Five conjectures on a diophantine equation involving two primes and a square of prime

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Abstract. In this paper I make five conjectures about the primes $q$, $r$ and the square of prime $p^2$, which appears as solutions in the diophantine equation $120*n*q*r + 1 = p^2$, where $n$ is non-null positive integer.

Conjecture 1:

For any $n$ non-null positive integer there exist $q$, $r$ primes such that $120*n*q*r + 1 = p^2$, where $p$ is prime or a power of prime.

Conjecture 2:

For any $q$ odd prime there exist $n$ non-null positive integer and $r$ prime such that $120*n*q*r + 1 = p^2$, where $p$ is prime or a power of prime.

Conjecture 3:

For any $q$, $r$ odd primes there exist $n$ non-null positive integer such that $120*n*q*r + 1 = p^2$, where $p$ is prime or a power of prime.

Conjecture 4:

For any $n$ non-null positive integer and any $q$ prime there exist $r$ prime such that $120*n*q*r + 1 = p^2$, where $p$ is prime or a power of prime.

Examples:

: For $[n, q] = [1, 5]$ there exist $r = 17$ such that $p = 101$ prime; also $r = 37$ such that $p = 149$ prime;
: For $[n, q] = [1, 7]$ there exist $r = 23$ such that $p = 139$ prime; also $r = 53$ such that $p = 211$ prime;
: For $[n, q] = [1, 11]$ there exist $r = 13$ such that $p = 131$ prime; also $r = 83$ such that $p = 331$ prime;
: For $[n, q] = [2, 5]$ there exist $r = 19$ such that $p = 151$ prime;
For \( [n, q] = [2, 7] \) there exist \( r = 3 \) such that \( p = 71 \) prime; also \( r = 17 \) such that \( p = 169 \) square of prime;

For \( [n, q] = [2, 11] \) there exist \( r = 3 \) such that \( p = 89 \) prime;

For \( [n, q] = [3, 7] \) there exist \( r = 13 \) such that \( p = 181 \) prime;

For \( [n, q] = [3, 11] \) there exist \( r = 3 \) such that \( p = 109 \) prime;

For \( [n, q] = [4, 5] \) there exist \( r = 67 \) such that \( p = 401 \) prime;

For \( [n, q] = [4, 7] \) there exist \( r = 17 \) such that \( p = 239 \) prime;

For \( [n, q] = [4, 11] \) there exist \( r = 11 \) such that \( p = 241 \) prime.

**Conjecture 5:**

For any \( n \) non-null positive integer there exist \( q \) prime such that \( 120nq^2 + 1 = p^2 \), where \( p \) is prime or a power of prime.

Note, for instance, the case from the examples below: \( 480\times11^2 + 1 = 241^2 \).