Two conjectures about the pairs of primes separated by a certain distance

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Abstract. In this paper I make two conjectures about the pairs of primes \([p_1, q_1]\), where the difference between \(p_1\) and \(q_1\) is a certain even number \(d\). I state that any such pair has at least one other corresponding, in a specified manner, pair of primes \([p_2, q_2]\), such that the difference between \(p_2\) and \(q_2\) is also equal to \(d\).

Conjecture 1:

For any pair of primes, greater than 3, \([p_1, q_1]\), where \(q_1 - p_1 = d\), there exist at least a pair of positive integers \([m, n]\), where \(n - m = d\), such that the numbers \(p_2 = p_1*q_1 - n + 1\) and \(q_2 = p_1*q_1 - m + 1\) are both primes.

Examples:

: For \([p_1, q_1] = [5, 7]\) there exist the pair \([m, n] = [5, 7]\) such that \(p_2 = 5*7 - 7 + 1 = 29\) and \(q_2 = 5*7 - 5 + 1 = 31\) are both primes;

: For \([p_1, q_1] = [5, 11]\) there exist the pair \([m, n] = [3, 9]\) such that \(p_2 = 5*11 - 9 + 1 = 47\) and \(q_2 = 5*11 - 3 + 1 = 53\) are both primes;

: For \([p_1, q_1] = [5, 13]\) there exist the pair \([m, n] = [5, 13]\) such that \(p_2 = 5*13 - 13 + 1 = 53\) and \(q_2 = 5*13 - 5 + 1 = 61\) are both primes;

: For \([p_1, q_1] = [7, 11]\) there exist the pair \([m, n] = [7, 11]\) such that \(p_2 = 7*11 - 11 + 1 = 67\) and \(q_2 = 7*11 - 7 + 1 = 71\) are both primes;

: For \([p_1, q_1] = [7, 13]\) there exist the pair \([m, n] = [7, 13]\) such that \(p_2 = 7*11 - 11 + 1 = 67\) and \(q_2 = 7*11 - 7 + 1 = 71\) are both primes;

: For \([p_1, q_1] = [11, 13]\) there exist the pair \([m, n] = [5, 7]\) such that \(p_2 = 11*13 - 5 + 1 = 137\) and \(q_2 = 11*13 - 7 + 1 = 139\) are both primes.
Conjecture 2:

For any even number \( d \) there exist an infinity of pairs of primes \([p_1, q_1]\), where \( q_1 - p_1 = d \), such that the numbers \( p_2 = p_1 * q_1 - p_1 + 1 \) and \( q_2 = p_1 * q_1 - q_1 + 1 \) are both primes.

Note: See, for instance, from the examples to the Conjecture 1 from above, the pair \([5, 7]\) for \( d = 2 \), the pair \([7, 11]\) for \( d = 4 \), the pair \([5, 13]\) for \( d = 8 \).