## LAUNCHING THE SIX-COLORING BARYON-ANTIBARYON ANTISYMMETRIC ISO-WAVEFUNCTIONS AND ISO-MATRICES\*

Nathan O. Schmidt Department of Mathematics Boise State University 1910 University Drive Boise, ID 83725, USA nathanschmidt@u.boisestate.edu

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### Abstract

In this work, we upgrade the Inopin-Schmidt quark confinement and baryonantibaryon duality proof with Santilli's new iso-mathematics. For a baryon-antibaryon pair confined to the six-coloring kagome lattice of the Inopin Holographic Confinement Ring (IHCR), we construct a cutting-edge procedure that iso-topically lifts the antisymmetric wavefunctions and matrices to iso-wavefunctions and iso-matrices, respectively. The initial results support our hypothesis that transitions between the energy and resonance states of the hadronic spectra may be rigorously characterized by properlycalibrated iso-topic liftings. In total, these rich developments suggest a promising future for this emerging iso-confinement framework, which must be subjected to additional scientific inquiry, scrutiny, and exploration.

**Keywords**: High energy particle physics; Higgs physics; Geometry and topology; Isomathematics; Complex systems; Quark confinement; Baryon iso-wavefunction; Kagome lattice theory; Spontaneous gauge symmetry breaking; Superfluids.

<sup>\*</sup>To the memory and honor of Dr. Andrej "Andy" Inopin.

### 1 Introduction

The new discipline of *iso-mathematics* [1, 2, 3, 4, 5] generates striking implications for science, technology, engineering, and mathematics. For example, its deployment has influenced novel innovations such as Santilli's hadronic mechanics [6], intermediate controlled nuclear synthesis [7, 8, 9, 10, 11, 12], and magnecular-based fuels [13, 14, 15, 16] for new, clean, sustainable, cost-efficient power sources that do not emit harmful radiation or toxic waste. Moreover, some of our recent work has initiated iso-fractals with chaos [17, 18, 19, 20], dynamic IHCR-based topological systems [21, 22], a topological iso-string theory in 4D iso-dual space-time [23], the iso-dual tesseract [24], the iso-electronium and magnecule order parameter upgrade hypotheses [25, 26, 27], and more.

In this paper, we focus on implementing iso-mathematics [1, 2, 3, 4, 5] in the Inopin-Schmidt quark confinement proof [28], where we will demonstrate that the *baryon-antibaryon antisymmetric wavefunction* can be iso-topically lifted to a new *iso-wavefunction* that establishes 2D and 3D *iso-matrices*. For this preliminary assessment, the objective is to launch an investigation of the following hypothesis:

The transitions between the energy and resonance states of the hadronic spectra may be rigorously characterized by properly-calibrated iso-topic liftings, including the "Higgs-like scalar massive amplitudeexcitations" [29] and the "Nambu-Goldstone pseudo-scalar massless phase-excitations" [30, 31, 32, 33] of the antisymmetric wavefunction's topological deformation order parameters for the simultaneous and spontaneous breaking of multiple gauge symmetries that are correlated with Legget's superfluid B phases [28, 34].

Thus, in the background of Section 2 we highlight the pertinent background results of the Inopin-Schmidt quark confinement proof [28], where we briefly review the IHCR-based topology and the fundamental complex structures for encoding the six-coloring kagome lattice's energy and resonance state space of the hadronic spectra. Next, in the procedures of Section 3 we explain how to iso-topically lift the IHCR (equipped with the six-coloring kagome lattice) and the topological deformation order parameters that encode the baryon states and transitions in the confinement proof [28]. Consequently, in the main results of Section 4 we employ the outcomes of Section 3 to define the cutting-edge iso-wavefunctions and iso-matrices for a baryon-antibaryon pair [28]. And finally we conclude with Section 5, where we briefly recapitulate these discoveries and suggest alignments for future modes of research.

## 2 Background review: the Inopin-Schmidt quark confinement scenario

In eq. (7) of [19]  $X = \mathbb{C}$  is the field of complex numbers that encodes a Euclidean 2D coordinate-vector space, where the complex number  $x \in X$ encodes a 2D coordinate-vector that is defined in eq. (6) of [19] as

$$x = x_{\mathbb{R}} + x_{\mathbb{I}} = (x_{\mathbb{R}}, x_{\mathbb{I}})_C = (|x|, \langle x \rangle)_P, \ \forall x \in X,$$
(1)

such that  $(x_{\mathbb{R}}, x_{\mathbb{I}})_C$  is the 2D Cartesian coordinate-vector and  $(|x|, \langle x \rangle)_P$  is the 2D polar coordinate-vector representation of the position of x that was deployed in the work of [19, 28, 35].

The 1-sphere IHCR  $T_R^1 \subset X$ —with the amplitude-radius R, the amplitude-curvature  $K = \frac{1}{R}$ , and the center of origin  $O = (0,0)_C$ —is isometrically embedded in X and is defined in eq. (16) of [19] as

$$T_R^1 = \{ x \in X : |x| = R \},$$
(2)

where  $T_R^1$  is the multiplicative group of all non-zero complex numbers with the modulus R; in terms of "topological circle trichotomy",  $T_R^1$  simultaneously delineates between the *interior* dynamical system of the *superluminal* "micro" 2-brane sub-space

$$X_{-} = \{ x \in X : |x| < R \}$$
(3)

and the *exterior* dynamical system of the *non-superluminal* "macro" 2-brane sub-space

$$X_{+} = \{ x \in X : |x| > R \}.$$
(4)

The initial results of the iso-fractal IHCR topology initiation of [19] strongly suggest that  $X_{-} \subset X$  and  $X_{+} \subset X$  may be strongly connected to Santilli's interior and exterior dynamical systems, respectively.

In the quark confinement proof of [28], the three colored quarks for the baryon are confined to a red-green-blue triangular sub-lattice in a quark bag and the three corresponding anticolored antiquarks for the antibaryon are confined to a complementary antired-antigreen-antiblue triangular sublattice in an antiquark bag, where the two sub-lattices are superimposed to the surface of  $T_R^1$  and form a six-coloring kagome lattice in the upgraded Gribov vacuum. In [28], the three quarks are point-particles located at  $r, g, b \in T_R^1$  (so |r| = |g| = |b| = R) with the red-green-blue triangular sub-lattice phase angle position constraint

$$\langle b \rangle = \langle g \rangle + \frac{2\pi}{3} = \langle r \rangle + \frac{4\pi}{3},\tag{5}$$

while the three antiquarks are point-antiparticles located at  $\bar{r}, \bar{g}, \bar{b} \in T_R^1$ (so  $|\bar{r}| = |\bar{g}| = |\bar{b}| = R$ ) with the complementary, antisymmetric antiredantigreen-antiblue triangular sub-lattice phase angle position constraint

$$\begin{array}{lll} \langle \bar{r} \rangle &=& \langle r \rangle \pm \pi \\ \langle \bar{g} \rangle &=& \langle g \rangle \pm \pi \\ \langle \bar{b} \rangle &=& \langle b \rangle \pm \pi, \end{array}$$

$$(6)$$

which are visually depicted in Figure 1 (also see Figures 3–4 of [28]).

In [28] the conventional "quantum numbers" are replaced with complexvalued order parameters for topological deformations of Laughlin quasiparticle fractional statistics [37] in the superfluidic space-time scenario for the spontaneous and simultaneous breaking of multiple gauge symmetries with antiferromagnetic ordering. Each quark and antiquark confined to  $T_R^1$ is assigned three distinct types of such order parameters that are "Cooperpaired" together with the position-dependent Leggett superfluid B phase angles [34] of eqs. (5–6):  $\forall x \in \{r, g, b, \bar{r}, \bar{g}, \bar{b}\}$  we have

1. the total angular momentum order parameter is [28]

$$\psi_J(x) \in \Phi_J(x) = \psi_J(x)_{\mathbb{R}} + \psi_J(x)_{\mathbb{I}} = (\psi_J(x)_{\mathbb{R}}, \psi_J(x)_{\mathbb{I}})_C = (|\psi_J(x)|, \langle \psi_J(x) \rangle)_P$$
(7)

for the spin-orbit coupling [38] of  $T_R^1$  and the spontaneously-generated antiferromagnetic ordering [37] of eqs. (29–31) in [28],



Fig. 1: The three quark-antiquark pairs of the six-coloring kagome lattice are confined to  $T_R^1$  in a bag [28] and located at  $\{r, g, b, \bar{r}, \bar{g}, \bar{b}\} \subset T_R^1$ . The interior dynamical system of  $T_R^1$  is the *superluminal* "micro" 2-brane sub-space  $X_-$ , while the exterior dynamical system of  $T_R^1$  is the *non-superluminal* "macro" 2-brane sub-space  $X_+$ , such that  $T_R^1$  simultaneously delineates  $X_-$  and  $X_+$  in accordance to M.C. Escher's famous reflecting sphere duality [28, 36].

2. the iso-spin order parameter is [28]

$$\psi_{I}(x) \in \Phi_{I}(x) = \psi_{I}(x)_{\mathbb{R}} + \psi_{I}(x)_{\mathbb{I}}$$
  
$$= (\psi_{I}(x)_{\mathbb{R}}, \psi_{I}(x)_{\mathbb{I}})_{C}$$
  
$$= (|\psi_{I}(x)|, \langle\psi_{I}(x)\rangle)_{P}, \qquad (8)$$

3. and the color charge order parameter is [28]

$$\psi_C(x) \in \Phi_C(x) = \psi_C(x)_{\mathbb{R}} + \psi_C(x)_{\mathbb{I}} = (\psi_C(x)_{\mathbb{R}}, \psi_C(x)_{\mathbb{I}})_C = (|\psi_C(x)|, \langle\psi_C(x)\rangle)_P.$$
(9)

To clarify the representation of eqs. (7–9) that are local to x, we refer to  $\psi_J(x)$ ,  $\psi_I(x)$ , and  $\psi_C(x)$  as order parameter states in the order parameter state spaces  $\Phi_J(x)$ ,  $\Phi_I(x)$ , and  $\Phi_C(x)$ , respectively [28]. Note that the order parameters of eqs. (7–9) take the same complex coordinate-vector notation form as the positions of eq. (1), where they have a synchronized 2D Cartesian-polar notation [28]—a complete list of these order parameters is listed in Table 1 of [28].

Thus from [28], we know that  $T_R^1$  acquires a Berry phase as the order parameters evolve and undergo transitions, where  $\Delta |\psi_J(x)|$ ,  $\Delta |\psi_I(x)|$ , and  $\Delta |\psi_C(x)|$  correspond to "Higgs-like" massive amplitude-excitations (characteristic of Nambu-Goldstone scalar boson fluctuations) for effective mass generation [29], and  $\Delta \langle \psi_J(x) \rangle$ ,  $\Delta \langle \psi_I(x) \rangle$ , and  $\Delta \langle \psi_C(x) \rangle$  correspond to massless phase-excitations (characteristic of Nambu-Goldstone pseudoscalar boson fluctuations) [30, 31, 32, 33]—see Figures 8–9 in [28]. The order parameters of eqs. (7–9) rotate freely in 2D and 3D space to generate correlated helices with Laughlin quasi-particle fractional statistics [37] while the Leggett superfluid B phases [34] of eqs. (5–6) remain constant and impose long range order (see Figure 10 in [28])—this serves as a baryon (and antibaryon) wavefunction constraint [28].

The r, g, and b implementations of eqs. (7-9) identify the respective quark wavefunctions from eqs. (34-36) of [28] as

$$\Psi(r) = \psi_C(r) \times \psi_J(r) \times \psi_I(r) \times r, \quad \Psi(r) \stackrel{def}{=} \langle r | \Psi \rangle, 
\Psi(g) = \psi_C(g) \times \psi_J(g) \times \psi_I(g) \times g, \quad \Psi(g) \stackrel{def}{=} \langle g | \Psi \rangle, \quad (10) 
\Psi(b) = \psi_C(b) \times \psi_J(b) \times \psi_I(b) \times b, \quad \Psi(b) \stackrel{def}{=} \langle b | \Psi \rangle,$$

and similarly the  $\bar{r}$ ,  $\bar{g}$ , and  $\bar{b}$  implementations of eqs. (7–9) identify the respective antiquark wavefunctions from eqs. (37–39) of [28] as

$$\Psi(\bar{r}) = \psi_C(\bar{r}) \times \psi_J(\bar{r}) \times \psi_I(\bar{r}) \times \bar{r}, \quad \Psi(\bar{r}) \stackrel{def}{=} \langle \bar{r} | \Psi \rangle, 
\Psi(\bar{g}) = \psi_C(\bar{g}) \times \psi_J(\bar{g}) \times \psi_I(\bar{g}) \times \bar{g}, \quad \Psi(\bar{g}) \stackrel{def}{=} \langle \bar{g} | \Psi \rangle, \quad (11) 
\Psi(\bar{b}) = \psi_C(\bar{b}) \times \psi_J(\bar{b}) \times \psi_I(\bar{b}) \times \bar{b}, \quad \Psi(\bar{b}) \stackrel{def}{=} \langle \bar{b} | \Psi \rangle,$$

such that the *full baryon and antibaryon states* confined to  $T_R^1$  from eqs. (32–33) of [28] are

$$\Psi_{total}(r, g, b) = \Psi(r) \times \Psi(g) \times \Psi(b) 
\Psi_{total}(\bar{r}, \bar{g}, \bar{b}) = \Psi(\bar{r}) \times \Psi(\bar{g}) \times \Psi(\bar{b})$$
(12)

for the antisymmetry of the two-particle cases

$$\Psi(r, \bar{r}) = -\Psi(\bar{r}, r) 
\Psi(g, \bar{g}) = -\Psi(\bar{g}, g) 
\Psi(b, \bar{b}) = -\Psi(\bar{b}, b)$$
(13)

for the confined quark and antiquark (two-particle) cases from eqs. (40–42) of [28]. Eqs. (10–13) permitted the definition of the 2D baryon-antibaryon antisymmetric matrix

$$\begin{pmatrix} 0 & \Psi_{total}(r, g, b) \\ \Psi_{total}(\bar{r}, \bar{g}, \bar{b}) & 0 \end{pmatrix}$$
(14)

from eq. (43) of [28] and the expanded 3D quark-antiquark antisymmetric matrix

$$\begin{pmatrix} 0 & \Psi(r) & \Psi(g) \\ \Psi(\bar{r}) & 0 & \Psi(b) \\ \Psi(\bar{g}) & \Psi(\bar{b}) & 0 \end{pmatrix}$$
(15)

from eq. (44) of [28]. Now, lets use iso-mathematics [1, 2, 3, 4, 5, 19] to attack the energy and resonance states of the hadronic spectrum.

## 3 Procedures: iso-topically lifting the complex confinement and encoding structures

Here, we propose the two distinct approaches for iso-topically [1, 2, 3, 4, 5, 19] upgrading the fundamental complex structures of the Inopin-Schmidt

quark confinement scenario [28]. Note: these procedures may be engaged independently or conjointly. Thus, we opt to demonstrate these procedures conjointly: we'll start by executing the approach of Section 3.1 and carry its results over to the approach of Section 3.2.

# 3.1 Inopin's holographic confinement ring and the six-coloring kagome lattice

The IHCR  $T_R^1 \subset X$  and the six-coloring kagome lattice positions of its confined quark-antiquark constituents  $\{r, g, b, \bar{r}, \bar{g}, \bar{b}\} \subset T_R^1$  in the dual baryon-antibaryon bag [28] are iso-topically lifted [1, 2, 3, 4, 5, 19] via the following procedure:

- 1. First, we select some positive-definite iso-unit  $\hat{\epsilon} > 0$  with the corresponding iso-unit inverse  $\hat{\kappa} = \frac{1}{\hat{\epsilon}} > 0$  [1, 2, 3, 4, 5, 19].
- 2. Second, we deploy  $\hat{\epsilon}$  to define the IHCR iso-topic lifting (and its inverse) as

$$\begin{array}{rcl}
f(\hat{\epsilon}): & T_R^1 & \to & T_{\hat{R}}^1 \\
f^{-1}(\hat{\epsilon}): & T_{\hat{R}}^1 & \to & T_R^1
\end{array}$$
(16)

to establish

$$\hat{x} = x \times \hat{\epsilon}, \ \forall x \in T_R^1 \to \forall \hat{x} \in T_{\hat{R}}^1, \tag{17}$$

so eq. (2) becomes

$$T^{1}_{\hat{R}} \equiv T^{1}_{R\hat{\epsilon}} \equiv \{ x \in X : |\hat{x}| = |x| \times \hat{\epsilon} = \hat{R} = R \times \hat{\epsilon} \},$$
(18)

such that  $T_{\hat{R}}^1$  is the *iso-1-sphere IHCR* centered at O with the positivedefinite amplitude-radius  $\hat{R} = R \times \hat{\epsilon} > 0$  and the amplitude-curvature  $\hat{K} = \frac{1}{\hat{R}} > 0$ . Here, note that because  $T_R^1 \subset X$  one could also isotopically lift the complete X via  $X \to X_{\hat{\epsilon}}$  to get a similar result of  $T_{\hat{R}}^1 \subset X_{\hat{\epsilon}}$ .

3. Third, given the obtained results of eqs. (16–18) and the recollection

of  $\{r, g, b, \bar{r}, \bar{g}, \bar{b}\} \subset T^1_R$ , we clarify that

$$\begin{aligned}
\hat{r} &\equiv r \times \hat{\epsilon} \quad \text{for} \quad r \to \hat{r} \\
\hat{g} &\equiv g \times \hat{\epsilon} \quad \text{for} \quad g \to \hat{g} \\
\hat{b} &\equiv b \times \hat{\epsilon} \quad \text{for} \quad b \to \hat{b} \\
\hat{r} &\equiv \bar{r} \times \hat{\epsilon} \quad \text{for} \quad \bar{r} \to \hat{r} \\
\hat{g} &\equiv \bar{g} \times \hat{\epsilon} \quad \text{for} \quad \bar{g} \to \hat{g} \\
\hat{b} &\equiv \bar{b} \times \hat{\epsilon} \quad \text{for} \quad \bar{b} \to \hat{b}
\end{aligned}$$
(19)

applies to the three iso-quark-iso-antiquark pairs confined to the six-coloring kagome lattice [28] with the uniform iso-amplitude iso-position constraint

$$|\hat{r}| \equiv |\hat{g}| \equiv |\hat{b}| \equiv |\hat{\bar{r}}| \equiv |\hat{\bar{g}}| \equiv |\hat{\bar{b}}|, \qquad (20)$$

the red-green-blue triangular sub-lattice iso-phase angle iso-position constraint

$$\langle \hat{b} \rangle \equiv \langle \hat{g} \rangle + \frac{2\pi}{3} \equiv \langle \hat{r} \rangle + \frac{4\pi}{3}$$
 (21)

preserved from eq. (5), and the complementary, antisymmetric antired-antigreen-antiblue triangular sub-lattice iso-phase angle position constraint

preserved from eq. (6).

4. Finally, given the acquired results of eqs. (16–22), we can, for example, write the iso-multiplication [1, 2, 3, 4, 5, 19] between the iso-positions of the three colored iso-quarks as

$$\hat{r} \,\,\hat{\times} \,\,\hat{g} \,\,\hat{\times} \,\,\hat{b} \tag{23}$$

and for the three anticoloring iso-antiquark iso-positions we can put

$$\hat{r} \times \hat{g} \times \hat{b}.$$
 (24)



Fig. 2: The three quark-antiquark pairs of the six-coloring kagome lattice are confined to  $T_R^1$  and located at  $\{r, g, b, \bar{r}, \bar{g}, \bar{b}\} \subset T_R^1$  [28]. When  $T_R^1$  is iso-topically lifted [1, 2, 3, 4, 5, 19] to  $T_{\hat{R}}^1$  via the iso-unit  $\hat{\epsilon}$ , then the quark-antiquark pairs are iso-topically lifted to  $\{\hat{r}, \hat{g}, \hat{b}, \hat{\bar{r}}, \hat{\bar{g}}, \hat{\bar{b}}\} \subset T_{\hat{R}}^1$  via the "magnification" (or "demagnification") iso-transition  $T_R^1 \to T_{\hat{R}}^1$ . The iso-topic lifting scales the amplitudes of the quark-antiquark pair positions (left). A slightly more sophisticated iso-topic lifting scales the amplitudes and rotates the azimuthal phase angles of the quark-antiquark pair positions (right). These iso-topic liftings could be calibrated to encode strictly continuous or non-strictly continuous energy and resonance states in the hadronic spectrum.

The example results of eqs. (23–24) merely exemplify one simple application of Santilli's iso-multiplication [1, 2, 3, 4, 5, 19].

At this point, the IHCR  $T_R^1 \subset X$  and the six-coloring kagome lattice positions of  $\{r, g, b, \bar{r}, \bar{g}, \bar{b}\} \subset T_R^1$  for the dual baryon-antibaryon bag have been iso-topically lifted [1, 2, 3, 4, 5, 19] to the iso-IHCR  $T_{\hat{R}}^1 \subset X$  and the six-coloring kagome lattice iso-positions of  $\{\hat{r}, \hat{g}, \hat{b}, \hat{r}, \hat{g}, \hat{b}\} \subset T_{\hat{R}}^1$ , respectively, for the dual iso-baryon-iso-antibaryon bag; For this, the obtained iso-complex iso-position results of eqs. (16–24) support the hypothesis that Santilli's iso-mathematics [1, 2, 3, 4, 5, 19] may upgrade the Inopin-Schmidt quark confinement proof [28] by interpreting the transitions between energy and resonance states in the hadronic spectrum as iso-topic liftings. We note that the said iso-topic liftings have the flexibility to be calibrated to encode strictly continuous or non-strictly continuous levels in the hadronic spectrum, which should be subjected to additional scrutiny and development via the scientific method in future work.

### 3.2 Topological deformation order parameters

The order parameter states  $\psi_J \in \Phi_J$ ,  $\psi_I \in \Phi_I$ , and  $\psi_C \in \Phi_C$  of eqs. (7–9)—that are now assigned to the six-coloring iso-positions at  $\{\hat{r}, \hat{g}, \hat{b}, \hat{r}, \hat{g}, \hat{b}\} \subset T^1_{\hat{R}}$  of the confined quark-antiquarks—are iso-topically lifted [1, 2, 3, 4, 5, 19] via the following procedure:

- 1. First, we select some positive-definite iso-unit  $\hat{\rho} > 0$  with the corresponding iso-unit inverse  $\hat{\rho} = \frac{1}{\hat{\rho}} > 0$  [1, 2, 3, 4, 5, 19]. Note that for illustration purposes we have opted to select the new iso-unit  $\hat{\rho}$  which may or may not be distinct from  $\hat{\epsilon}$ : we could select  $\hat{\rho} \neq \hat{\epsilon}$  or the simpler case  $\hat{\rho} = \hat{\epsilon}$ .
- 2. Second, we deploy  $\hat{\rho}$  to define the order parameter iso-topic liftings

(and their inverses) as

$$\begin{aligned}
f_{J}(\hat{\rho}) &: \quad \Phi_{J}(\hat{x}) \quad \to \quad \Phi_{J_{\hat{\rho}}}(\hat{x}) \\
f_{J}^{-1}(\hat{\rho}) &: \quad \Phi_{J_{\hat{\rho}}}(\hat{x}) \quad \to \quad \Phi_{J}(\hat{x}) \\
f_{I}(\hat{\rho}) &: \quad \Phi_{I}(\hat{x}) \quad \to \quad \Phi_{I_{\hat{\rho}}}(\hat{x}) \\
f_{I}^{-1}(\hat{\rho}) &: \quad \Phi_{I_{\hat{\rho}}}(\hat{x}) \quad \to \quad \Phi_{I}(\hat{x}) \\
f_{C}(\hat{\rho}) &: \quad \Phi_{C}(\hat{x}) \quad \to \quad \Phi_{C_{\hat{\rho}}}(\hat{x}) \\
f_{C}^{-1}(\hat{\rho}) &: \quad \Phi_{C_{\hat{\rho}}}(\hat{x}) \quad \to \quad \Phi_{C}(\hat{x})
\end{aligned}$$
(25)

to establish

$$\begin{aligned}
\hat{\psi}_J(\hat{x}) &\equiv \psi_J(\hat{x}) \times \hat{\rho}, \quad \forall \psi_J(\hat{x}) \in \Phi_J(\hat{x}) \to \forall \psi_J(\hat{x}) \in \Phi_{J_{\hat{\rho}}}(\hat{x}) \\
\hat{\psi}_I(\hat{x}) &\equiv \psi_I(\hat{x}) \times \hat{\rho}, \quad \forall \psi_I(\hat{x}) \in \Phi_I(\hat{x}) \to \forall \hat{\psi}_I(\hat{x}) \in \Phi_{I_{\hat{\rho}}}(\hat{x}) \\
\hat{\psi}_C(\hat{x}) &\equiv \psi_C(\hat{x}) \times \hat{\rho}, \quad \forall \psi_C(\hat{x}) \in \Phi_C(\hat{x}) \to \forall \hat{\psi}_C(\hat{x}) \in \Phi_{C_{\hat{\rho}}}(\hat{x}),
\end{aligned}$$
(26)

 $\forall \hat{x} \in T^1_{\hat{R}}$ . Here, note that we used the same iso-unit to iso-topically lift the three  $\psi_J \in \Phi_J$ ,  $\psi_I \in \Phi_I$ , and  $\psi_C \in \Phi_C$  to  $\hat{\psi}_J \in \Phi_{J_{\hat{\rho}}}$ ,  $\hat{\psi}_I \in \Phi_{I_{\hat{\rho}}}$ , and  $\hat{\psi}_C \in \Phi_{C_{\hat{\rho}}}$ , respectively, but another option is that we could also select three different iso-units for independent liftings. Such selections really just depend on what we want to do in a given scenario. In the limited context of this paper, we are interested in the simultaneous, uniform scaling of all order parameters with a single value of  $\hat{\rho}$ .

3. Third, given the obtained results of eqs. (25–26) and the recollection of  $\{\hat{r}, \hat{g}, \hat{b}, \hat{\bar{r}}, \hat{\bar{g}}, \hat{\bar{b}}\} \subset T^1_{\hat{R}}$ , we clarify that

$$\begin{aligned}
\hat{\psi}_{J|I|C}(\hat{r}) &\equiv \psi_{J|I|C}(\hat{r}) \times \hat{\rho} \quad \text{for} \quad \psi_{J|I|C}(\hat{r}) \to \hat{\psi}_{J|I|C}(\hat{r}) \\
\hat{\psi}_{J|I|C}(\hat{g}) &\equiv \psi_{J|I|C}(\hat{g}) \times \hat{\rho} \quad \text{for} \quad \psi_{J|I|C}(\hat{g}) \to \hat{\psi}_{J|I|C}(\hat{g}) \\
\hat{\psi}_{J|I|C}(\hat{b}) &\equiv \psi_{J|I|C}(\hat{b}) \times \hat{\rho} \quad \text{for} \quad \psi_{J|I|C}(\hat{b}) \to \hat{\psi}_{J|I|C}(\hat{b}) \\
\hat{\psi}_{J|I|C}(\hat{r}) &\equiv \psi_{J|I|C}(\hat{r}) \times \hat{\rho} \quad \text{for} \quad \psi_{J|I|C}(\hat{r}) \to \hat{\psi}_{J|I|C}(\hat{r}) \\
\hat{\psi}_{J|I|C}(\hat{g}) &\equiv \psi_{J|I|C}(\hat{g}) \times \hat{\rho} \quad \text{for} \quad \psi_{J|I|C}(\hat{g}) \to \hat{\psi}_{J|I|C}(\hat{g}) \\
\hat{\psi}_{J|I|C}(\hat{b}) &\equiv \psi_{J|I|C}(\hat{b}) \times \hat{\rho} \quad \text{for} \quad \psi_{J|I|C}(\hat{b}) \to \hat{\psi}_{J|I|C}(\hat{b})
\end{aligned} \tag{27}$$

applies to the spontaneous antiferromagnetic ordering [37] constraints of eqs. (29–31) in [28] for the iso-quark-iso-antiquark pairs confined to

the iso-topically lifted six-coloring kagome lattice. In eq. (27), observe that we've started to use the consolidated notation of  $\hat{\psi}_{J|I|C}$  to denote any of the three order parameters, thereby reducing the number of repeated definitions.

4. Finally, given the acquired results of eqs. (25–26), we can, for example, write the iso-multiplication [1, 2, 3, 4, 5, 19] between the three distinct iso-order parameters for some quark at  $\hat{x} \in T^1_{\hat{R}}$  as

$$\hat{\psi}_J(\hat{x}) \stackrel{\times}{\times} \hat{\psi}_I(\hat{x}) \stackrel{\times}{\times} \hat{\psi}_C(\hat{x}) \tag{28}$$

and for some antiquark at  $\hat{\bar{x}} \in T^1_{\hat{R}}$ , such that  $\hat{\bar{x}} = -\hat{x}$ , we have

$$\hat{\psi}_J(\hat{\bar{x}}) \stackrel{\times}{\times} \hat{\psi}_I(\hat{\bar{x}}) \stackrel{\times}{\times} \hat{\psi}_C(\hat{\bar{x}}).$$
(29)

At this point, the order parameter states  $\psi_J \in \Phi_J, \ \psi_I \in \Phi_I$ , and  $\psi_C \in \Phi_C$ —assigned to the six-coloring iso-positions at  $\{\hat{r}, \hat{g}, \hat{b}, \hat{\bar{r}}, \hat{\bar{g}}, \hat{\bar{b}}\} \subset T^1_{\hat{\mu}}$ of the confined quark-antiquarks—have been iso-topically lifted to the isoorder parameter states  $\hat{\psi}_J \in \Phi_{J_{\hat{o}}}, \ \hat{\psi}_I \in \Phi_{I_{\hat{o}}}, \ \text{and} \ \hat{\psi}_C \in \Phi_{C_{\hat{o}}}, \ \text{respec-}$ tively. For this, the obtained iso-complex iso-topological deformation results of eqs. (25–29) support the hypothesis that Santilli's iso-mathematics [1, 2, 3, 4, 5, 19] may upgrade the Inopin-Schmidt quark confinement proof [28] by interpreting the transitions between energy and resonance states in the hadronic spectrum as iso-topic liftings. Additionally, the iso-topic liftings exemplify *amplitude variations* that are dependent on the selected iso-unit  $\hat{\rho}$ , where such variations are indeed "Higgs-like" massive amplitudeexcitations [29] for order parameter scaling as  $T^1_{\hat{R}}$  iteratively acquires Berry phases [28]. Hence, if one were to carefully select a proper iso-unit that generates phase variations [28], then such iso-topic liftings could furthermore exemplify massless phase-excitations that are characteristic of Nambu-Goldstone pseudo-scalar boson fluctuations [30, 31, 32, 33]. Such iso-order parameters rotate freely in 2D and 3D space to generate correlated helices with Laughlin quasi-particle fractional statistics [37] while the Leggett superfluid B phases [34] remain *constant* and impose *long range order* [28] this serves as a contraint for the iso-topically lifted iso-baryon (and isoantibaryon) antisymmetric iso-wavefunctions in the upcoming Section 4.



Fig. 3: Note: in this diagram only the green iso-quark at iso-position  $\hat{g} \in T_{\hat{R}}^1$  with one order parameter is displayed for the sake of illustration simplicity. The isounit  $\hat{\rho}$  is deployed to iso-topically lift the  $\hat{g}$ 's order parameter  $\psi_J(\hat{g}) \in \Phi_J(\hat{g})$  to the iso-order parameter  $\hat{\psi}_J(\hat{g}) \in \Phi_{J_{\hat{\rho}}}(\hat{g})$  via the iso-transition  $\Phi_J(\hat{g}) \to \Phi_{J_{\hat{\rho}}}(\hat{g})$ . The iso-topic lifting scales the amplitude of the iso-quark's order parameter, where the change of scale indicates a "Higgs-like" massive scalar amplitude-excitation [28, 29] (top). A slightly more sophisticated iso-topic lifting scales the amplitudes and rotates the azimuthal phase angle of the iso-quark's order parameter, where the change of direction indicates a "Nambu-Goldstone" massless pseudo-scalar phaseexcitation [28, 30, 31, 32, 33] (bottom). Such iso-topic liftings are simultaneously applied to each of the order parameters at each (iso-quark and iso-antiquark) iso-position of the six-coloring kagome lattice.

## 4 Main results: the baryon-antibaryon antisymmetric isowavefunctions and iso-matrices

The conjoint iso-topic lifting procedural results of eqs. (16–24) from Section 3.1 and eqs. (25–29) from Section 3.2 permit us to upgrade the confined baryon-antibaryon antisymmetric wavefunctions and matrices [28] constructed in eqs. (10–15) with Santilli's iso-multiplication [1, 2, 3, 4, 5, 19]. Here, for illustration purposes, we assume the simple case of  $\hat{\rho} = \hat{\epsilon}$  to preserve relative scaling between  $T_{\hat{R}}^1$  and its six-coloring iso-order parameters of  $\hat{\psi}_J \in \Phi_{J_{\hat{\rho}}}, \, \hat{\psi}_I \in \Phi_{I_{\hat{\rho}}}$ , and  $\hat{\psi}_C \in \Phi_{C_{\hat{\rho}}}$ .

First, given the established iso-units  $\hat{\epsilon}$  and  $\hat{\rho}$ , the quark wavefunctions of eq. (10) are enhanced with iso-multiplication [1, 2, 3, 4, 5, 19] to define the quark iso-wavefunctions for  $\hat{r}$ ,  $\hat{g}$ ,  $\hat{b}$  as

$$\hat{\Psi}(\hat{r}) \equiv \hat{\psi}_J(\hat{r}) \times \hat{\psi}_I(\hat{r}) \times \hat{\psi}_C(\hat{r}) \times \hat{r}, \qquad \hat{\Psi}(\hat{r}) \stackrel{def}{\equiv} \langle \hat{r} | \hat{\Psi} \rangle, 
\hat{\Psi}(\hat{g}) \equiv \hat{\psi}_J(\hat{g}) \times \hat{\psi}_I(\hat{g}) \times \hat{\psi}_C(\hat{g}) \times \hat{g}, \qquad \hat{\Psi}(\hat{g}) \stackrel{def}{\equiv} \langle \hat{g} | \hat{\Psi} \rangle, 
\hat{\Psi}(\hat{b}) \equiv \hat{\psi}_J(\hat{b}) \times \hat{\psi}_I(\hat{b}) \times \hat{\psi}_C(\hat{b}) \times \hat{b}, \qquad \hat{\Psi}(\hat{b}) \stackrel{def}{\equiv} \langle \hat{b} | \hat{\Psi} \rangle,$$
(30)

and the antiquark wavefunctions of eq. (11) are similarly adjusted to define the *antiquark iso-wavefunctions* for  $\hat{r}$ ,  $\hat{g}$ , and  $\hat{b}$  as

$$\hat{\Psi}(\hat{r}) \equiv \hat{\psi}_{J}(\hat{r}) \times \hat{\psi}_{I}(\hat{r}) \times \hat{\psi}_{C}(\hat{r}) \times \hat{r}, \qquad \hat{\Psi}(\hat{r}) \stackrel{def}{\equiv} \langle \hat{r} | \hat{\Psi} \rangle 
\hat{\Psi}(\hat{g}) \equiv \hat{\psi}_{J}(\hat{g}) \times \hat{\psi}_{I}(\hat{g}) \times \hat{\psi}_{C}(\hat{g}) \times \hat{g}, \qquad \hat{\Psi}(\hat{g}) \stackrel{def}{\equiv} \langle \hat{g} | \hat{\Psi} \rangle 
\hat{\Psi}(\hat{b}) \equiv \hat{\psi}_{J}(\hat{b}) \times \hat{\psi}_{I}(\hat{b}) \times \hat{\psi}_{C}(\hat{b}) \times \hat{b}, \qquad \hat{\Psi}(\hat{b}) \stackrel{def}{\equiv} \langle \hat{b} | \hat{\Psi} \rangle.$$
(31)

Hence, eqs. (30–31) authorize us to rewrite eq. (12) as the full baryon and antibaryon iso-states confined to  $T^1_{\hat{R}}$  as

$$\hat{\Psi}_{total}(\hat{r}, \hat{g}, \hat{b}) \equiv \hat{\Psi}(\hat{r}) \times \hat{\Psi}(\hat{g}) \times \hat{\Psi}(\hat{b}) 
\hat{\Psi}_{total}(\hat{r}, \hat{g}, \hat{\bar{b}}) \equiv \hat{\Psi}(\hat{r}) \times \hat{\Psi}(\hat{\bar{g}}) \times \hat{\Psi}(\hat{\bar{b}}),$$
(32)

where the antisymmetry of the two-particle cases in eq. (13) becomes

$$\begin{aligned}
\hat{\Psi}(\hat{r},\hat{r}) &\equiv -\hat{\Psi}(\hat{r},\hat{r}) \\
\hat{\Psi}(\hat{g},\hat{g}) &\equiv -\hat{\Psi}(\hat{g},\hat{g}) \\
\hat{\Psi}(\hat{b},\hat{b}) &\equiv -\hat{\Psi}(\hat{b},\hat{b}),
\end{aligned}$$
(33)

while eq. (14) is iso-topically lifted to the 2D baryon-antibaryon antisymmetric iso-matrix

$$\begin{pmatrix} 0 & \hat{\Psi}_{total}(\hat{r}, \hat{g}, \hat{b}) \\ \hat{\Psi}_{total}(\hat{r}, \hat{g}, \hat{\bar{b}}) & 0 \end{pmatrix},$$
(34)

thereby enabling us to upgrade eq. (15) with the expanded 3D quarkantiquark antisymmetric iso-matrix

$$\begin{pmatrix} 0 & \hat{\Psi}(\hat{r}) & \hat{\Psi}(\hat{g}) \\ \hat{\Psi}(\hat{r}) & 0 & \hat{\Psi}(\hat{b}) \\ \hat{\Psi}(\hat{b}) & \hat{\Psi}(\hat{b}) & 0 \end{pmatrix}.$$
 (35)

#### 5 Conclusion and outlook

In total, the initial "iso-discoveries" of this preliminary assessment support our hypothesis. We found that upgrading the six-coloring antisymmetric wavefunctions and matrices of the confined baryon-antibaryon pair [28] with Santilli's iso-mathematics [1, 2, 3, 4, 5] to install the corresponding isowavefunctions and iso-matrices was a relatively straightforward process that yielded cutting-edge results. These outcomes are significant because they advance the Inopin-Schmidt quark confinement proof [28] to new heights by equipping it with the power, flexibility, and capability of iso-topically interpreting and encoding the hadronic energy and resonance states in a generalized fashion.

Therefore, in order to further investigate our hypothesis and probe this domain, results and claims, we plan to continue our assault along this research trajectory. Hence, there is much work to be done in the near future. In particular, it should be highly beneficial to further analyze this emerging framework in terms of Santilli's hadronic mechanics [6] and magnecules [8, 16, 26, 27, 39, 40, 41, 42, 43, 44], along with the Yuan-Mo-Wang nonets model [45]. Also, the next step should be to introduce a more rigorous iso-mathematical treatment with properly-calibrated iso-topic liftings that generalize and match the experimentally-verified hadronic spectra characteristics of the said models with a greater degree of precision to expand the predictive accuracy and capability; for this, we'll need to

investigate iso-topic implementations for both the strictly continuous and non-strictly continuous cases. More precisely, we must theoretically and experimentally prove (or disprove) that the iso-order parameters of the antisymmetric iso-wavefunctions and iso-matrices for the topological deformations do in fact quantify "Higgs-like scalar massive amplitude-excitations" [29] and the "Nambu-Goldstone pseudo-scalar massless phase-excitations" [30, 31, 32, 33] in the superfluidic space-time with Laughlin quasi-particle [37] fractional statistics [28]. Furthermore, we must define a series of isotopic liftings that correspond to, for example, the periodic table of the elements.

If the above listed future research objectives can be achieved, then it will be imperative to exercise such developments in new, clean, sustainable, cost-efficient power sources such as Santilli's intermediate controlled nuclear synthesis [7, 8, 9, 10, 11, 12] and magnecular-based fuels [13, 14, 15, 16], which both have a *significant* potential for a wide-range of industrial applications and do not emit harmful radiation or toxic waste.

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