Wilson and Polyakov loops of gravitational gauge fields in Rindler space

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Abstract: We will study the gravitational forces acting between static massive sources generated by exchange of massless gravitons within the framework of Quantum Gauge Theory Gravity .The gravitational force will be determined via Wilson loops and Polyakov loop correlation functions. This method will enable us to separate the contribution of the quantum mechanical transverse graviton from that of the classical longitudinal field. It will be the method of choice if one attempts to determine the gravitational static force in simulations of Quantum Gauge Theory Gravity on a Rindler space lattice.

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1. Introduction

The study of quantum fields in Rindler space has played an important role in developing our understanding of quantum fields in non-trivial space-times. The importance of these studies derives to a large extent from the existence of a horizon in Rindler space. The kinematics of non-interacting quantum fields in Rindler space [1], [2], [3] and their relation to fields in Minkowski space together with the interpretation in terms of quantum fields at finite temperature [4], [5] are well understood [6].

The relation between acceleration and finite temperature remains an important element of the thermodynamics of black holes.

Various workers have attempted to derive General Relativity from a gauge-like principle, involving invariance of physics under transformations of the locally (i.e. in the tangent space at each point) acting Lorentz or Poincare group. ([7], [8], [9]).

N. Wu [10−13] proposed a Quantum Gauge Theory of Gravity (QGTG) based on the gravitational gauge group (G). In Wu's theory, the gravitational interaction is considered as a fundamental interaction in a flat Minkowski space-time and not as space-time geometry.

A model of interacting massive gauge gravitons and a possible heavy gauge graviton resulting from shell decay of Higgs bosons and the De Broglie-Bohm approach of gravitational gauge fields have been developed recently by the author within the framework of QGTG [14,15,16,17,18].

Dynamical issues of quantum gravitational gauge fields in Rindler space are in the center of the present work. We will study the gravitational forces acting between static massive sources generated by exchange of massless gravitons within the framework of QGTG [14, 15, 16, 17, 18].

The gravitational force will be determined via Wilson loops and Polyakov loop correlation functions. This method will enable us to separate the contribution of the quantum mechanical transverse graviton from that of the classical longitudinal field. It will be the method of choice if one attempts to determine the gravitational static force in simulations of QGTG on a Rindler space lattice.

2. Fundamentals of Quantum Gauge Theory of Gravity

Following, N. Wu, the infinitesimal transformations of the gravitational gauge group G are of the form [10]:

$$U_\varepsilon = 1 - i\varepsilon^\alpha P_\alpha ,\ \alpha = 0,1,2,3, \tag{1}$$

where ε^α are the infinitesimal parameters of the group and $P_\alpha = -i\partial / \partial x^\alpha$ are the generators of the gauge group.

It is known that these generators commute each other

$$[P_\alpha, P_\beta] = 0. \tag{2}$$

However, this property of the generators does not mean that the gravitational gauge group is an Abelian group, because the elements of the gravitational group do not commute [10]:

$$[U_{\varepsilon_2}, U_{\varepsilon_2}] \neq 0. \tag{3}$$

The gravitational gauge-covariant derivative is defined by

$$D_\mu = \partial_\mu - igC_\mu(x), \tag{4}$$

where $C_\mu(x)$ is the gravitational gauge field and g is the gravitational gauge coupling constant. $C_\mu(x)$ is a Lorentz vector. Under gravitational gauge transformation, $C_\mu(x)$ transforms as

$$C_{\mu}(x) \to C_{\mu}'(x) = \hat{U}_{\varepsilon}(x) C_{\mu}(x) \hat{U}_{\varepsilon}^{-1}(x) + \frac{i}{g} \hat{U}_{\varepsilon}(x)(\partial_{\mu} \hat{U}_{\varepsilon}^{-1}(x)), \tag{5}$$

and D_μ transforms covariantly as,

$$D_{\mu}(x) \to D_{\mu}'(x) = \hat{U}_{\varepsilon}(x) D_{\mu}(x) \hat{U}_{\varepsilon}^{-1}(x). \tag{6}$$

Gravitational gauge field $C_\mu(x)$ can be expanded as linear combinations of generators of gravitational gauge group,

$$C_{\mu}(x) = C_{\mu}^{\alpha}(x) \cdot \hat{P}_{\alpha}, \tag{7}$$

where C_{μ}^{α} is the component field of gravitational gauge field.

C_{μ}^{α} looks like a second-rank tensor, but it is not a tensor field. The index α is not an ordinary Lorentz index, but a gauge group index. Since gravitational gauge field C_{μ}^{α} has only one Lorentz index, it is a kind of vector field. The strength of gravitational gauge field is defined by the second-order Lorenz tensor,

$$F_{\mu\nu} = \frac{1}{-ig} \left[D_{\mu}, D_{\nu} \right], \tag{8}$$

or

$$F_{\mu\nu} = \partial_{\mu} C_{\nu}(x) - \partial_{\nu} C_{\mu}(x) - ig C_{\mu}(x) C_{\nu}(x) + ig C_{\nu}(x) C_{\mu}(x), \tag{9}$$

$F_{\mu\nu}$ is a vector in group space; therefore, it can be expanded in group space:

$$F_{\mu\nu}(x) = F_{\mu\nu}^{\alpha}(x) \cdot \hat{P}_{\alpha}. \tag{10}$$

The explicit form of component field strength is

$$F_{\mu\nu}^{\alpha} = (\partial_{\mu} C_{\nu}^{\alpha}) - (\partial_{\nu} C_{\mu}^{\alpha}) - g C_{\mu}^{\beta} (\partial_{\beta} C_{\nu}^{\alpha}) + g C_{\nu}^{\beta} (\partial_{\beta} C_{\mu}^{\alpha}) \,. \tag{11}$$

The strength of gravitational gauge field transforms covariantly under gravitational gauge transformation. Similar to traditional gauge field theory, the kinematical term for gravitational gauge field can be written as:

$$\Im_0 = -\frac{1}{4} \eta^{\mu\rho} \eta^{\nu\sigma} g_{\alpha\beta} F_{\mu\nu}^{\alpha} F_{\rho\sigma}^{\beta}. \tag{12}$$

We can easily prove that this Lagrangian is not invariant under gravitational gauge transformation. It transforms covariantly as follows:

$$\Im_0 \to \Im_0' = (\hat{U}_{\varepsilon} \Im_0). \tag{13}$$

In order to resume the gravitational gauge symmetry of the action, we introduce an essential factor in the form of:

$$e^{I(C)} = e^{g\eta_{1\alpha}^{\mu} C_{\mu}^{\alpha}}, \;\; I(C) = g\eta_{1\alpha}^{\mu} C_{\mu}^{\alpha}. \tag{14}$$

The full Lagrangian $\Im$ is then given by:

$$\Im = e^{I(C)} \Im_0, \tag{15}$$

and the action S is defined by;

$$S = \int d^4 x \Im. \tag{16}$$

It can be proven that this action has local gravitational gauge symmetry [10]. According to the gauge principle, the global symmetry gives out a conserved current, which is:

$$T_{i\alpha}^{\mu} = e^{I(C)} \left(-\frac{\partial \Im_0}{\partial(\partial_{\mu} C_{\nu}^{\alpha})} \partial_{\alpha} C_{\nu}^{\alpha} + \eta_{1\alpha}^{\mu} \Im_0 \right). \tag{17}$$

We call this quantity inertial energy-momentum tensor [10].

The Euler-Lagrange equations for and C_{μ}^{α} gauge fields are:

$$\partial_{\mu} \frac{\partial \Im}{\partial(\partial_{\mu} C_{\nu}^{\alpha})} = \frac{\partial \Im}{\partial C_{\nu}^{\alpha}}. \tag{18}$$

These forms are identical with those that occur in quantum field theory [10]. By inserting equation (15) into (18), we get:

$$\partial_{\mu} \frac{\partial \Im_0}{\partial(\partial_{\mu} C_{\nu}^{\alpha})} = \frac{\partial \Im_0}{\partial C_{\nu}^{\alpha}} + g\eta_{1\alpha}^{\nu} \Im_0 - g\partial_{\mu}(\eta_{1\beta}^{\mu} C_{\mu}^{\alpha}) \frac{\partial \Im_0}{\partial(\partial_{\mu} C_{\nu}^{\alpha})} \quad . \tag{19}$$

Suppose that the gravitational gauge field C_{μ}^{α} is very weak in vacuum $gC_{\mu}^{\alpha} \approx 0$. Then, in leading order approximation, by substituting equation (14) to equations (19) we obtain:

$$\partial_{\mu} \frac{\partial \Im_0}{\partial(\partial_{\mu} C_{\alpha\nu})} = -\partial_{\mu} F_{\alpha}^{0\mu\nu} = \frac{\partial \Im_0}{\partial C_{\alpha\nu}} = 0. \tag{20}$$

The equations of motion for gravitons, thus, become:

$$\partial_{\mu} F_{\alpha}^{0\mu\nu} = 0. \tag{21}$$

We define

$$F_{ij}^{\alpha} = -\varepsilon_{ijk} B_k^{\alpha}, \quad F_{0i}^{\alpha} = E_i^{\alpha}. \tag{22}$$

Equation (21) is then changed into:

$$\nabla\vec{B}^{\alpha} = 0, \tag{23}$$

$$\frac{\partial}{\partial t}E^{\alpha} - \nabla\times B^{\alpha} = 0, \tag{24}$$

From definitions (22), we can prove that

$$\nabla\vec{E}^{\alpha} = 0, \tag{25}$$

$$\frac{\partial}{\partial t}B^{\alpha} + \nabla\times E^{\alpha} = 0. \tag{26}$$

Without superscript α, equations (23-26) would be the ordinary Maxwell equations. In conventional quantum field theory, the strength of gravitational field in vacuum is extremely weak, so the gravitational wave in vacuum is composed of four independent vector waves. Though the gravitational gauge field is a vector field, its component fields C^{α}_{μ} have one Lorentz index μ and one group index α. Both indexes have the same behavior under Lorentz transformation, a behavior that makes the gravitational field to resemble a tensor field. We thus call the gauge gravitational field a pseudo-tensor field. The spin of gauge gravitational field, determined by its behavior under Lorentz transformation, is 2.

In conventional quantum field theory, spin-1 field is a vector field, and vector field is a spin-1 field. In gravitational gauge field theory, this correspondence is violated. The reason is that, in gravitational gauge field theory, the group index contributes to the spin of a field, while in conventional gauge field theory; the spin of a field is independent of the group index.

3. Wilson loops of gravitational gauge fields in Rindler space

The Rindler space metric [19]

$$ds^2 = e^{2a\xi}\left(d\tau^2 - d\xi^2\right) - dx_{\perp}^2 \tag{27}$$

is the (Minkowski) metric seen by a uniformly accelerated observer (acceleration α in x_1-direction). Rindler space (τ,ξ) and Minkowski space $\left(t,x^1\right)$ coordinates are related by

$$t(\tau,\xi) = \frac{1}{a}e^{a\xi}\sinh a\tau,\ \ x^1(\tau,\xi) = \frac{1}{a}e^{a\xi}\cosh a\tau \tag{27}$$

The range of τ,ξ is

$$-\infty \le \tau,\zeta \le \infty, \tag{28}$$

while the preimage of the Rindler space covers only part of Minkowski space, the right Rindler wedge,

$$R_{+} = \{x^{\mu} \,|\, |t| \le x^1\} \tag{29}$$

The restriction of the preimage of the Rindler space to the right Rindler wedge gives rise to a horizon, the boundary $t = \pm x^1, \xi = -\infty$. With this property the Rindler metric can be identi_ed with other static metrics in the near horizon limit. In particular this is the case for the Schwarzschild metric which can be approximated in the limit that the distance from the horizon is small in comparison to the Schwarzschild radius and if the spherical Schwarzschild horizon is replaced by a tangential plane.

Following, F. Lenz et.al [6] we consider the gravitational gauge field coupled to external massive sources given by the action

$$S = -\frac{1}{4}\int d\tau d\xi d^2 x_\perp g^{\alpha\mu} g^{\beta\nu} F^{\alpha\mu} F_{\beta\nu} + S_{\text{int}}, \qquad S_{\text{int}} = \int d\tau d\xi d^2 x_\perp g_{\alpha\beta} C_\mu^\alpha T^{\beta\mu}. \tag{30}$$

The computation of Wilson loops [20, 21, 6] constitute the preferred techniques in analytical and numerical studies of interaction energies of static sources in gauge theories. Wilson loops are defined as integrals over the gauge field along a closed curve C in spacetime

$$e^{iW_C} = e^{ig\oint dx^\mu C_\mu^\alpha} \tag{31}$$

The invariance of the Wilson loop under gauge and (general) coordinate transformations and reparameterization which is explicit in Eq. (31) makes the Wilson loop a particularly useful tool for our purpose. Up to self energy contributions, the interaction energy of two oppositely massive sources is given by the expectation value (e.g. in the Minkowski space ground state) of a rectangular Wilson loop in a time-space plane with side lengths T and R

$$\Sigma_\pm = \lim_{T\to\infty} \frac{1}{T} \tilde{W}_{C[R,T]}, \tag{32}$$

with the ground state expectation value $\tilde{W}_{C[R,T]}$

$$e^{\tilde{W}_{C[R,T]}} = i\left\langle 0_M \mid e^{iW_{C[R,T]}} \mid 0_M \right\rangle, \tag{33}$$

The gravitational gauge fields along the loop can be interpreted as resulting from two opposite massive sources which are separated in an initial phase from distance 0 to R (for a rectangular loop this initial phase is reduced to one point in time), remain separated at this distance for the time T and recombine in a final phase. In order to make the contributions from the turning-on period negligible, the gravitational interaction energy of static mass is defined by the $T \to \infty$ limit. In terms of the graviton propagator

$$D_{\mu\nu}^{\alpha\beta}(x,x') = i\left\langle 0_M \mid T\left[C_\mu^\alpha(x) C_\nu^\beta(x')\right] \mid 0_M \right\rangle, \tag{34}$$

the Wilson loop is given by (cf. [20])

$$\tilde{W}_C = \frac{g^2}{2}\int ds \int ds' \frac{dx_C^\mu}{ds}\frac{dx_C'^\nu}{ds} D_{\mu\nu}^{\alpha\beta}(x_C(s), x_C'(s')). \tag{35}$$

In Lorenz gauge,

$$\partial^\mu C_\mu = 0, \tag{36}$$

The Minkowski space graviton propagator is expressed in terms of the scalar propagator as

$$D_{\mu\nu}^{\alpha\beta(M)}(x,x') = g^{\alpha\beta}\eta_{\mu\nu} D(x,x') = \frac{g^{\alpha\beta}\eta_{\mu\nu}}{4i\pi^2[(x-x')^2 - i\delta]}, \tag{37}$$

with the Minkowski space metric $\eta_{\mu\nu}$. The Rindler space graviton propagator is obtained by the change in coordinates (27)

$$D_{\mu\nu}^{\alpha\beta}(\tau-\tau',\xi,\xi' x_\perp - x'_\perp) = g^{\alpha\beta}\lambda_{\mu\nu}D(x(\upsilon),x'(\upsilon'))$$
$$= \frac{a^2 e^{-a(\xi+\xi')}}{8i\pi^2}\frac{g^{\alpha\beta}\lambda_{\mu\nu}(\upsilon,\upsilon')}{\cosh a(\tau-\tau')-\cosh\eta - i\delta}, \tag{38}$$

where we have used the notation

$$\upsilon^{(')} = \{\tau^{(')},\xi^{(')},\upsilon_\perp^{(')}\},\ \lambda_{\mu\nu}(\upsilon,\upsilon') = \frac{dx^\rho}{d\upsilon^\mu}\frac{dx'^\sigma}{d\upsilon'^\nu}\eta_{\mu\nu} \tag{39}$$

Under the coordinate transformation, the Lorenz gauge condition becomes

$$\nabla_\mu C_\alpha^\mu = \partial_\tau C_\alpha^\tau(\partial_\xi + 2a)C_\alpha^\xi + \partial_\perp C^\perp = 0, \tag{40}$$

with the covariant derivative ∇_μ.

Under the following coordinate transformation (27) the shape of a loop changes. With the change in shape also the value of the gravitational interaction energy changes which is defined with respect to two different limits ($t\to\infty$ or $\tau\to\infty$) [6]. We compute the gravitational interaction energy for a rectangular loop with 2 of the 4 segments of the loop varying in time τ and $\xi, x_\perp$ kept fixed while the other two segments are computed at fixed τ. The sum W_0 of the two contributions from the integration along the τ axis can be carried out without specifying the segments in the spatial coordinates. Inserting (cf. Eq. (38))

$$D_{00}^{00}(\tau-\tau',\xi,\xi' x_\perp - x'_\perp) = g^{00}\,\frac{a^2}{8i\pi^2}\frac{\cosh a(\tau-\tau')}{\cosh a(\tau-\tau')-\cosh\eta - i\delta}, \tag{41}$$

into Eq. (35) we find

$$W_0 = \frac{g^2a^2}{8i\pi^2}\int_0^T ds\int_0^T ds'\left[\frac{\cosh a(s-s')}{\cosh a(s-s')-(1+i\delta)} - \frac{\cosh a(s-s')}{\cosh a(s-s')-(\cosh\eta+i\delta)}\right] \tag{42}$$

Here $\cosh\eta$ is given by

$$\cosh\eta(\xi,\xi',x_\perp - x'_\perp) = 1 + \frac{(e^{a\xi}-e^{a\xi'})^2 + a(x_\perp - x'_\perp)}{2e^{a(\xi+\xi')}} = 1+\sigma^2(\xi,\xi',x_\perp - x'_\perp). \tag{43}$$

with the spatial coordinates $\xi^{(')}, x_\perp^{(')}$ of the vertices of the rectangle. Introducing $s\pm s'$s as integration variables W_0 can be rewritten as

$$W_0 = \frac{g^2a^2}{8i\pi^2}\int_0^T ds\left[I_0(s,1+i\delta) - I_0(s,\cosh\eta + i\delta)\right], \tag{44}$$

With

$$I_0(s,\cosh\eta+\delta)=2\int_0^s ds'\frac{\cosh as'}{\cosh as'-\cosh\eta-i\delta}=2s+\frac{2i\pi\cosh\eta}{a\sqrt{\sinh^2\eta+2i\delta}}$$
$$\times\left[1-\frac{i}{\pi}\left(\ln\frac{e^{as}-\cosh\eta-\sqrt{\sinh^2\eta+2i\delta}}{e^{as}-\cosh\eta+\sqrt{\sinh^2\eta+2i\delta}}+\ln\frac{1-\cosh\eta+\sqrt{\sinh^2\eta+2i\delta}}{\cosh\eta+\sqrt{\sinh^2\eta+2i\delta-1}}\right)\right] \tag{45}$$

The last step can be verified by differentiation. The contribution to the interaction energy V of two oppositely massive sources can be extracted in the $T\to\infty$ limit from the two s' - independent terms in (45) (the integrals of the s' - dependent terms converge). For large T, the integration along the spatial segments of the rectangle (the horizontal segments of the loop [6]) yields a T-independent term and does therefore not contribute to the gravitational interaction energy. The non-diagonal element D_{01} gives rise to two space-time contributions to the Wilson loop which in the large T limit become independent of the spatial coordinates and cancel each other. Thus we obtain the asymptotic value Σ of the Wilson loop expressed in terms σ [6]

$$\Sigma_{\pm}(\sigma)\lim_{T\to\infty}\frac{1}{T}W_0=-\frac{g^2}{4\pi}s\coth\eta\left[1+\frac{i\eta}{\pi}\right]+U_0=V(\sigma)+U_0, \tag{46}$$

with the gravitational interaction energy

$$V(\sigma)=-\frac{g^2}{4\pi}a\frac{1+\sigma^2}{\sqrt{2\sigma^2+\sigma^4}}\left[1+\frac{i}{\pi}\ln\frac{\sqrt{2\sigma^2+\sigma^4}+\sigma^2}{\sqrt{2\sigma^2+\sigma^4}-\sigma}\right]. \tag{47}$$

The integration “constant" U_0 arises from the first term in (42) and represents the selfenergy of the static mass. Regularizing the divergent integrals by point splitting, U_0 is given in terms of the proper distance in AdS4 [6]

$$U_0=\frac{g^2a}{8\pi}\left(\frac{1}{\sqrt{2}\delta\sigma(\xi)}+\frac{1}{\sqrt{2}\delta\sigma(\xi')}+\frac{2i}{\pi}\right),\quad \delta\sigma^2(\xi')=\frac{a^2}{2}\left(\delta\xi^2+e^{-2a\xi'}\delta x_\perp^2\right). \tag{48}$$

Here,Eq. (30) generated by two mass moving along the trajectories is parametrized as

$$T_i^{\alpha\mu}=\sum_i m_i\int ds_i\,\frac{d\upsilon_i^{\alpha\mu}(s_i)}{ds_i}\delta^4(\upsilon-\upsilon_i(s_i)) \tag{49}$$

resulting in the graviton-charge vertex

$$S_{\text{int}}=\sum_i m_i\int ds_i C_\mu^\alpha(\upsilon_i(s_i))\frac{d\upsilon_{\alpha i}^\mu}{ds_i}. \tag{50}$$

The relevant quantity to be computed is the effective action which, for mass at rest in Rindler space, yields the sum of interaction and self energies

$$W_{vc}=\frac{1}{2}\sum_{i=1,2}m_i m_j\int ds_i\int ds_j D_{\mu\nu}^{\alpha\beta}(\upsilon_i(s_i),\upsilon_j(s_j))\frac{d\upsilon_{\alpha i}^\mu(s_i)}{ds_i}\frac{d\upsilon_{\beta j}^\nu(s_j)}{ds_j} \tag{51}$$

where the propagators in different coordinates are obtained from each other by the corresponding coordinate transformations (cf. Eq. (38)). We define the Fourier transform in time of the 00-component of the propagator (Eq. (41)) by the limit

$$\begin{aligned}\tilde{D}_{00}^{00}(\omega,\xi,\xi',x_\perp) &= \lim_{T\to\infty}\int_{-T}^{T} d\tau e^{i\omega\tau} D_{00}^{00}(\tau,\xi,\xi',x_\perp)\\ &= \frac{a^2\sin\omega T}{4i\pi^2\omega} + \frac{a^2}{4\pi}\coth\eta\left[e^{-i\frac{\omega\eta}{a}} + \frac{2i\sin\frac{\omega\eta}{a}}{1-e^{-2\pi\frac{\omega}{a}}}\right],\end{aligned} \tag{52}$$

and disregarding the (divergent) constant, for opposite massive sources m = m_1 = -m_2, the result (47)

$$-m^2\tilde{D}_{00}^{00}(0,\xi,\xi',x_\perp) = V(\sigma), \tag{53}$$

is reproduced.

For small distances the imaginary part is constant and agrees with the constant in the finite temperature propagator in Minkowski space

$$\lim_{\sigma\to 0} V(\sigma) = \frac{m^2 a}{4\pi}\left[\frac{1}{\sqrt{2}\sigma}(1+\frac{3}{2}\sigma^2) + \frac{i}{\pi}(1+\frac{2}{3}\sigma^2)\right]. \tag{54}$$

With increasing distance the inertial forces become important. The weaken graviton interaction energies and change completely the asymptotic behavior

$$\lim_{\sigma\to 0} V(\sigma) = -\frac{m^2 a}{4\pi}\left(1 + \frac{1}{2\sigma^4} + \frac{i}{\pi}\ln 2\sigma^2\right). \tag{55}$$

Of particular interest is the behavior of the interaction energy if one of the sources approaches the horizon while the position of the other source is kept fixed. For

$$\xi_h \to -\infty, \sigma^2 \to \frac{1}{2}e^{-a(\xi_h+\xi)}(e^{2a\xi} + a^2(x_\perp - x'_\perp)^2) \to \infty \tag{56}$$

the interaction energies are given by

$$\lim_{\xi_h\to\infty} V(\sigma) = -\frac{m^2 a}{4\pi}\left(1 - \frac{a(\xi_h+\xi)}{\pi}\right) \tag{57}$$

We also find in analogy with the ‘’no hair" theorem [22], [23], [24] for Schwarzschild black holes that a scalar source close to the horizon cannot be observed asymptotically while vector sources are visible [6]. when approaching the horizon while the gravitational coupling remains constant. In detail the results for Rindler and Schwarzschild metrics are different. A significant improvement can be obtained by modifying the Rindler metric

$$ds^2 = e^{2a\xi}(d\tau^2 - d\xi^2) - dx_\perp^2 \to e^{2a\xi}(d\tau^2 - d\xi^2) - \frac{1}{4a^2}d\Omega^2 \text{ [6]} \tag{58}$$

The dynamics in this space is very close to the dynamics in Rindler space unless the difference between the S^2 and R^2 matters. This is the case when deriving the “no hair" theorem where only the *l = 0* waves matter [25].

4. Polyakov loop correlator of gravitation gauge fields in Rindler space

In numerical studies of gauge theories at finite temperature on the lattice an important quantity for characterizing the interaction energy of static charges is the correlation function of Polyakov loops [21, 6]. These studies are carried out on a Euclidean lattice and have confirmed the existence of a transition in Yang-Mills theories from the confining to the deconfined phase. It would be of great interest to extend these studies to Rindler space and to follow the fate of the confined phase when approaching the horizon. Here, we will calculate the Polyakov loop correlator in Rindler space with imaginary (Rindler) time ([6]) which due to the acceleration is a periodic coordinate. Up to a multiplication with i, the ‘’Euclidean" propagators are obtained from the real time propagators (38)) by this change of the time coordinate. In particular the relevant component of the gravitational gauge field propagator (41) is given by

$$D_{00}^{E}(\tau_E-\tau'_E,\xi,\xi',x_\perp-x'_\perp)=\frac{a^2}{8\pi^2}\frac{\cos a(\tau_E-\tau'_E)}{\cos a(\tau_E-\tau'_E)-\cosh\eta-i\delta}. \tag{59}$$

The periodicity in imaginary time expresses the similarity of acceleration and finite temperature. Unlike the temperature, the acceleration a also appears together with the spatial coordinates. The Polyakov loop is defined by

$$P(m,\xi,x_\perp)=\exp\{im\oint d\tau_E C_0^{0(E)}(\tau_E,\xi,x_\perp)\}, \tag{60}$$

and the Polyakov loop correlator associated with two static charges $m_{1,2}$ located at $\xi_{1,2}$ $x_{\perp 1,2}$ is given by

$$C_P(\xi_1,\xi_2,x_{\perp 1},x_{\perp 2})=\langle 0_M\,|\,C_P(m_1,\xi_1,x_{\perp 1})C_P(m_2,\xi_2,x_{\perp 2})\,|\,0_M\rangle \tag{61}$$

Written as a path integral, this correlation function is easily evaluated with the result

$$C_P(\xi_1,\xi_2,x_{\perp 1},x_{\perp 2})=e^{-\frac{2\pi}{a}(f_{11}+f_{22}+2f_{21})}, \tag{62}$$

where the self and interaction energy contributions to the ‘’free energy" f_{ij} [6]

$$\begin{aligned} f_{ij}&=\frac{am_i m_j}{32\pi^3}\int_0^{2\pi}ds\int_0^{2\pi}ds'\frac{\cos(s-s')}{\cosh\eta(\xi_i,\xi_j,x_{\perp i},x_{\perp j})-\cos(s-s')}\\ &=\frac{am_i m_j}{8\pi}(\coth\eta(\xi_i,\xi_j,x_{\perp i},x_{\perp j})-1). \end{aligned} \tag{64}$$

Regularization by point splitting (cf. Eq. (48)) yields the gravitational self-energies

$$f_{ii}=\frac{m_i^2}{8\pi}\frac{1}{\sqrt{\delta\xi^2+e^{-2a\xi_i}\delta x_\perp^2}}. \tag{65}$$

The gravitational interaction energy of the two masses

$$V_P(\xi_i,\xi_j,x_{\perp i},x_{\perp j}) = \frac{am_1m_2}{4\pi}\coth\eta(\xi_i,\xi_j,x_{\perp i},x_{\perp j})-1), \tag{66}$$

In agreement with the results of [26] concerning the equivalence of propagators defined in static space-time with either real or imaginary times, the Rindler space propagator (cf. Eq. (41)) can be reconstructed given the imaginary time propagator (cf. Eq. (59)). However, unlike in Minkowski space, this reconstruction may not work separately for single Fourier components such as the static component of the propagators. Apparently, the difference between imaginary and real time static propagators of the gravitational field is due to the non-trivial (imaginary) contributions to the propagator from zero energy gravitons.

5. Conclusion

The gravitational force will be determined via Wilson loops and Polyakov loop correlation functions. This method will enable us to separate the contribution of the quantum mechanical transverse graviton from that of the classical longitudinal field. It will be the method of choice if one attempts to determine the gravitational static force in simulations of QGTG on a Rindler space lattice.

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