Six conjectures on primes based on the study of 3-Carmichael numbers and a question about primes

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Abstract. In this paper are stated six conjectures on primes, more precisely on the infinity of some types of pairs of primes, all of them met in the study of 3-Carmichael numbers.

Conjecture 1:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*(m + 1) – n and y = q*(n + 1) – m are both primes.

Examples:
: for [p, q] = [3, 3] we have [x, y] = [5, 13] for [m, n] = [2, 4];
: for [p, q] = [7, 11] we have [x, y] = [29, 73] for [m, n] = [4, 6].

Conjecture 2:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*(m - 1) + n and y = q*(n - 1) + m are both primes.

Examples:
: for [p, q] = [7, 7] we have [x, y] = [11, 23] for [m, n] = [2, 4];
: for [p, q] = [5, 13] we have [x, y] = [11, 67] for [m, n] = [2, 6].

Conjecture 3:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p + (m + 1)*n and y = q + m*n are both primes.

Examples:
: for [p, q] = [5, 5] we have [x, y] = [17, 13] for [m, n] = [2, 4];

: for [p, q] = [5, 7] we have [x, y] = [29, 23] for [m, n] = [2, 8].

Conjecture 4:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*m – 2*n and y = q*n + 2*m are both primes.

Examples:
: for [p, q] = [11, 11] we have [x, y] = [23, 61] for [m, n] = [3, 5];
: for [p, q] = [11, 13] we have [x, y] = [23, 71] for [m, n] = [3, 5].

Conjecture 5:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*m – 2*n and y = q*n - 2*m are both primes.

Examples:
: for [p, q] = [3, 3] we have [x, y] = [7, 17] for [m, n] = [11, 13];
: for [p, q] = [3, 5] we have [x, y] = [13, 61] for [m, n] = [17, 19].

Conjecture 6:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*m + 2*n and y = q*n + 2*m are both primes.

Examples:
: for [p, q] = [5, 5] we have [x, y] = [29, 41] for [m, n] = [3, 7];
: for [p, q] = [5, 11] we have [x, y] = [19, 79] for [m, n] = [1, 7].

Question:

Are there an infinity of primes with the property that can be written as p*m + n – q as well as q*n + m – p, where p, q are distinct primes and m, n are distinct positive integers? But under the condition that m, n, p, q are all four primes? Such number is, for instance, 397 = 13*31 + 7 – 13 = 61*7 + 31 – 61.

Note:

Like I already said in Abstract, I met these pairs of primes in the study of 3-Carmichael numbers: see my previous paper “Connections between the three prime factors of a 3-Carmichael number”.