t - px - qz)\,\mathrm{d}z,\\ B_r = &\quad a_r p\sin(\omega t - px - qz)\,\mathrm{d}x\wedge\mathrm{d}y - a_r q\sin(\omega t - px - qz)\,\mathrm{d}y\wedge\mathrm{d}z,\\ \Delta_r = &\quad -a_r\omega\epsilon\sin(\omega t - px - qz)\,\mathrm{d}z\wedge\mathrm{d}x \end{aligned}$$

for the reflected wave and

$$\begin{aligned} E_t = &\quad a_t\omega\sin(\omega t - px + q_t z)\,\mathrm{d}y,\\ H_t = &\quad -\frac{a_t q}{\mu_-}\sin(\omega t - px + qz)\,\mathrm{d}x - \frac{p}{\mu_-}\sin(\omega t - px + qz)\,\mathrm{d}z,\\ B_t = &\quad -a_t p\sin(\omega t - px + qz)\,\mathrm{d}x\wedge\mathrm{d}y - a_t q\sin(\omega t - px + qz)\,\mathrm{d}y\wedge\mathrm{d}z,\\ \Delta_t = &\quad a_t\omega\epsilon\sin(\omega t - px + qz)\,\mathrm{d}z\wedge\mathrm{d}x \end{aligned}$$

for the refracted wave.

In this case boundary conditions have a more complicated form than (11) and (16) because the total electric strength produces electric current density. Since Δ has only zx-component, according to specific notation of exterior calculus on a plane, according to the equations (3,4), the current density produced by it also has only zx-component. Hence, the complete boundary condition consists of five equations

$$\begin{aligned} (E_i + E_r)_y = (E_t)_y, \qquad & (B_i + B_r)_{xy} = (B_t)_{xy}, \\ & (\Delta_i + \Delta_r)_{xy} - (\Delta_t)_{xy} = 0, \\ \kappa(\Delta_i + \Delta_r + \Delta_t)_{zx} = I_{zx}, \quad & (H_i + H_r - H_t)_x = -4\pi I_{zx} \end{aligned} \tag{19}$$

where I use the equation (4). Due to the equation (14) which expresses the surface charge density through current density, these conditions yield only three non-coinciding equations which are

$$a_r + a_t = a_i, \quad \epsilon\kappa\omega\,(a_i - a_r + a_t) = j \quad \frac{q}{\mu}(a_i + a_r - a_t) = 4\pi j.$$

If the amplitude a_i is specified, three other unknowns come out of these equations after excluding j:

$$a_r = \frac{4\pi a_i \epsilon\mu\omega\kappa}{q + 4\pi\epsilon\mu\omega\kappa}, \quad a_t = \frac{a_i q}{q + 4\pi\epsilon\mu\omega\kappa}, \quad j = \frac{2a_i q\epsilon\kappa\omega}{q + 4\pi\epsilon\mu\omega\kappa}$$

or, in terms of incident angle,

$$a_r = \frac{4\pi a_i n\omega\kappa}{4\pi\kappa n + \cos i}, \quad a_t = \frac{a_i \cos i}{4\pi\kappa n + \cos i}, \quad j = \frac{2a_i\epsilon\kappa\omega\cos i}{4\pi\kappa n + \cos i} \tag{20}$$

The current surface density on the plane is given by

$$I = j\sin(\omega t - px)\,\mathrm{d}z \wedge \mathrm{d}x.$$

Now, let us consider another polarization.

7 Reflection and transmission at a conducting plane. H tangent to the plane

In case of another polarization everything is as in the previous section, but now H is tangent to the plane. The corresponding strengths and inductions of the three waves in the form

$$\begin{aligned} E_i =\ & \frac{a_i q}{\epsilon}\sin(\omega t - px + qz)\,\mathrm{d}x + \frac{a_i p}{\epsilon}\sin(\omega t - px + qz)\,\mathrm{d}z, \\ H_i =\ & a_i\omega\sin(\omega t - px + qz)\,\mathrm{d}y \\ B_i =\ & a_i\omega\mu\sin(\omega t - px + qz)\,\mathrm{d}z \wedge \mathrm{d}x, \\ \Delta_i =\ & a_i p\sin(\omega t - px + qz)\,\mathrm{d}x \wedge \mathrm{d}y + a_i q\sin(\omega t - px + qz)\,\mathrm{d}y \wedge \mathrm{d}z \end{aligned}$$

for the incident wave,

$$
\begin{aligned}
E_r = &\ -\frac{a_r q}{\epsilon}\sin(\omega t - px - qz)\,\mathrm{d}x + \frac{a_r p}{\epsilon}\sin(\omega t - px - qz)\,\mathrm{d}z,\\
H_r = &\ a_r\omega\sin(\omega t - px - qz)\,\mathrm{d}y,\\
B_r = &\ a_r\omega\mu_-\sin(\omega t - px - qz)\,\mathrm{d}z\wedge\mathrm{d}x\\
\Delta_r = &\ a_r p\sin(\omega t - px - q_t z)\,\mathrm{d}x\wedge\mathrm{d}y - a_r q_t\sin(\omega t - px - q_t z)\,\mathrm{d}y\wedge\mathrm{d}z
\end{aligned}
$$

for the reflected wave and

$$
\begin{aligned}
E_t = &\ \frac{a_t q}{\epsilon}\sin(\omega t - px + qz)\,\mathrm{d}x + \frac{a_t p}{\epsilon}\sin(\omega t - px + qz)\,\mathrm{d}z,\\
H_t = &\ a_t\omega\sin(\omega t - px + qz)\,\mathrm{d}y,\\
B_t = &\ a_t\omega\mu_-\sin(\omega t - px + qz)\,\mathrm{d}z\wedge\mathrm{d}x,\\
\Delta_t = &\ a_t p\sin(\omega t - px + qz)\,\mathrm{d}x\wedge\mathrm{d}y + a_t q\sin(\omega t - px + qz)\,\mathrm{d}y\wedge\mathrm{d}z
\end{aligned}
$$

for the refracted wave.

In this case boundary conditions have a more complicated form than (19) because charge density σ_{xy} is non-zero and correspondingly, Δ has non-zero xy-component. Hence, the complete boundary condition consists of five equations

$$
\begin{aligned}
(E_i + E_r)_x = (E_t)_x, &\qquad (B_i + B_r)_{xy} = (B_t)_{xy},\\
(\Delta_i + \Delta_r)_{xy} - (\Delta_t)_{xy} = 4\pi\sigma_{xy}, &\\
\kappa(\Delta_i + \Delta_r + \Delta_t)_{yz} = I_{yz}, &\qquad (H_i + H_r - H_t)_y = 4\pi I_{yz}.
\end{aligned}
$$

These conditions yield only three non-coinciding equations which are

$$
\begin{aligned}
a_i - a_r - a_t = 0, &\qquad p(a_i + a_r - a_t) = 4\pi\sigma,\\
\omega(a_i + a_r - a_t) = 4\pi j, &\quad q\kappa(a_i - a_r + a_t) = j.
\end{aligned}
$$

The second equation of the first line and the first equation of the second line together constitute the charge conservation law represented as the equation (15). Therefore one of them can be ignored together with the unknown surface charge density σ which can be obtained from this law if necessary. If the amplitude a_i is specified, three other unknowns come out of these equations after excluding j:

$$
a_r = \frac{4\pi a_i q\epsilon\kappa}{\omega + 4\pi p\epsilon\kappa},\quad a_t = \frac{a_i\omega}{\omega + 4\pi p\epsilon\kappa},\quad j = \frac{2a_i q\epsilon\kappa\omega}{\omega + 4\pi p\epsilon\kappa}
$$

or, in terms of incident angle,

$$
a_t = \frac{a_i}{4\pi n\kappa\cos i + 1},\quad a_r = \frac{4\pi a_i\kappa n\cos i}{4\pi n\kappa\cos i + 1},\quad j = \frac{2a_i\kappa\omega n\cos i}{4\pi n\kappa\cos i + 1} \tag{21}
$$

The current surface density on the plane is given by

$$
I = j\sin(\omega t - px)\,\mathrm{d}z\wedge\mathrm{d}x. \tag{22}
$$

Note that under straight incidence $i = 0$ amplitudes of reflected and transmitted waves for both polarizations coincide as it should be geometrically. The corresponding surface charge density can also be found if needed.

8 Conducting interface

In this section we assume that the interface has non-zero conductivity κ and incident wave produces a non-zero current density on it. The phenomenon to be studied is transformation of an incident wave into an evanescent one that is equivalent to partial absorption of it by the conducting layer between two media. The effect of absorption is stronger for waves with electric strength parallel to the plane, therefore we consider only one polarization. Applying boundary conditions (19) yields the following system of algebraic equations

$$\begin{aligned}
& a_i - a_r = a_t \\
& \frac{q_i}{\mu_+}(a_i + a_r) - \frac{q_t}{\mu_-}\,a_t = 4\pi j \\
& 4\pi\kappa\omega\epsilon_+(a_i + a_r) - 4\pi\kappa\omega\epsilon_- a_t = -4\pi j
\end{aligned}$$

Exclusion of j and denoting

$$P_\pm = \frac{q_i}{\mu_+} \pm 4\pi\kappa\omega\epsilon_+, \quad Q = \frac{q_t}{\mu_-} + 4\pi\kappa\omega\epsilon_-$$

allows one to represent the remaining equations in the form

$$a_i - a_r - a_t = 0, \quad P_+ a_i + P_- a_r - Q a_t = 0$$

and solve them as

$$a_t = a_i\,\frac{P_+ + P_-}{Q + P_-}, \quad a_r = a_i\,\frac{Q - P_+}{Q + P_-} \tag{23}$$

and current density can also be obtained from one or another equation above.

If $\epsilon_+\mu_+$ where the wave propagates from, is greater than $\epsilon_-\mu_-$ where it does to, then under some angles of incidence, the Snell law reads that $\sin r \geq 1$ that is impossible. Under these conditions, the wave in the less dense medium should be represented by

$$\begin{aligned}
E_t = &\ a_t\omega e^{-q_t z}\sin(\omega t - px)\,\mathrm{d}y, \\
H_t = &\ -\frac{a_t q_t}{\mu_+}e^{-q_t z}\cos(\omega t - px)\,\mathrm{d}x - \frac{a_t p}{\mu_+}e^{-q_t z}\sin(\omega t - px)\,\mathrm{d}z, \\
B_t = &\ -a_t p e^{-q_t z}\sin(\omega t - px)\,\mathrm{d}x\wedge\mathrm{d}y - a_t q_t e^{-q_t z}\cos(\omega t - px)\,\mathrm{d}y\wedge\mathrm{d}z, \\
\Delta_t = &\ a_t\omega\epsilon_+ e^{-q_t z}\sin(\omega t - px)\,\mathrm{d}z\wedge\mathrm{d}x
\end{aligned}$$

where q_t satisfies the equation (8).

9 Conclusion

Unlike an interface between two distinct media, conductors interact with electromagnetic waves by currents which they carry. However, simplest problems of interactions of this kind, in which both the wave and conductor have planar symmetry and besides, thickness of the latter is much smaller than the wavelength, admit complete solution which provide amplitudes of reflected and transmitted waves, phase shifts and angles in case of jump of dielectric factors on it. This solution is obtained in explicit form. Besides formal quantitative results we have some qualitative ones which would make sense for practical applications. One of them is well-known effect of polarization of waves when being reflected and refracted at an interface, which now is accompanied by certain values of amplitudes (20,21) for each polarization. Another quantitative results seems to be more interesting.

In case of total internal reflection at a conducting interface the value of amplitude of the reflected wave as a function of all the rest parameters, shows that under to some special conditions this amplitude may become equal to zero. In this case total internal reflection actually yields no reflection and the wave is completely absorbed by the conducting plane. This phenomenon exists if the difference $Q - P_+$ in the equation (23) vanishes under some angles of incidence. Existence of the phenomenon of total absorption of electromagnetic wave is interesting by itself, but also can be interesting from practical point of view.

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