Black Hole Entropy and the Scale Principle

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Earlier this year I wrote a paper entitled Scale Factors and the Scale Principle. In this paper I formulated a new Law which describes a number of fundamental quantum mechanical laws. This paper shows that the black hole entropy obeys this new formulation.

Keywords: *Black hole entropy, Boltzmann's constant, Planck length, event horizon.*

1. Introduction

This paper investigates the relationship between the *Black hole entropy* and the Scale Law.

2. The Scale Principle or Scale Law (Summary)

In May 2014 I published a paper entitled the Scale Factors and the Scale Principle. In that paper this principle was called the *Quantum Scale Principle*. However after finding that Einstein's relativistic energy also obeys this Law, I changed its name to the *Scale Principle* (or Scale Law). Since the first version the principle has evolved to the present form given by the following relationships:

(1)

Meta Law: *Scale Principle* or *Scale Law*	
(1a) Implicit form (exponents and scale factor)	(1b) Explicit form (ratio, exponents and scale factor)
$D_1^n \left[< \middle| \leq \middle| = \middle| \geq \middle| > \right] S \; D_2^m$ D_1 = Dimensionless quantity D_2 = Dimensionless quantity	$\left(\frac{Q_1}{Q_2}\right)^n \left[< \middle| \leq \middle| = \middle| \geq \middle| > \right] S \left(\frac{Q_3}{Q_4}\right)^m$ (See details below)

The above symbols stand for

a) Quantities:

(i) Q_1, Q_2, Q_3 and Q_4 are physical quantities of identical dimension (such as Length, Time, Mass, Temperature, etc), or
ii) Q_1 and Q_2 are physical quantities of dimension 1 or dimensionless constants while Q_3 and Q_4 are physical quantities of dimension 2 or dimensionless constants. However, if Q_1 and Q_2 are dimensionless constants then Q_3 and Q_4 must have dimensions and viceversa.
(e.g.: Q_1 and Q_2 could be quantities of Mass while Q_3 and Q_4 could be quantities of Length).
The physical quantities can be variables (including differentials, derivatives, Laplacians, divergence, integrals, etc.), constants, dimensionless constants, any mathematical operation between the previous quantities, etc.

b) Relationship type: The relationship is one of five possibilities: **less than or equal to** inequation ($\le$), or **less than** inequation (<), or **equal to** - equation (=), or a **greater than or equal to** inequation ($\ge$), or a **greater than** inequation (>).

c) Scale factor*:* S is a dimensionless *scale factor*. This factor could be a real number, a complex number, a real function or a complex function (strictly speaking real numbers are a particular case of complex numbers). The scale factor could have more than one value for the same relationship. In other words a scale factor can be a quantum number.

d) Exponents: n and m are integer exponents: 0, 1, 2, 3, …
Some examples are:
example 1: $n = 0$ and $m = 1$;
example 2: $n = 0$ and $m = 2$;
example 3: $n = 1$ and $m = 0$;
example 4: $n = 1$ and $m = 1$;
example 5: $n = 1$ and $m = 2$;
example 6: $n = 2$ and $m = 0$;
example 7: $n = 2$ and $m = 1$;
It is worthy to remark that:
i) The exponents, n and m, cannot be both zero in the same relationship.
ii) The number n is the exponent of both Q_1 and Q_2 while the number m is the exponent of both Q_3 and Q_4 regardless on how we express the equation or inequation (1c). This means that the exponents will not change when we express the relationship in a mathematically equivalent form such as

$$\left(\frac{Q_4}{Q_3}\right)^m \left[< \middle| \le \middle| = \middle| \ge \middle| > \right] S \left(\frac{Q_2}{Q_1}\right)^n$$

iii) So far these integers are less than 3. However we leave the options open as we don't know whether we shall find higher exponents in the future.

The scale law (1b) can also be written (mixed form) as

$$Q_1^n Q_4^m \left[< \middle| \leq \middle| = \middle| \geq \middle| > \right] S \, Q_2^n \, Q_3^m$$

The Scale Law describes fundamental laws such as the Heisenberg's uncertainty principle, the black hole entropy, the fine structure constant, Einstein's relativistic energy equation, the formula for the Schwazschild radius, the Bohr Postulate, the De Broglie wavelength-momentum relationship, Newton's law of universal gravitation, the Schrödinger equation, the Friedmann equation, and maybe many others.

References [1] and [2] provide a more complete explanation on the Scale Law.

3. The Black Hole Entropy

I shall show that the black hole entropy is a special case of the *Scale Law*. To find the exact scale factor in this special case we need to do a much more detailed investigation (either as Stephen Hawking did or as the author did [3]).

Let us consider the following scale table

Length *(Exponent = 2)*	**Constant** *(Exponent = 1)*	**Entropy** *(Exponent = 1)*	**Length** (Planck Scale) *(Exponent = 2)*
R	k_B	S_{BH}	L_P
R^2	k_B	S_{BH}	L_P^2

Table 1: *Scale Table used to find the equation for the black hole entropy.*

Where
R = black hole radius
k_B = Boltzmann's constant
S_{BH} = Berkenstein-Hawking's black hole entropy

According to the scale law we write

$$R^2 k_B = S \, S_{BH} \, L_P^2 \tag{2}$$

Solving this equation for the entropy, S_{BH}, yields

$$S_{BH} = \frac{1}{S} \frac{k_B R^2}{L_P^2} \tag{3}$$

Because we already know the equation for the black hole entropy we know that the scale factor is 1/π, therefore we write

$$S_{BH} = \frac{k_B \pi \ R^2}{L_P^2} \tag{4}$$

If we multiply the second side by 4/4 we shall get the area of a sphere of radius R in the numerator, thus

$$S_{BH} = \frac{k_B 4\pi \ R^2}{4 L_P^2} \tag{5}$$

Considering that the Planck length is given by

$$L_P \equiv \sqrt{\frac{\hbar G}{c^3}} \tag{6}$$

Substituting the Planck length in equation (5) with the value given by equation (6) gives

$$S_{BH} = \frac{k_B c^3 4\pi \ R^2}{4\hbar G} \tag{7}$$

Substituting the area of the *event horizon*, $4\pi \ R^2$, with A_H yields

$$S_{BH} = \frac{k_B c^3}{4\hbar G} A_H \quad \text{(Berkenstein-Hawking's formula for the black hole entropy)} \tag{8}$$

This is the Berkenstein-Hawking formula for the black hole entropy. Thus we have proved that black hole entropy is a special case of the Scale Principle.

4. Conclusions

This paper shows that the *black hole entropy* obeys the present formulation: the Scale Law.

5. Notes

This investigation was published on line for the first time in May 2014 as part of another paper. Now it is published separately for clarity reasons.

REFERENCES

[1] R. A. Frino, *Scale Factors and the Scale Principle*, viXra:1405:0207, (2014) (Version 1 was published on May 2014).
[2] R. A. Frino*, Where Do the Laws of Physics Come From?*, viXra: xxxx.xxxx, (2014)

[3] R. A. Frino, *The Quantum Theory of Black Holes*, viXra: 1406.0036, (2014).