Maximal Acceleration Perspective Problems.

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Abstract

We determine nonlinear transformations between coordinate systems which are mutually in a constant symmetrical accelerated motion. The maximal acceleration limit follows from the kinematical origin. Maximal acceleration is an analogue of the maximal velocity in special relativity. We derive the dependence of mass, length, time, Doppler effect, Čerenkov effect and transition radiation angle on acceleration as an analogue phenomena in special theory of relativity. The derived addition theorem for acceleration can play crucial role in modern particle physics and cosmology.

1 Introduction

The problem of acceleration of charged particles or systems of particles is the permanent and the most prestige problem in the accelerator physics. Particles can be accelerated by different ways. Usually by the classical electromagnetic fields, or, by light pressure of the laser fields (Baranova et al., 1994; Pardy, 1998, 2001, 2002). The latter method is the permanent problem of the laser physics for many years.

Here, we determine transformations between coordinate systems which moves mutually with the same acceleration. We determine transformations between non relativistic and relativistic uniformly accelerated systems.

Let us remind the special theory of relativity velocity and acceleration The Lorentz transformation between two inertial coordinate systems $S(0, x, y, z)$ and $S'(0, x', y', z')$ (where system S'

moves in such a way that x-axes converge, while y and z-axes run parallel and at time $t = t' = 0$ for the origin of the systems O and O' it is $O \equiv O'$) is as follows:

$$x' = \gamma(v)(x - vt), \quad y' = y, \quad z' = z', \quad t' = \gamma(v)\left(t - \frac{v}{c^2}x\right), \tag{1}$$

where

$$\gamma(v) = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}. \tag{2}$$

The infinitesimal form of this transformation is evidently given by differentiation of the every equation. Or,

$$dx' = \gamma(v)(dx - vdt), \quad dy' = dy, \quad dz' = dz, \quad dt' = \gamma(v)\left(dt - \frac{v}{c^2}dx\right). \tag{3}$$

It follows from eqs. (3) that if v_1 is velocity of the inertial system 1 with regard to S and v_2 is the velocity of the inertial systems 2 with regard to 1, then the relativistic sum of the two velocities is

$$u_2 = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}. \tag{4}$$

The mathematic object called four-velocity follows from the Lorentz transformation after some additional operations. From the ordinary three-dimensional velocity vector one can form a four-vector. This four-dimensional velocity (four-velocity) of a particle is the vector

$$u^\mu = \frac{dx^\mu}{ds}, \tag{5}$$

where, according to Landau et al. (1987)

$$ds = cdt\sqrt{1 - \frac{v^2}{c^2}} \tag{6}$$

with v being the ordinary three-dimensional velocity of the particle and c being the velocity of light. Thus

$$u^1 = \frac{dx^1}{ds} = \frac{dx}{cdt\sqrt{1 - \frac{v^2}{c^2}}} = \frac{v_x}{c\sqrt{1 - \frac{v^2}{c^2}}}. \tag{7}$$

Then,

$$u^\mu = \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \frac{\mathbf{v}}{c\sqrt{1 - \frac{v^2}{c^2}}}\right). \tag{8}$$

Note, that the four-velocity is a dimensionless quantity. The components of the four-velocity are not independent. Noting that $dx^\mu dx_\mu = ds^2$, we have

$$u^\mu u_\mu = 1. \tag{9}$$

Geometrically, u^μ is a unit four-vector tangent to the world line of the particle.

Similarly to the definition of the four-velocity, the second derivative

$$a^\mu = \frac{d^2 x^\mu}{ds^2} = \frac{du^\mu}{ds} \tag{10}$$

may be called the four-acceleration. Differentiating formula (9), we find:

$$u^\mu a_\mu = 0, \tag{11}$$

i.e. the four-vectors of velocity and acceleration are "mutually perpendicular".

Now, let us determine the relativistic uniformly accelerated motion, i.e. the rectilinear motion for which the acceleration a^μ in the proper reference frame (at each instant of time) remains constant. We proceed as follows.

In the reference frame in which the particle velocity is $v = 0$, the components of the four-acceleration $a^\mu = (0, a/c^2, 0, 0)$ (where $\mathbf{a}$ is the ordinary three-dimensional acceleration, which is directed along the x axis). The relativistically invariant condition for uniform acceleration must be expressed by the constancy of the four-scalar which coincides with a^2 in the proper reference frame:

$$a^\mu a_\mu = const = -\frac{a^2}{c^4}. \tag{12}$$

In the "fixed" frame, with respect to which the motion is observed, writing out the expression for $a^\mu a_\mu$ gives the equation:

$$\frac{d}{dt}\frac{v}{\sqrt{1-\frac{v^2}{c^2}}} = a, \tag{13}$$

or,

$$\frac{v}{c\sqrt{1-\frac{v^2}{c^2}}} = at + const. \tag{14}$$

Setting $v = 0$ for $t = 0$, we find that $const = 0$, so that

$$v = \frac{at}{\sqrt{1+\frac{a^2t^2}{c^2}}}, \tag{15}$$

Integrating once more and setting $x = 0$ for $t = 0$, we find:

$$x = \frac{c^2}{a}\left(\sqrt{1+\frac{a^2t^2}{c^2}} - 1\right). \tag{16}$$

For $at \ll c$, these formulas go over the classical expressions $v = at, x = \frac{a}{2}t^2$. For $at \to \infty$, the velocity tends toward the constant value c.

The proper time of a uniformly accelerated particle is given by the integral (Landau et al., 1987)

$$\int_0^t \sqrt{1+\frac{v^2(t)}{c^2}}dt = \frac{c}{a}\mathrm{arcsinh}\frac{at}{c}. \tag{17}$$

At the limit $t \to \infty$ it increases much more slowly than t, according to the law

$$\frac{c}{a}\ln\frac{2at}{c}. \tag{18}$$

The infinitesimal form of Lorentz transformation (3) evidently gives the Lorentz length contraction and time dilation. Namely, if we put $dt = 0$ in the first equation of system (3), then the Lorentz length contraction follows in the infinitesimal form $dx' = \gamma(v)dx$. Or, in other words, if in the system S' the infinitesimal length is dx', then the relative length with regard to the system S is $\gamma^{-1}dx'$. Similarly, from the last equation of (3) it follows the time dilatation for $dx = 0$. Historical view on this effect is in the Selleri article (Selleri, 1997).

2 Uniformly accelerated frames with space-time symmetry

Let us take two systems $S(0,x,y,z)$ and $S'(0,x',y',z')$, where system S' moves in such a way that x-axes converge, while y and z-axes run parallel and at time $t = t' = 0$ for the beginning of the systems O and O' it is $O \equiv O'$. Let us suppose that system S' moves relative to some basic system B with acceleration $a/2$ and system S' moves relative to system B with acceleration $-a/2$. It means that both systems moves one another with acceleration a and are equivalent because in every system it is possibly to observe the force caused by the acceleration $a/2$. In other words no system is inertial.

Now, let us consider the formal transformation equations between two systems. At this moment we give to this transform only formal meaning because at this time, the physical meaning of such transformation is not known. On the other hand, the consequences of the transformation will be shown very interesting. The first published derivation of such transformation by the standard way was given by author (Pardy, 2003; 2004; 2005), and the same transformations were submitted some decades ago (Pardy, 1974). The old results can be obtained if we perform transformation

$$t \to t^2, \quad t' \to t'^2, \quad v \to \frac{1}{2}a, \quad c \to \frac{1}{2}\alpha \tag{19}$$

in the original Lorentz transformation (1). We get:

$$x' = \Gamma(a)(x - \frac{1}{2}at^2), \quad y' = y, \quad z' = z, \quad t'^2 = \Gamma(a)\left(t^2 - \frac{2a}{\alpha^2}x\right) \tag{20}$$

with

$$\Gamma(a) = \frac{1}{\sqrt{1 - \frac{a^2}{\alpha^2}}}. \tag{21}$$

We used practically new denotation of variables in order to get the transformation (20) between accelerated systems.

The transformations (20) form the one-parametric group with the parameter a. The proof of this mathematical statement can be easy performed if we perform the transformation T_1 from S to S', transformation T_2 from S' to S'' and transformation T_3 from S to S''. Or,

$$x' = x'(x,t;a_1), \quad t' = t'(x,t;a_1), \tag{22}$$

$$x'' = x''(x',t';a_2), \quad t'' = t''(x',t';a_2), \tag{23}$$

After insertion of transformations (22) into (23), we get

$$x'' = x''(x,t;a_3), \quad t'' = t''(x,t;a_3), \tag{24}$$

where parameter a_3 is equal to

$$a_3 = \frac{a_1 + a_2}{1 + \frac{a_1 a_2}{\alpha^2}}. \tag{25}$$

The inverse parameter is $-a$ and parameter for identity is $a = 0$ It may be easy to verify that the final relation for the definition of the continuous group transformation is valid for our transformation. Namely (Eisenhart, 1943):

$$(T_3T_2)\,T_1 = T_3\,(T_2T_1)\,. \tag{26}$$

The physical interpretation of this nonlinear transformations is the same as in the case of the Lorentz transformation only the physical interpretation of the invariant function $x = (1/2)\alpha t^2$ is different. Namely it can be expressed by the statement. If there is a physical signal in the system S with the law $x = (1/2)\alpha t^2$, then in the system S' the law of the process is $x' = (1/2)\alpha t'^2$, where α is the constant of maximal acceleration. It is new constant and cannot be defined by the game with known physical constants.

Let us remark, that it follows from history of physics, that Lorentz transformation was taken first as physically meaningless mathematical object by Larmor, Voigt and Lorentz and later only Einstein decided to put the physical meaning to this transformation and to the invariant function $x = ct$. We hope that the derived transformation will appear as physically meaningful.

Using relations $t \leftarrow t^2$, $t' \leftarrow t'^2$, $v \leftarrow \frac{1}{2}a$, $c \leftarrow \frac{1}{2}\alpha$, the nonlinear transformation is expressed as the Lorentz transformation forming the one-parametric group. This proof is equivalent to the proof by the above direct calculation. The integral part of the group properties is the so called addition theorem for acceleration.

$$a_3 = \frac{a_1 + a_2}{1 + \frac{a_1 a_2}{\alpha^2}}. \tag{27}$$

where a_1 is the acceleration of the S' with regard to the system S, a_2 is the acceleration of the system S'' with regard to the system S' and a_3 is the acceleration of the system S'' with regard to the system S. The relation (27), expresses the law of acceleration addition theorem on the understanding that the events are marked according to the relation (20).

If $a_1 = a_2 = a_3 = ... + a_n = a$, for n accelerated carts which rolls in such a way that the first cart rolls on the basic cart, the second rolls on the first cart and so on, then we get for the sum of n accelerated carts the following formula

$$a_{sum} = \frac{1 - \left(\frac{1-a/\alpha}{1+a/\alpha}\right)^n}{1 + \left(\frac{1-a/\alpha}{1+a/\alpha}\right)^n}, \tag{28}$$

which is an analogue of the formula for the inertial systems (Lightman et al., 1975).

In this formula as well as in the transformation equation (20) appears constant α which cannot be calculated from the theoretical considerations, or, constructed from the known physical constants (in analogy with the velocity of light). What is its magnitude can be established only by experiments. The notion maximal acceleration was introduced some decades ago by author (Pardy, 1974). Caianiello (1981) introduced it as some consequence of quantum mechanics and Landau theory of fluctuations. Revisiting view on the maximal acceleration was given by Papini (2003). At present time it was argued by Lambiase et al. (1999) that maximal acceleration determines the upper limit of the Higgs boson and that it gives also the relation which links the mass of W-boson with the mass of the Higgs boson. The LHC and HERA experiments presented different answer to this problem.

3 Transformation with constant acceleration in the fixed frame

In the "fixed" frame, with respect to which the motion is observed, we use the equation (13) to derive the adequate transformation: Or,

$$\xi(a,t) = \frac{c^2}{a}\left(\sqrt{1 + \frac{a^2t^2}{c^2}} - 1\right). \tag{29}$$

For $at \ll c$, these formulas go over the classical expressions $v = at, x = \frac{1}{2}at^2$. For $at \to \infty$, the velocity tends toward the constant value c.

The transformation equations between S and S' can be easily derived. Let us give some instructions.

It may be easy to see, that

$$x' = \Gamma(a)(x - \xi(a,t)), \quad y' = y, \quad z' = z, \tag{30}$$

with

$$\Gamma(a) = \frac{1}{\sqrt{1 - \frac{a^2}{\alpha^2}}}. \tag{31}$$

Then,

$$x = \Gamma(a)(x + \xi(a,t')) \tag{32}$$

and

$$\xi(a,t') = \Gamma^{-1}x - x' = x/\Gamma - \Gamma x + \Gamma\xi(a,t) \tag{33}$$

It follows from the last equation the variable t' and the identity $\xi(\alpha,t') = \xi(\alpha,t)$.

Let us remark, that if we use the infinitesimal transformation (3) with the velocity depending on time (15), then we obtain after integration the new original transformation for ccelerated systems (Pardy, 2003, 2004, 2005) with the new physical meaning.

4 Dependence of mass, length, time, the Doppler effect, the Čerenkov effect and the transition radiation angle on acceleration

If the maximal acceleration is the physical reality, then it should have the similar consequences in a dynamics as the maximal velocity of motion has consequences in the dependence of mass on velocity. We can suppose in analogy with the special relativity that mass depends on the acceleration for small velocities, in the similar way as it depends on velocity in case of uniform motion. Of course such assumption must be experimentally verified and in no case it follows from special theory of relativity, or, general theory of relativity (Fok, 1961). So, we postulate ad hoc, in analogy with special theory of relativity:

$$m(a) = \frac{m_0}{\sqrt{1 - \frac{a^2}{\alpha^2}}}; \quad v \ll c, \quad a = \frac{dv}{dt}. \tag{34}$$

Let us derive as an example the law of motion when the constant force F acts on the body with the rest mass m_0. Then, the Newton law reads (Landau et al., 1997):

$$F = \frac{dp}{dt} = m_0\frac{d}{dt}\frac{v}{\sqrt{1 - \frac{a^2}{\alpha^2}}}. \tag{35}$$

The first integral of this equation can be written in the form:

$$\frac{Ft}{m_0} = \frac{v}{\sqrt{1 - \frac{a^2}{\alpha^2}}}; \quad a = \frac{dv}{dt}, \quad F = const.. \tag{36}$$

Let us introduce quantities

$$v = y, \quad a = y', \quad A(t) = \frac{F^2t^2}{m_0^2\alpha^2}. \tag{37}$$

Then, the quadratic form of the equation (36) can be written as the following differential equation:

$$A(t)y'^2 + y^2 - A(t)\alpha^2 = 0, \tag{38}$$

which is nonlinear differential equation of the first order. The solution of it is of the form $y = Dt$, where D is some constant, which can be easily determined. Then, we have the solution in the form:

$$y = v = Dt = \frac{t}{\sqrt{\frac{m_0^2}{F^2} + \frac{1}{\alpha^2}}}. \tag{39}$$

For F $\to \infty$, we get $v = \alpha t$. This relation can play substantial role at the beginning of the big-bang, where the accelerating forces can be considered as infinite, however the law of acceleration has finite nonsingular form.

At this moment it is not clear if the dependence of the mass on acceleration can be related to the energy dependence on acceleration similarly to the Einstein relation uniting energy, mass and velocity (Okun, 2001, 2002; Sachs, 1973).

The infinitesimal form of author transformation (20) evidently gives the length contraction and time dilation. Namely, if we put $dt = 0$ in the first equation of system (20), then the length contraction follows in the infinitesimal form $dx' = \Gamma(a)dx$. Or, in other words, if in the system S' the infinitesimal length is dx', then the relative length with regard to the system S is $\Gamma^{-1}dx'$. Similarly, from the last equation of (20) it follows the time dilatation for $dx = 0$.

The relativistic Doppler effect is the change in frequency (and wavelength) of light, caused by the relative motion of the source and the observer (as in the classical Doppler effect), when taking into account effects described by the special theory of relativity.

The relativistic Doppler effect is different from the non-relativistic Doppler effect as the equations include the time dilation effect of special relativity and do not involve the medium of propagation as a reference point (Rohlf, 1994).

The Doppler shift caused by acceleration can be also derived immediately from the original relativistic equations for the Doppler shift. We only make the transformation $v \to a/2, c \to \alpha/2$ to get

$$\frac{\lambda'}{\lambda} = \sqrt{\frac{1 - a/\alpha}{1 + a/\alpha}} \tag{40}$$

when the photons of the wave length λ are measured toward photon source, and

$$\frac{\lambda'}{\lambda} = \sqrt{\frac{1 + a/\alpha}{1 - a/\alpha}} \tag{41}$$

when the photons of the wave length λ are measured in the frame that is moving away from the photon source. Different approach used Friedman et al. (2010).

Concerning the Čerenkov radiation, it is based on the fact that the speed of light in the medium with the index of refraction n is c/n. A charged particle moving in such medium can have the speed greater than it is the speed of light in this medium. When a charged particle moves faster than the speed of light in this medium, a portion of the electromagnetic radiation emitted by excited atom along the path of the particle is coherent. The coherent radiation is emitted at a fixed angle with respect to the particle trajectory. This radiation was observed by Čerenkov in 1935. The characteistic angle was derived by Tamm and Frank in the form (Rohlf, 1994)

$$\cos\theta = \frac{c}{vn}. \tag{42}$$

The Čerenkov angle caused by acceleration can be also derived immediately from the original Frank-Tamm equations for this effect. We only make the transformation $v \to a/2, c \to \alpha/2$ to get

$$\cos\theta = \frac{\alpha}{an}. \tag{43}$$

In case of the Ginzburg transition radiation the radiation in concentraced in the angle

$$1/\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{44}$$

The transition radiation angle caused by acceleration can be also derived immediately from the original Ginzburg formula for this effect. We only make the transformation $v \to a/2, c \to \alpha/2$ to get

$$1/\Gamma = \frac{1}{\sqrt{1-\frac{a^2}{\alpha^2}}}. \tag{45}$$

5 The rotating systems

It is defined by equations

$$x = r\cos(\varphi + \omega t), \quad y = r\sin(\varphi + \omega t). \tag{46}$$

The corresponding space-time element is as follows:

$$ds^2 = \left(1 - \frac{\omega^2 r^2}{c^2}\right)(cdt)^2 - \frac{2\omega r^2}{c} d\varphi(cdt) - dz^2 - dr^2 - r^2 d\varphi^2. \tag{47}$$

Although the rotating system cannot be considered as equivalent to the linear accelerated system, nevertheless, the radial component of every part of this system is in the permanent acceleration. The application in the galactic space is evident. In other words, if the radial coordinate of Earth with regard to Sun is r_E and its radial acceleration w_E and the radial coordinate of Moon with regard to Earth is r_M and acceleration w_M, then the relative acceleration w_r of Moon with regard to Sun is not $w_E + w_M$, but it is given by the formula

$$w_r = \frac{w_E + w_M}{1 + \frac{w_E w_M}{\alpha^2}}. \tag{48}$$

The last formula is an analogue of the formula which determines the relative velocities in case of the inertial motion in the special theory of relativity. The last formula is true only if the transverse effects do not influence the radial effects. It can be verified optically, because we know that the optical frequency of the emission source is influenced by acceleration.

Similarly, it is possible to verify the dependence of mass on acceleration, also by the ultracentrifuge, or immediately by physics in LHC, or ELI.

6 Discussion

The maximal acceleration constant which was derived here is kinematical one and it differs from the Caianiello (1981) definition following from quantum mechanics. Our constant cannot be determined by the system of other physical constants. It is an analogue of the numeric velocity of light which cannot be composed from others physical constants, or, the Heisenberg fundamental length in particle physics. The nonlinear transformations (20) changes the Minkowski metric

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 \tag{49}$$

to the new metric with the Riemann form. Namely:

$$ds^2 = \alpha^2 t^2 dt^2 - dx^2 - dy^2 - dz^2 \tag{50}$$

and it can be investigated by the methods of differential geometry. So, equations (20) and (50) can form the preamble to investigation of accelerated systems.

If some experiment will confirm the existence of kinematical maximal acceleration α, then it will have certainly crucial consequences for Einstein theory of gravity because this theory does not involve this factor. Also the cosmological theories constructed on the basis of the original Einstein equations will require modifications. The so called Hubble constant will be changed and the scenario of the accelerating universe modified.

Also the standard model of particle physics and supersymmetry theory will require generalization because they does not involve the maximal acceleration constant. It is not excluded that also the theory of parity nonconservation will be modified by the maximal acceleration constant. In such a way the particle laboritories have perspective programes involving the physics with maximal acceleration.

The prestige problem in the modern theoretical physics - the theory of the Unruh effect, or, the existence of thermal radiation detected by accelerated observer - is in the development (Fedotov et al., 2002) and the serious statement, or comment to the relation of this effect to the maximal acceleration must be elaborated. The analogical statement is valid for the Hawking effect in the theory of black holes.

It is not excluded that the maximal acceleration constant will be discovered by ILC. The unique feature of the International Linear Collider (ILC) is the fact that its CM energy can be increased gradually simply by extending the main linac.

Let us remark in conclusion that it is possible to extend and modify quantum field theory by maximal acceleration. It is not excluded that the kinematical maximal acceleration constant will enable to reformulate the theory of renormalization.

REFERENCES

Baranova, N. B. and Zel'dovich, B. Ya. (1994). Acceleration of charged particles by laser beams, JETP **78**, (3), 249-258.

Caianiello, E. R. (1981). Is There Maximal Acceleration? Lett. Nuovo Cimento, **32**, 65 ; ibid. (1992). Revista del Nuovo Cimento, **15**, No. 4.

Eisenhart, L. P. *Continuous Groups of Transformations*, (Princeton, 1943).

Fedotov, A. M., Narozhny, N. B. , Mur, V. D. and Belinski, V. A. (2002). An Example of a Uniformly Accelerated Particle Detector with non-Unruh Response, arXiv: hep-th/0208061.

Fok, V. A. *The Theory of Space, Time and Gravity*, second edition, (GIFML, Moscow, 1961). (in Russian).

Friedman, Y. (2010). The Maximal Acceleration, Extended Relativistic Dynamics and Doppler type Shift for an Accelerated Source, arXiv:0910.5629v3 [physics.class-ph] 19 Jul 2010.

Lambiase, G., Papini, G. and Scarpetta, G. (1998). Maximal Acceleration Limits on the Mass of the Higgs Boson, arXiv:hep-ph/9808460; ibid. (1999). Nuovo Cimento B, **114**, 189-197.

Landau, L. D. and Lifshitz, E. M., *The Classical Theory of Fields*, 7-th ed., (Pergamon Press, Oxford, 1987).

Lightman, A. P., Press W. H., Price, R. H. and Teukolsky S. A. *Problem Book in Relativity and Gravitation*, (Princeton University Press, 1975).

Okun, L. B. (2000). Photons, Gravity and Comcept of Mass, arXiv:physics/0111134v1, ibid. (2002). Nucl. Phys. Proc. Suppl. **110**, 151-155.

Papini, G. (2003). Revisiting Caianiello's Maximal Acceleration, e-print quant-ph/0301142.

Pardy, M. (1974). The Group of Transformations for Accelerated Systems, Jan Evangelista Purkyně University, Brno (unpublished).

Pardy, M. (1998). The Quantum Field Theory of Laser Acceleration, Phys. Lett. A **243**, 223-228.

Pardy, M. (2001). The Quantum Electrodynamics of Laser Acceleration, Radiation Physics and Chemistry **61**, 391-394.

Pardy, M. (2002). Electron in the Ultrashort Laser Pulse, arXiv:hep-ph/02372; ibid International Journal of Theor. Physics, **42**, No.1, 99.

Pardy, M. (2003). The Space-Time Transformation and the Maximal Acceleration, arXiv:grqc/0302007.

Pardy, M. (2004). The Space-Time Transformation and the Maximal Acceleration, Space-time&Substance Journal, 1(21), (2004), pp. 17-22.

Pardy, M. (2005). Creativity Leading to Discoveries in Particle Physics and Relativity, arXiv:physics/0509184

Rohlf, J. W. *Moderm Physics from α to Z^0* (John Wiley & Sons, Inc. New York, 1994).

Sachs, M. (1973). On the Meaning of $E = mc^2$, International Journal of Theoretical Physics, **8**, No. 5, 377 (1973).

Selleri, F. (1997). The Relativity Principle and Nature of Time, Foundations of Physics, **27**, No. 11, 1527.