First-Order Analysis of Eye Imaging

The diagram above shows a generic lens train for digital imaging of the human retina. I am using this diagram to carry out a first-order analysis from the elementary diffraction theory of image formation. The system comprises three lenses, an aperture stop conjugate to the eye pupil and an imager. For simplicity, I assume aberration free imaging with an air-equivalent model eye whose focal length measures 17 mm (which corresponds to a normal unaccommodated eye having $1000/17 = 59$ Diopters). I also assume that these are the main inputs to the optics design:

1) The required angular resolution at the imager plane is $R$ (# of pixels/degree of the field of view).

2) The angular Field Of View (FOV) is $\text{FOV} = \theta$ degrees.
3) The central wavelength is placed at $\lambda_c$.

4) The system must be able to resolve a minimum line width of $\delta (mm)$.

5) The pixel footprint is $\Delta (mm)$ per side.

6) The diameter of the eye pupil is $D (mm)$.

**Computation of the angular resolution $R$**

The numerical aperture in image space is given by

$$NA_{im} = \sin \theta'$$  \hspace{1cm} (1)

and the diameter of the diffraction blur (Airy disk) is

$$\Phi = \frac{1.22 \lambda_c}{NA_{im}}$$  \hspace{1cm} (2)

The optical invariant (étendue) for this system can be written as

$$Et = h \times NA_{obj} = h' \times NA_{im}$$  \hspace{1cm} (3)

Using (1) and (2) yields

$$h' = \frac{E \Phi}{1.22 \lambda_c}$$  \hspace{1cm} (4)

which automatically fixes the image height $h'$, that is, the radius of the image circle. The lateral magnification is also fixed by

$$m = \frac{h'}{h} = \frac{NA_{obj}}{NA_{im}}$$  \hspace{1cm} (5)
For a real optical system whose performance in diffraction limited, it is natural to assume that the diameter of the diffraction blur matches the linear size of the pixel, that is,

\[ \Phi \approx \Delta \] (6)

The angular resolution \( R \) is related to the diameter of the image circle via

\[ 2h'(mm) = FOV(\text{deg}) \times R(\text{pix/deg}) \times \Delta(mm) \] (7)

From (4) and (7) we obtain

\[ E_t = 0.61\lambda_c \times FOV \times R \] (8)

On the other hand, the optical invariant can be expressed as

\[ E_t = h \times NA_{obj} = EFL_{eye} \times \tan u \times \frac{D}{2 \times EFL_{eye}} \] (9)

or

\[ E_t = \frac{D \tan u}{2} \] (10)

Combining (8) and (10) leads to

\[ R = \frac{D \tan u}{1.22\lambda_c \theta} \] (11)

Taking a 40 deg. FOV and a 4 mm diameter non-mydriatic pupil \( (D = 4) \), returns an angular resolution of \( R = 50.8 \) pixels per degree. If the pupil diameter measures 3 mm instead, the angular resolution becomes \( R = 38 \) pixels per degree.
Bottom line is that, when the FOV is 40 deg., the circle image needs to cover an array of 1520 x 1520 = 2.3 Mpix for a non-mydriatic 3 mm diameter pupil and an array of 2032 x 2032 = 4.1 Mpix for a 4 mm diameter pupil.

**Theoretical modulation required to resolve \( \delta \)**

The Modulation Transfer Function (MTF) for an aberration-free monochromatic optical system is given by

\[
MTF(\nu) = \frac{2}{\pi} (\Psi - \cos \Psi \sin \Psi)
\]  

(12)

where \( \nu \) is the spatial frequency in line-pairs/mm and

\[
\Psi(\nu) = \arccos\left(\frac{\lambda}{2NA_{im}}\right) \text{ (radians)}
\]

(13)

The limiting cases of (12) are

1) \( MTF(0) = 1 \)

2) Cutoff frequency \( \nu_c = \frac{2NA_{im}}{\lambda_c} \Rightarrow MTF(\nu_c) = 0 \)

Since the system must be able to resolve a retinal detail whose line-width measures \( \delta \) (mm), the corresponding spatial frequency in line-pairs/mm may be computed from

\[
\nu_0 = \frac{1}{2\delta}
\]

(14)
in which \(2\delta\) represents the length of a black and a white bar forming the line-pair. From (2), (6) and (14) we get

\[
\Psi(v_o) = \arccos \left( \frac{\Delta}{4.88 \times \delta} \right)
\]  

(15)

Further assuming that the line-width \(\delta\) is comparable in size with the pixel footprint (\(\delta \approx \Delta\)) leads to

\[
\Psi(v_o) = 1.36 \text{(rad)} = 0.8658 \times \frac{\pi}{2} \text{(rad)}
\]  

(16)

and a modulation of

\[
MTF(v_o) = 0.738 = 73.8\%
\]  

(17)