Is it possible to apply Godel's incompleteness theorem to scientific theories?

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Abstract

Godel’s incompleteness theorem is normally applied to mathematics. But i just found an article by Michael Goodband who argues that GIT can also be applied to scientific theories, see http://www.mjgoodband.co.uk/papers/Godel-science-theory.pdf.

Introduction

Godel’s incompleteness theorem is normally applied to mathematics. But i just found an article by Michael Goodband who argues that GIT can also be applied to scientific theories, see http://www.mjgoodband.co.uk/papers/Godel-science-theory.pdf.

My own idea can be expressed generally as follows: any theory boils down to an exposition of a statement/proposition. According to GIT, in any theory there is at least one statement which is unprovable, and therefore any theory can be considered as incomplete or has indeterminacy. One implication of this deduction is that any theory should be made falsifiable (Popper), and also perhaps we can use conditional Bayesian probability to describe acceptance of a theory.

Answers:

[1] Vitaly Voloshin
I think it is implicitly applied to every theory, even more than that. In simplified version GIT can be understood as that fact that any finite system of statements is never a complete truth. It is a mathematical proof of the more general philosophical statement by Hegel "the truth is the whole".

[2] Liudmila Pozhar
Absolutely. Any first-principle physical theory (model) must be based on several axioms (first principles) and utilize them to prove useful results mathematically (theorems). One can and SHOULD apply Godel's theorem to any first-principle physics theory. Unfortunately, many theoretical approaches to physics problems are semi-empirical, so they do not formulate their axioms mathematically. Thus, it's difficult to apply Godel's theorem in such cases. I would not call such approaches theories, though. They are simply half-empirical reasoning.

[3] Victor Christianto
@Vitaly and Liudmila. Thanks you for your answers. There are two questions in this regard:
a. Considering GIT, then is it possible for human to achieve a Theory of Everything (see a paper by Stanley Jaki, http://www.sljaki.com/JakiGodel.pdf);

b. how can we introduce the logic of GIT into scientific methods? I think one possible way in this regard is by considering Bayesian acceptance of a theory, to complement Popperian falsifiability. That is because Bayesian probability seems to be more operational than falsifiability.

What do you think? Thanks

[4] Liudmila Pozhar

a. Again, according to Godel, one cannot prove rigorously (being in the framework of a given theory), that there is no other theory that would not be more general than the one considered. If one wants a math description of something, then one has to formulate several axioms and derive theorems that should deliver desirable results. One may keep generalizing axioms and develop more general theories, but one cannot prove rigorously, that there is no other system of axioms that would not contain a given one as a sub-system (meaning, that a given system of axioms is all-inclusive).

b. I believe, mathematical methods are applicable to everything, but it takes time (decades, centuries, millenia?) to develop proper math methods and apply them properly. In every case, to reach a destination one must take the first step and go on.

[5] jean claude Dutailly

We need first to define what a scientific theory is. It can be describd as a set of universal assertions (laws that are deemed to be always true, and can be checked) and a rule of inference (which enables to build other laws, notably for specific systems). What the Gödel theorem says is that a mathematical theory, strong enough to account for arithmetics, there are always some theorems which are true but cannot be proven. Actually it means that one cannot build a theory of mathematics starting only from obvious axioms and logical rules of inference : we need some axioms, but their choice is not obvious, one can add other axioms (but usually they are useless for any practical purpose).

This can be generalized for any scientific theory : one needs some basic hypotheses, and a rule of inference, but the choice of the hypotheses is not dictated by a simple logical reasoning. So we have to decide what theory to choose, and there are several criteria in this choice.

For more see on this site my paper on "common structures in scientific theories"

[6] Liudmila Pozhar

To all interested in this discussion: logic IS MATH. There are a number of logics, classical and non-classical (note: the name classical here has nothing to do with classical
physics). All are studied by math's discipline called mathematical logic. Any logic is based on a system of axioms, just like any math. theory. In physics (not in math) a hypothesis is a concept, not necessarily mathematically formulated (usually it is not). Many phenomenological physical theories are developing some concepts whose relations to the first principles (axioms) are not revealed, and in most cases, questionable.

[7] jean claude Dutailly

To Ludmilla

I disagree: there is a mathematical logic, whose aim is to study the consistence of formal theories. It encompasses predicates logic. So one can study mathematical theories, such as arithmetics and set theories. Mathematical logic use a set of rules, notably a rule of inference, and one can conceive different kind of logic, but this is always external to mathematical theories. This is necessary to be able to tell something about mathematics itself.

From this point of view mathematics is a science, it is based on axioms, and there is no obvious criteria to choose one set of axioms or another. This is the true meaning of Goedem’s theorem! we want an efficient mathematics, so we need at least to incorporate arithmetics and set theory, but to do so there are always some theorems which cannot be proven, this is not critical (there is no inconsistency) just it tells us that efficient mathematics are more than a simple language: it incorporates deep structures, which come from the way mathematicians have, along centuries, invented mathematics.

[8] Vitaly Voloshin

From my point of view, formal logic is logic which avoids contradictions. It is a part of most general, dialectical logic which explains how the whole world is developing. This is based on contradictions. GIT explains the special case of it which fits into the "thesis -> antithesis ->synthesis" law.

Thesis: we have a system of axioms. Anti-thesis: there is a fact which cannot be explained using this system of axioms (contradiction). Synthesis: we add a new axiom to the system and obtain a better system. The history of math and physics serves as good examples to this.

[9] Liudmila Pozhar

Dr. Voloshin, you are right, in a way. Anything, that is not based on mathematical logic is not a theory, rigorously speaking - it is just a quasi-scientific reasoning. Returning to physics: selection of axioms for physical theories are based on mathematical analysis of known physical facts. Again, anything that is not based on math logics is not a physical theory (regardless of what people who love phenomenology think).

[10] jean claude Dutailly
To Vitaly

Your question: what is a theory? is part of epistemology. And one can give a quite decent definition of a scientific theory. In a given field (not necessarily in natural sciences) there can be several theories, and one issue is to choose between them. The usual criteria are simplicity (generalisation of the Occam's razor rule), extension (positivism), conservatism, pragmatism (the new must explain the old). But there is no scientific truth, and even if we must assume the existence of a real world, we cannot claim that we know what it is.

[11] Liudmila Pozhar

Dr. Voloshin, if you are talking about physics, it is a science by definition (not just a collection of reasoning called philosophy). A physics theory is a mathematical hypothesis that rests on a set of axioms and draws "results" (theorems) using those axioms. All kinds of considerations, like "where is a boundary between a theory and not a theory" have a simple answer above. A theory is a set of axioms and theorems that is derivable from those axioms. The rest (things like phenomenological theories, etc.) is a scholastic exercise that is designed to justify some ad hoc "results" their authors use for inability to develop a real theory.

[12] Jean Claude Dutailly

A scientific theory comprises a set of objects, with precise definitions and properties related to the measures which can be done, a set of fundamental hypotheses, a set of basic laws, which are falsifiable, and a method of inference which enables to deduce new laws, for specific occurrences, from the basic laws.

It requires some formalism, but this is not necessarily a mathematical formalism. In Chemistry the atomistic representation deals with most of the usual problems. And in Economics the different accounting systems (for companies, states, health services...) provide another kind of formalism, which does not require a high level of mathematics.

In social sciences one can conceive scientific theories based on purely logical laws (predicates).

In a given "science", meaning a given field of study, there are usually different theories (in Physics Newtonian, Relativity, Quantum Mechanics,...) which can be or not complementary. One of the big issue in science is to choose among these theories, according to some criteria (simplicity, extensivity,...).

Concluding remarks
The contributors all agree that it is possible to apply Godel’s incompleteness theorem to scientific theories. Therefore it implies there is always incompleteness in any theory.

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