# Modified Unruh Temperature, Entropy and Entropic Force on Schwarzschild Black Hole

Yong Bao

Postbox 777, 100 Renmin South Road, Luoding 527200, Guangdong, China E-mail: baoyong9803@163.com

The Unruh temperature, entropy and entropic force on the Schwarzschild black hole (SBH) can be modified. The general modified equipartition law of energy (ELOE) is proposed by unification, the modified Unruh temperature on event horizon and it quantization are obtained by quantization method. And the modified entropy and entropic force are derived. These are significant to solve the black hole modified question.

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#### I. Introduction

It is a significant question that how to modify the equipartition law of energy (ELOE), entropy and entropic force [1] in theoretical physics. Recently some modified scenarios are proposed. It can be classified three methods approximately. One is introducing the Debye function by C. Gao [2], X. Li and Z. Chang [3], V. V. Kiselev, S. A. Timofeev [4]. Another is recommending zero-point energy by L. Cheng [5]. The third is presenting fundamental length by J. A. Neto [6], F. R. Klinkhamer [7], and H. Sahlmann [8]. But they didn't modify the Unruh temperature, entropy and entropic force of black holes.

In this paper, we propose the general modified ELOE; obtain the modified Unruh temperature on event horizon and it quantization of Schwarzschild black hole (SBH); derive the modified entropy and entropic force of SBH.

## **II. General Modified ELOE**

In this section, we review briefly [2] and [6]; propose the general modified ELOE.

First let us review briefly [2] and [6]. In [2], the

modified ELOE is assumed (we work with  $\hbar = c = k = 1$  unites)

$$E = NTD(x) / 2 \tag{1}$$

where E is the total energy, T is the Unruh temperature, the Debye function is defined by

$$D(x) \equiv (3 / x^3) \int_0^x y^3 / (e^y - 1) dy$$
 (2)

x is related to T as follows

$$x \equiv T_D / T \tag{3}$$

where  $T_D$  is the Debye critical temperature. For a gravitational system,  $T_D$  is the critical Unruh temperature. Note that when  $x \ll 1$ , i.e. in the limit of strong gravitational field, we have

$$D(x) = 1 \tag{4}$$

Classical ELOE E = NT / 2 is retrieved.

In [6] an assumption is proposed

$$N = A / l^2 \tag{5}$$

where N is the number of degrees of freedom, A is the area of the holographic screen, l is a fundamental length which is proportional to Planck length  $l_P$ 

$$l_P^2 = (5 - 3q) l^2 / 2 \tag{6}$$

where  $l_P \equiv \sqrt{\hbar G/c^3}$ ,  $0 \le q \le 5/3$  is a dimensionless

parameter, when q = 1,  $l = l_P$ . The modified ELOE is E = NT / (5-3q) (7)

When q = 1, the classical ELOE is retrieved.

Uniting (1) and (7), we obtain the general modified ELOE

$$E = NT D(x) / (5-3q)$$
 (8)

When D(x) = 1, that is (7); q = 1, it is (1).

# III. Modified Unruh Temperature, Entropy and Entropic Force

In this section, we modify the Unruh temperature, entropy and entropic force of SBH.

#### A. Modified Unruh temperature

For simplicity we only consider SBH. It belongs to the strong gravitational field, so taking D(x) = 1 to (8), we have

$$M_R = NT_R / (5 - 3q) \tag{9}$$

where  $M_R$  is the total energy of SBH,  $T_R$  is the Unruh temperature at the horizon of SBH. It is the modified ELOE on horizon of SBH. Taking  $A = A_R = 4\pi R_H^2$ ,  $R_H = 2GM$  and  $M_R = M$  to (9), we obtain

$$T_R = (5 - 3q) / 16\pi GM \tag{10}$$

where  $A_R$  is the area of horizon of SBH,  $R_H$  is its radius, M is the mass of SBH. When q = 1,  $T_R = 1 / 8\pi GM$ , so (10) is the modified Unruh temperature at the horizon. Taking  $M = \sqrt{2n+1} M_P / 2$  [9] to (10), we find

 $T_R = (5-3q) / 8\pi\sqrt{2n+1}GM_P$ (11) where  $M_P = \sqrt{\hbar c/G}$  is Planck mass,  $n = 0, 1, 2 \cdots$ . When q = 1,  $T_R = 1 / 4\pi\sqrt{2n+1}GM_P$ , so the modified Unruh temperature on horizon is quantized.

From (11) we obtain

$$T_R A_R = (5 - 3q)\sqrt{2n + 1}GM_P / 2$$
 (12)

So the horizon area of SBH is inversely proportional to its Unruh temperature.

#### **B.** Modified entropy

From [6] we obtain the modified entropy

$$S = (5 - 3q)A / 8l_P^2$$
(13)

When q = 1,  $S = A / 4l_P^2$ . Substituting  $A = A_R = 4\pi R_H^2$  $R_H = 2GM$ , and  $M = \sqrt{2n+1} M_P / 2$  into (13), we have

$$S_R = 2(5-3q)\pi GM^2 = (5-3q)(2n+1)\pi GM_P^2 / 2$$
  
= (5-3q)(2n+1)\pi / 2 (14)

where  $S_R$  is the entropy of SBH. When q = 1,  $S_R = (2n + 1)\pi$ . It is the modified entropy of SBH.

#### C. Modified entropic force

Taking (10) and (14) to  $F = T\delta S / \delta x$  [1], we obtain

 $F_R = T_R \delta S_R / \delta R_H = (5-3q)^2 / 8G$  (15) where  $F_R$  is the entropic force of SBH. This is the modified entropic force of SBH. When q = 1,  $F_R = 1 / 2G$ . We list Table 1.

TABLE 1: Modified Unruh temperature, entropy and entropic force

SBH	$R_H = 2GM$
Unruh	$T_R = (5 - 3q) /$
temperature T	$8\pi\sqrt{2n+1} GM_P$
entropy S	$S_R = (5-3q)(2n+1)\pi / 2$
Entropic force F	$F_R = (5-3q)^2 / 8G$

#### VI. Conclusion

In this paper, we have reviewed briefly [2] and [6], proposed the general modified ELOE, obtained the modified ELOE on horizon of SBH, the modified Unruh temperature at the horizon and it quantization, the inverse relation between horizon area of SBH and its Unruh temperature, the modified entropy of SBH, and the modified entropic force of SBH. The modified formulas give expression to the universality including the former results. Because the mass of macroscopic SBH is greater than  $3.2M_{\odot}$ , where  $M_{\odot}$  is solar mass, n in inverse relation and modified formulas is larger than  $10^{39}$ , it is different from general quantized numbers. We only consider SBH, but these are significant to solve the black hole modified question.

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