# The Mass of the Proton

## (The Equation for the Mass of the Proton Confirmed by the Latest Atomic Interferometry Measurements of the Newton's Gravitational Constant)

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**Abstract**

*This paper is concerned with the discovery of a new theoretical equation for the mass of the proton. Since the equation depends, among other things, on the gravitational constant G, I found that the formula produces the correct value of the proton mass if and only if we use the latest experimental limits (due to the experimental error of 150 ppm) for this constant. The latest and most accurate value of G obtained so far comes from atomic interferometry. This new and revolutionary experimental method was devised by an Italian team of scientists and the results were published earlier this year in Nature. Due to the previous lack of accuracy and large discrepancies in the measurement of G, this experimental result is a scientific breakthrough that allow us and shall allow us to confirm, among other things, the validity of equations for the mass of particles, the latest fundamental particle formulations and cosmological theories.*

***Keywords***: *Atomic interferometry, gravitational constant, fine structure constant, Planck mass.*

## 1. Introduction

An accurate value of the gravitational constant $G$ is of paramount importance in the assessment of any theory of everything and any other unified theory where $G$ plays an important role. Scientists measured this elusive constant for over 200 years but they found their results were so far apart that the value of $G$ was, until recently, very unreliable. The discrepancy was relatively so large that it made practically impossible to assess the correctness of some formulas and theories beyond reasonable doubt. Fortunately things have recently changed. Early this year a team of Italian scientists (G. Rosi et al [1]) measured $G$ with an unprecedented accuracy through a new method known as atomic interferometry. Most previous measurements were superseded by their measurements and science once again took a giant leap forward.

The equation introduced in this paper comes from my 2012's investigations. The value of $G$ I used at that time was the value published by NIST in 2010: 6.67384 $\times 10^{-11} m^3 Kg^{-1} S^{-2}$ . However, with this relatively high value of $G$, the corresponding value for the mass of the proton I obtained from equation (4) turned out to be in disagreement with the experiment. Given the disparity, I thought that the equation was incorrect and I did not paid any more attention to it.

Since 2012 things have changed. The latest measurements carried out by the Italian team showed that the value of $G$ is lower than previously thought, and in particular, is lower that the 2010 CODATA value. This is how an almost forgotten equation was revived and became the subject of this paper.

## 2. The Equation for the Mass of the Proton

The theoretical equation for the mass of the proton I discovered is

$$m_p = \frac{m_e}{A(1-A)} \quad \text{(Equation for the mass of the proton – form 1)} \qquad (1)$$

where

$$A = \alpha^{12}\left(\frac{M_P}{m_e}\right) \quad \text{(Dimensionless constant)} \qquad (2)$$

$$M_P = \sqrt{\frac{hc}{2\pi G}} \quad \text{(Planck mass)} \qquad (3)$$

$$\alpha = \frac{e^2}{2\epsilon_0 h c} \quad \text{(Fine structure constant)} \qquad (4)$$

$m_p =$ proton rest mass
$m_e =$ electron rest mass
$M_P =$ Planck mass
$\alpha =$ fine structure constant
$e$ = elementary chrge
$\epsilon_0 =$ permitivity of vacuum
$h$ = Planck's constant
$c$ = speed of light in vacuum

Substituting equation (2) into equation (1) gives

$$m_p = \frac{m_e}{\alpha^{12}\frac{M_P}{m_e}\left(1-\alpha^{12}\frac{M_P}{m_e}\right)} \quad \text{(Equation for the mass of the proton – form 2)} \qquad (5)$$

This equation tells us that the mass of the proton is a function of the mass of the electron, the Planck mass and the fine structure constant.

Equation (5) can be expressed as follows

$$m_p = \frac{m_e^2}{\alpha^{12} M_P\left(1-\alpha^{12}\frac{M_P}{m_e}\right)} \quad \text{(Equation for the mass of the proton – form 3)} \qquad (6)$$

  

Another way of writing this equation is by defining the gravitational constant for the electron, $\alpha_{Ge}$ , as

$$\alpha_{Ge} = \left(\frac{m_e}{M_P}\right)^2 \qquad \text{(Gravitational constant for the electron)} \qquad (7)$$

Finally equation (5) can be re-written in terms of the fine structure constant and the gravitational constant for the electron defined by equation (7). The result is

$$m_p = \frac{m_e}{\frac{\alpha^{12}}{\sqrt{\alpha_{\mathrm{Ge}}}}\left(1 - \frac{\alpha^{12}}{\sqrt{\alpha_{\mathrm{Ge}}}}\right)} \qquad \text{(Equation for the mass of the proton – form 4)} \qquad (8)$$

Equations (1), (5), (6) and (8) are four different forms of the equation for the mass of the proton. However, equation (8) seems to be the most elegant of them.

In summary the equation for the mass of the proton depends on six fundamental physical constants and one mathematical constant. This is shown in Table 1

| Fundamental physical constants | Mathematical constants |
|---|---|
| 1) electron rest mass $(m_e)$<br>2) Newton's gravitational constant $(G)$<br>3) Planck's constant $(h)$<br>4) speed of light in vacuum $(c)$<br>5) elementary charge $(e)$<br>6) permitivity of vacuum $(\epsilon_0)$ | 7) number pi $(\pi)$ |

**Table 1:** This table shows the 7 constants that are part of the equation for the mass of the proton.

## 3. Experimental Errors

In this section we shall consider both the lower and upper limits of the experimental values of the gravitational constant *G* according to the error of 150 ppm reported by Rosi at al. Thus the possible range of values of *G* are given by the interval [G_min, G_max], where the lower and upper limits are, respectively:

### Lower experimental limit

G_min = $(6.67191 - 6.67191 \times 0.00015) \times 10^{-11} m^3 Kg^{-1} S^{-2}$ =
$6.670\,909\,214 \times 10^{-11} m^3 Kg^{-1} S^{-2}$

  

### Upper experimental limit

G_max= $(6.67191 + 6.67191 \times 0.00015) \times 10^{-11} m^3 Kg^{-1} S^{-2}$ =
$6.672\,910\,787 \times 10^{-11} m^3 Kg^{-1} S^{-2}$

Table 2 shows the values for the mass of the proton calculated with equation (4). The results of these calculations are shown in column 4. The values of the Planck mass used to calculate the mass of the proton are shown in column 3. The values of the gravitational constant used to calculate the Planck mass of column 3 are shown in column 1.

| Source | Gravitational constant<br>G<br>$\times 10^{-11} m^3 Kg^{-1} S^{-2}$ | Comment | Planck Mass<br>$M_P$<br>$\times 10^{-8} Kg$ | Mass of the proton<br>$m_p$<br>$\times 10^{-27} Kg$ |
|---|---|---|---|---|
| CODATA 2010 | G_nist=6.673 84 | Lowest accuracy | 2.176 509 | 1.673 04 |
| Rosi et al [1] Atomic interferometry method | G_max=6.672 910 787 | Upper experimental limit | 2.176 661 | 1.672 784 |
| Rosi et al [1] Atomic interferometry method | G_mean=6.671 91 | Experimental arithmetic mean<br>G_mean=<br>(G_min + G_max) / 2 | 2.176 824 | 1.672 659 |
| Author's proposed theoretical value of G | G_the = 6.671 614 932 | Theoretical value of G that yields the observed proton mass | 2.176 872 191 | 1.672 621 777<br>(Observed value) |
| Rosi et al [1] Atomic interferometry method | G_min=6.670 909 214 | Lower experimental limit | 2.176 987 | 1.672 534 |

**Table 2:** This table shows the values for the mass of the proton calculated with equation (4) using the values of the gravitational constant shown on the first column.

The reader could probably have realized that the value of *G* shown on the fourth row, first column of the above table, $6.671\,614\,932 \times 10^{-11} m^3 Kg^{-1} S^{-2}$ , was chosen to reproduce the observed value of the proton mass.

This could look like a trick, however, taking into consideration the latest measurement based on atomic interferometry, we see that the experimental value of *G* is much closer to the corresponding proposed theoretical value than ever before. This evidence strongly suggests that the equation for the mass of the proton introduced in this paper must be correct. Thus, if this is the case, the exact value of the Newton's gravitational constant, accurate to nine decimal places, should be $6.671\ 614\ 932 \times 10^{-11} m^3 Kg^{-1} S^{-2}$ .

## 4. Conclusions

The conclusions drawn from this paper are: a) we shall adopt the value of $6.671\ 614\ 932 \times 10^{-11} m^3 Kg^{-1} S^{-2}$ as the correct value for the gravitational constant, and b) this value can be used as a theoretical reference for future measurements comparisons.

**REFERENCES**

[1] G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli & G. M. Tino, *Precision measurement of the Newtonian gravitational constant using cold atoms*, Nature, VOL 510, p. 518-521, Macmillan Publishers Limited, (2014).