Title: Goldbach Conjecture; Proof(?)

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Abstract: The Goldbach Conjecture may be stated as follows:

Every even number greater than 4 can be written as the sum of two prime numbers.

Thus 6 = 3+3
     8 = 3+5
     10 = 3+7 or 5+5

This attempt does not require knowledge of the distribution of primes.
**Proof**  
Consider the ordered set $S_P$ of all primes $\leq P$.  
\[ S_P = \{3, 5, 7 \ldots P\} \]  
The ordered set $S_E$ of distinct even numbers that can be made as the sum of 2 primes from $S_P$ is:  
\[ S_E = \{6, 8, \ldots, 2P\} \]  
Assume the Goldbach Conjecture is true; thus all the even numbers in the range $[6, 2P]$ are in $S_E$.  

Example  
$P=7$; $S_P = \{3, 5, 7\}$; $S_E = \{6, 8, 10, 12, 14\}$  
The even number after $2P$ is $2P+2$: we will show it is also the sum of 2 primes.  

Any pair of primes summing to $2P+2$ must be in the interval $[3, 2P-1]$ with the larger one $>P$.  

Assume  
\[ A+B = 2P+2 \quad \{A, B \text{ prime; } A>B; \} \]  
\[ (B\neq A \text{ as this would mean } A \text{ and } B \text{ are even}) \]  
thus  
\[ P<A<=2P-1 \quad \{A \text{ is not in } S_P\} \]  
and  
\[ 3<=B<=P \quad \{B \text{ is in } S_P\} \]  

**Bertrand’s Theorem** confirms there is at least one prime number between $P$ and $2P$.  
\[ \therefore \text{ a prime number } A \text{ exists.} \]  

There are 2 cases to consider:  

(i)  
$2P-1$ is prime so $2P+2 = 3 + (2P-1)$  
\[ \therefore A=2P-1, B=3 \text{ are a "Goldbach Pair", or,} \]  

(ii)  
$2P-1$ is not prime so that $3<B<=P<A<2P-1$.  
\[ \therefore B+1 \text{ is in } S_E \text{ and } B \text{ is in } S_P. \]  
Thus $(A, B)$ are a Goldbach Pair for case (ii).  
And so on, ... , extending $S_P$ and $S_E$.  

Thus Goldbach’s Conjecture is true.