Two sequences of primes whose formulas contain the powers of the number 2

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Abstract. In this paper I present two possible infinite sequences of primes, having in common the fact that their formulas contain the powers of the number 2.

Conjecture 1:

There exist an infinity of primes of the form $2^m + n^2$, where m is non-null positive integer and n odd integer.

The first few such primes for [m, n] = [2, n]:
: $3^2 + 4 = 13$ for $n = 3$;
: $5^2 + 4 = 29$ for $n = 5$;
: $7^2 + 4 = 53$ for $n = 7$;
: $13^2 + 4 = 173$ for $n = 13$;
: $17^2 + 4 = 293$ for $n = 17$.

The first few such primes for [m, n] = [4, n]:
: $5^2 + 16 = 41$ for $n = 5$;
: $11^2 + 16 = 137$ for $n = 11$;
: $29^2 + 16 = 857$ for $n = 29$;
: $31^2 + 16 = 977$ for $n = 31$;
: $41^2 + 16 = 1697$ for $n = 41$.

The first few such primes for [m, n] = [8, n]:
: $5^2 + 256 = 281$ for $n = 5$;
: $19^2 + 256 = 617$ for $n = 19$;
: $29^2 + 256 = 1097$ for $n = 29$;
: $31^2 + 256 = 1217$ for $n = 31$;
: $71^2 + 256 = 5297$ for $n = 71$.

The first few such primes for [m, n] = [m, 1]:
: $2^1 + 1 = 3$ for $m = 1$;
: $2^2 + 1 = 5$ for $m = 2$;
: $2^4 + 1 = 17$ for $m = 4$;
: $2^8 + 1 = 257$ for $m = 8$;
: $2^{16} + 1 = 65537$ for $m = 16$.

The first few such primes for [m, n] = [m, 3]:
: $2^1 + 9 = 11$ for $m = 1$;
: $2^2 + 9 = 13$ for $m = 2$;
: $2^3 + 9 = 17$ for $m = 3$;
: $2^5 + 9 = 41$ for $m = 5$;
: $2^6 + 9 = 73$ for $m = 6$.

Conjecture 2:

There exist an infinity of primes of the form (2^n)^k + 2^n + 1, where n is non-null positive integer and k positive integer.

The first few such primes for [n, k] = [n, 1]:
: 5 for n = 1;
: 17 for n = 3;
: 257 for n = 7.

The first few such primes for [n, k] = [n, 2]:
: 7 for n = 1;
: 73 for n = 3;
: 262657 for n = 9.

The first few such primes for [n, k] = [n, 3]:
: 11 for n = 1;
: 521 for n = 3;
: 32801 for n = 5.

The first few such primes for [n, k] = [1, k]:
: 5 for k = 1;
: 7 for k = 2;
: 11 for k = 3.

The first few such primes for [n, k] = [3, k]:
: 17 for k = 1;
: 73 for k = 2;
: 521 for k = 3.

The first few such primes for [n, k] = [5, k]:
: 32801 for k = 3;
: 1048609 for k = 4;
: 1073741857 for k = 6.