Two sequences of primes whose formulas contain the powers of the number 2

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Abstract. In this paper I present two possible infinite sequences of primes, having in common the fact that their formulas contain the powers of the number 2.

Conjecture 1:

There exist an infinity of primes of the form $2^m + n^2$, where $m$ is non-null positive integer and $n$ odd integer.

The first few such primes for $[m, n] = [2, n]$:
- $3^2 + 4 = 13$ for $n = 3$
- $5^2 + 4 = 29$ for $n = 5$
- $7^2 + 4 = 53$ for $n = 7$
- $13^2 + 4 = 173$ for $n = 13$
- $17^2 + 4 = 293$ for $n = 17$.

The first few such primes for $[m, n] = [4, n]$:
- $5^2 + 16 = 41$ for $n = 5$
- $11^2 + 16 = 137$ for $n = 11$
- $29^2 + 16 = 857$ for $n = 29$
- $31^2 + 16 = 977$ for $n = 31$
- $41^2 + 16 = 1697$ for $n = 41$.

The first few such primes for $[m, n] = [8, n]$:
- $5^2 + 256 = 281$ for $n = 5$
- $19^2 + 256 = 617$ for $n = 19$
- $29^2 + 256 = 1097$ for $n = 29$
- $31^2 + 256 = 1217$ for $n = 31$
- $71^2 + 256 = 5297$ for $n = 71$.

The first few such primes for $[m, n] = [m, 1]$:
- $2^1 + 1 = 3$ for $m = 1$
- $2^2 + 1 = 5$ for $m = 2$
- $2^4 + 1 = 17$ for $m = 4$
- $2^8 + 1 = 257$ for $m = 8$
- $2^{16} + 1 = 65537$ for $m = 16$.

The first few such primes for $[m, n] = [m, 3]$:
- $2^1 + 9 = 11$ for $m = 1$
- $2^2 + 9 = 13$ for $m = 2$
- $2^3 + 9 = 17$ for $m = 3$
- $2^5 + 9 = 41$ for $m = 5$
- $2^6 + 9 = 73$ for $m = 6$. 
**Conjecture 2:**

There exist an infinity of primes of the form \((2^n)^k + 2^n + 1\), where \(n\) is non-null positive integer and \(k\) positive integer.

The first few such primes for \([n, k] = [n, 1]\):
  : 5 for \(n = 1\);
  : 17 for \(n = 3\);
  : 257 for \(n = 7\).

The first few such primes for \([n, k] = [n, 2]\):
  : 7 for \(n = 1\);
  : 73 for \(n = 3\);
  : 262657 for \(n = 9\).

The first few such primes for \([n, k] = [n, 3]\):
  : 11 for \(n = 1\);
  : 521 for \(n = 3\);
  : 32801 for \(n = 5\).

The first few such primes for \([n, k] = [1, k]\):
  : 5 for \(k = 1\);
  : 7 for \(k = 2\);
  : 11 for \(k = 3\).

The first few such primes for \([n, k] = [3, k]\):
  : 17 for \(k = 1\);
  : 73 for \(k = 2\);
  : 521 for \(k = 3\).

The first few such primes for \([n, k] = [5, k]\):
  : 32801 for \(k = 3\);
  : 1048609 for \(k = 4\);
  : 1073741857 for \(k = 6\).