The following paper consists of two parts. The first one is the translation of the 21st chapter "The Pioneer anomaly" of my book "Aether theory with experimental verification". I chose this particular chapter for the reason that the Pioneer anomaly is a very interesting issue to be discussed. Here I have preserved the original numeration of the equations given in the book. The book has been published by the publishing house ‘Lulu’ (www.lulu.com) but it is also available for reading at viXra [1]. There have been different interpretations of the Pioneer anomaly, but mine is from my aether theory. For the purpose of a more exhaustive explanation of this particular occurrence, the second part of this paper presents some fundamental results of the aether theory, which are closely related to the Doppler effect.

**Keywords:** Pioneer anomaly, aether, Doppler effect.

**Part one**

The Pioneer anomaly

One of the unsolved mysteries of astrophysics is the so-called Pioneer anomaly. What is it all about? The Pioneer space mission can be said to have been quite successful regardless of occasional problems. The success of the mission prompted NASA to send two more spacecraft which would conduct research on the outer solar system, which is comprised of the following planets: Jupiter, Saturn, Uranus and Neptune. The two spacecraft were Pioneer 10 and Pioneer 11.

A more complete analysis of the Pioneer anomaly encompasses some basic data on the missions. Pioneer 10 was launched in March 1972 and it reached Jupiter in December 1973. Pioneer 11 was launched in March 1973. It reached Jupiter first and was later redirected towards Saturn. Both Pioneers exceeded their designers’ expectations since they had been designed to operate for 21 months only and remained functional for 30 years (Pioneer 10). Both spacecraft proved that the asteroid belt
could be passed, even though there was always a realistic chance of failure. Having passed by Jupiter and Saturn, both spacecraft left the Solar System moving along hyperbolic orbits in diametrically opposite directions.

Communication with spacecraft is done via radio connections. The communication with Pioneer 10 and Pioneer 11 was done via Deep Space Network (DSN). The last successful radio signal transmission from Pioneer 10 was recorded on April 27, 2002. In January 2003, powerful DSN radio antennas managed to catch a very weak signal from Pioneer 10. At that point, the distance between the Sun and the spacecraft was approximately 82.1 astronomical units (au). Two more contacts with the spacecraft were attempted, but unsuccessfully. The last time NASA contacted Pioneer 10 was on November 11, 1995.

The analysis of the DSN radio data can be used for determining the velocity of the spacecraft and its trajectory in that way as well as the distance within a time frame. Regardless of all this, spacecraft’s velocity, trajectory and distance are calculated through Newton’s law of gravitation and Einstein’s general theory of relativity. In calculating the trajectory, apart from these two fundamental theories, other non-gravitational effects are also considered. One such all-encompassing analysis gave the results for both the velocity and the trajectory of the spacecraft. Thus obtained velocity is represented as $v_{mod}$. It refers to the real velocity a spacecraft has at the particular point on the trajectory. This velocity is important for further analysis. Since it represents the real value of velocity, I will equate it to $v_{real}$, and will be using the latter throughout the paper.

$$v_{mod} = v_{real}$$ (21.1)

Apart from the $v_{mod}$ value, the trajectory of the spacecraft was also determined. The spacecraft was really moving along that mathematically calculated trajectory, so it represents the real trajectory of the spacecraft.

The trajectory obtained from the radio received data is different from the real trajectory of the spacecraft.

As an illustration of the difference between the two trajectories (distance), the following example will be considered. The mean value of the velocity of Pioneer 10 during the mission was 12.96km/s. The mean value of the velocity of Pioneer 11 was 11.42 km/s. In my calculations I will be using the value of 12.2 km/s for the Pioneer velocity. Both Pioneer spacecraft travelled mostly along hyperbolic trajectories within the Solar system, but for the simplicity of the illustration of the Pioneer anomaly we will presume that the Pioneer spacecraft makes a uniform linear motion within a year. During this time it will cover the distance $S_{real} = v_{real}t$, $v_{real}$ being the actual velocity of the spacecraft. However, the radio calculations of the velocity point to the distance value which is by 5000 kilometers lower that the real value ($S_{real}$). Here I represent the most simplified way of moving, that is, the uniform linear motion to illustrate this anomaly, but the same conclusion can be reached even when a non-uniform motion is considered. The radio wave analysis shows that each year the Pioneer spacecraft covers a distance the value of which is by 5000 kilometers lower of the real value. This occurrence has got a cumulative effect. Since each year the spacecraft is in the position different from the real position by 5000 kilometers, in 30 years’ time Pioneer 10 will be in the position which differs from the real one approximately by the value of the distance between the Earth and the Moon.

The first to notice the anomaly in the movement of the two spacecraft was John D. Anderson in 1980. At that time he was working in the Jet Propulsion Lab (JPL). Owing to his work of many years and his dedication, this anomaly has gained its world-wide attention. Its existence is no longer disputed. What it lacks is a plausible explanation. The Pioneer anomaly was first detected after radio data analysis. Anderson did not publish his results right away. He was looking for the influences that would cause the spacecraft’s slowdown so he had to reanalyze the results obtained from the radio wave signals. After some time, Anderson and his team were convinced that there really was the anomaly in the spacecraft acceleration.
In 1994 a physicist Michael Martin Nieto asked Anderson’s opinion on the Solar system gravity issue. Anderson informed him of the anomaly he had discovered. Scientific community became informed about it, as well. Immediately there was some criticism relating to the faulty software, the fact that not all the influences had been accounted for, etc. Actually, scientific community was taken aback by this research for a result like that one could essentially change the notions of gravity. If there was the deviation from the acceleration of the spacecraft, that could mean that there might be a deviation from Newton’s law of universal gravitation and Einstein’s general theory of relativity.

The scientific paper on the Pioneer anomaly was published in 1998, and a detailed analysis of the anomaly in 2002. The analysis of the radio signals which came from the distance of Pioneer 10 of 40 au to 70.5 au to us (that is, of 22.4 au to 31.7 au in the case of Pioneer 11) led to the conclusion that there was a deviation from the spacecraft acceleration, that is, that there was a slowdown.

I have already pointed out that when the movement of the spacecraft is followed via radio signals, the data obtained lead to the conclusion that the value of the distance that the spacecraft covers is lower by 5000 kilometers from the real value $S_{\text{real}}$. In some papers this has been explained as the effect of the real force which slows down the spacecraft. The analysis of the radio data as well as the fact that the trajectory of the spacecraft deviates from the mathematically calculated one yield for the introduction of the appropriate value of acceleration $a_p$. It is an approximately constant sunward acceleration

$$a_p = (8.74 \pm 1.33) \times 10^{-10} \text{ m/s}^2 \quad (21.2)$$

The anomaly was constant in both spacecraft, therefore, the physical influence that would cause the slowdown given in equation (21.2) had to be present throughout the whole mission.

A lot of effort has been put in the original data from Pioneer 10 and 11 reconstruction. It needs to be clear that those data are 40 years old and that they have been preserved on magnetic tapes which have probably started decaying slowly. Everything has changed over time, hardware, software, the way of preserving data, even the DSN structure. Scientists have managed to recover most of the old data and transform the into modern formats. In 2011, an analysis of the Pioneer anomaly was published which was based on a lot more data than the 2002 analysis. The 2011 analysis gathered the data of 23,1 years of the Pioneer 10 mission and 10,75 years of the Pioneer 11 mission. Most of the data refer to the period of the two missions within the Solar system when Pioneer 10 was at the distance of 18 to 80 au and Pioneer 11 at 9 to 32 au. The enormous effort invested in getting the most complete analysis of the anomaly resulted in the confirmation of all the previous researches.

There are numerous factors that can cause a spacecraft slowdown, such as solar radiation, solar wind, the force of the radio beam used for communicating with the Earth, the gravitational forces from the Kuiper belt, etc. Along with all these influences, there are some factors that could be ascribed to the technical malfunction of the spacecraft. It can be a gas leak from the thermoelectric generators, anisotropic thermal radiation of the spacecraft, etc. Over the time each of the factors has been separately analyzed and it has been concluded that none of them can be the cause of the Pioneer anomaly.

This occurrence of the deceleration of the spacecraft some authors relate to the effect of the real force which causes the slowdown. One such real force could have been brought about by the gas leak or the anisotropic thermal radiation of the spacecraft. A lot of attempts have been made to explain the source of this force.

Different approaches have been used to explain the Pioneer anomaly, such as the application of cosmology results. However, with all the approaches taken and the assumptions made with the aim of resolving the mystery of the Pioneer anomaly, there is not a single all-encompassing and consistent explanation. It was and still is a mystery that perplexes physicists.

The data from the other spacecraft such as Galileo, Ulysses, Voyager and Cassini have been used. In the case of the Galileo spacecraft it is obvious that the effect of solar radiation and anomalous
acceleration cannot be separated. However, the data analysis of the Ulysses spacecraft showed
acceleration \( a_p = (12 \pm 3) \times 10^{-10} \text{m/s}^2 \). The analyses of the received data of other spacecraft are
useful for dismissing the possibility of a systemic error in the DSN system. The data analyses of other
spacecraft are beneficial in that those spacecraft have similar constructions to the ones of Pioneer 10
and Pioneer 11, so the same effect could have appeared in both spacecraft causing the anomaly.

Not only does the anomaly exist in some spacecraft but it has also been observed in
astronomical objects. The scientist, Gary Page, has discovered 15 asteroids exhibiting the movement
anomaly using radio astronomical methods. The asteroids are in the outer Solar System. The most
suitable candidate for examining the anomaly is the asteroid 1995SN55, which is a 370-kilometer-wide
rock and has been in that part of the Solar system for the last 54 years.

The Pioneer tracking is based on a complex exchange of the radio signals between the stations
on the Earth, that is, the DSN system, and the spacecraft. The DSN system is the network of gigantic
antennas and communication facilities used in interplanetary space missions. The system supports
the spacecraft within the Earth’s orbit. It is used in observational radar astronomy in the research of the
Solar System and the space. The antennas are located on Earth in such a way that when a spacecraft is
at a certain distance from the planet it can establish and maintain contact with at least one antenna.
They are located in California, on the outskirts of Madrid and Canberra. Such positioning is necessary
because if there were just one antenna, the Earth’ rotation would unable any communication with a
spacecraft. This system is the most powerful of the kind in the world, and scientists have been working
for years on its improvement and upgrading.

Despite its complexity, the way of determining the velocity of a spacecraft is similar to the way
car speed is determined with the use of the radar. Velocity determination is based on the use of the
relations for the relativistic Doppler effect. This paper presents the way for determining the velocity of a
spacecraft [2]. 2-way Doppler tracking is used. A radio signal of the \( f_0 \) frequency is sent from the
Earth. The usual value of that frequency is 2.29 GHz. The velocity of the spacecraft is marked as \( v_{str1} \). The STR is used to show that relations from the special theory of relativity are used as well. The
movement of the spacecraft causes the occurrence of the Doppler effect so the frequency registered on
the spacecraft has the following value

\[
\nu' = \frac{1}{\sqrt{1 - (v_{str1}/c)^2}} \left(1 - \frac{v_{str1}}{c}\right) f_0
\]  \hspace{1cm} (21.3)

The spacecraft sends the signal back to the Earth thus becoming the source of the radio wave of the
\( \nu' \) frequency.

Following the rules of the special theory of relativity, the following radio wave frequency is
registered on the Earth

\[
\nu'' = \frac{1}{\sqrt{1 - (v_{str1}/c)^2}} \left(1 - \frac{v_{str1}}{c}\right) \nu'
\]  \hspace{1cm} (21.4)

From the equations given in (21.3) and (21.4), we arrive at the equation which determines the
\( \nu'' \) frequency

\[
\nu'' = \frac{1 - \frac{v_{str1}}{c}}{1 + \frac{v_{str1}}{c}} f_0
\]  \hspace{1cm} (21.5)

The returning signal (of the \( \nu'' \) frequency) is directly compared to the transmitted signal from
the DSN system. Having in mind that the spacecraft is moving away from the Earth, the value of the \( \nu'' \)
frequency will be lower than the \( f_0 \) frequency. From the equation given in (21.5), the relative change of
the frequency can be formulated
\[ \frac{v'' - v_0}{v_0} = 1 - \frac{u_{str1}}{c} - 1 \]  

(21.6)

This equation becomes approximate because the velocity of the spacecraft is smaller than the \( c \) velocity

\[ \frac{v'' - v_0}{v_0} = -2 \frac{u_{str2}}{c} \]  

(21.7)

In the (21.7) equation, the velocity is marked as \( u_{str2} \) because it differs from the \( u_{str1} \) velocity. The analyses of the Pioneer anomaly use the (21.7) equation.

It is my assumption that the anomaly in the Pioneer movement was caused by the use of the inadequate Doppler effect relations. The Doppler effect equations are based on the special theory of relativity. I will use my equations from the ether theory for the Doppler effect in order to explain the Pioneer anomaly. To prove or refute the claim that the equations from the ether theory can account for the movement of spacecraft, I would have to have the original mission data. Since I do not have them, I will be using numerical values of certain physical concepts that can be found in scientific studies.

There is some evidence that supports the thesis that inadequate Doppler effect relations were used in the Pioneer movement analysis. The anomaly appeared when the spacecraft was at the 9 au distance but it also appeared at the distance of 80 au, which was a wide range. It cannot be localized to a specific interval of distance from the Earth. Assuming that the inadequate Doppler relations theory was true, then the anomaly would always appear when any communication with the spacecraft was established, regardless of the distance between the spacecraft and the Earth. The Pioneer anomaly sometimes cannot be observed at smaller distances due to the solar radiation or the solar wind. It becomes measurable at larger distances. The anomaly appears in the movement of different spacecraft, whatever the construction or the mass of the spacecraft be. The velocity of those spacecraft is determined through the relations of the special theory of relativity for the Doppler effect, resulting in the occurrence of the anomaly. As I have already pointed out, this anomaly appears in the asteroid movement as well, but asteroid movement tracking is done through the principle of radio waves recoiling from the asteroid and the use of the Doppler effect equations.

The following fact needs to be emphasized. The Pioneer anomaly is sometimes explained as occurring due to the effect of some additional gravitational forces which cause the slowdown. It has been presumed that those gravitational forces stem from the dark matter, or from the deviation from the Newton’s law of gravitation that has been observed at larger distances from the Sun. The additional gravitational force would affect the movement of the Pioneer spacecraft causing their slowdown, but the effect of such additional gravitational forces on big astronomical objects such as planets has not been observed. They would certainly cause perturbations in those planets’ movements.

It is my presumption that the Doppler effect relations from the special theory of relativity are inadequate and that they produce wrong results. It is my belief that instead of them, the Doppler effect relations from the ether theory should be used. In order to prove or refute this presumption, original data from the Pioneer missions have to be used.

The (21.6) and (21.7) equations describe the way of determining the \( v' \) frequency. First a radio wave of the \( v_0 \) frequency is emitted from the DSN system. After the interaction with the spacecraft, it recoils and returns to the Earth where it becomes registered as the \( v' \) frequency. The latter has the lower value than the former, due to the spacecraft’s moving away from the Earth. I have dealt with this physical situation in the chapters 13 and 17 and derived the relation which links the frequencies of the emitted and recoiled radio wave from the standpoint of the ether theory. In order to get the clear overview of the analysis, I will repeat the derivation.

A radio signal of the \( v_0 \) frequency is emitted from the Earth. I will mark the absolute velocity of the Earth as \( u_1 \) and the absolute velocity of the spacecraft as \( u_2 \). The absolute velocity of the spacecraft
is bigger and it moves away from the Earth. I will mark the relative velocity of the spacecraft in relation to the Earth as $v_{te}$. The $te$ subscript marks the relations from the ether theory. The Doppler effect will appear and the frequency of the radio wave registered on the spacecraft from the ether theory has got the following value

$$v' = v_0 \left(1 - \frac{v_{te}}{c}\right) \frac{\sqrt{1 - (u_1/c)^2}}{\sqrt{1 - (u_2/c)^2}} \quad (21.8)$$

The spacecraft recoils the signal and sends it back to the Earth, and it can be said to be the source of the radio wave of the $v'$ frequency. In concord with the ether theory (13.33 equation), the following radio wave frequency is registered on the Earth

$$v^e = v' \left(1 - \frac{v_{te}}{c}\right) \frac{\sqrt{1 - (u_2/c)^2}}{\sqrt{1 - (u_1/c)^2}} \quad (21.9)$$

The (21.8) and (21.9) equations describe the way of determining the $v^e$ frequency

$$v^e = \left(1 - \frac{v_{te}}{c}\right)^2 v_0 \quad (21.10)$$

As it was the case with the special theory of relativity, here too the equation for determining the relative change of frequency can be formed, drawing on the (21.10) equation

$$\frac{v^e - v_0}{v_0} = \left(1 - \frac{v_{te}}{c}\right)^2 - 1 \quad (21.11)$$

The frequencies measured when tracking the spacecraft are $v^e$ and $v_0$. The $v_0$ frequency has got a constant value. Based on these two frequencies a relative change of the frequencies can be determined. But when you get that value the question is which equation (21.6), (21.7) or (21.11)) should be used to determine the speed of the spacecraft. There are three equations in total, two from the special theory of relativity (21.6 and 21.7)) and one from the ether theory (21.11). The one most commonly used in practice is (21.7).

Now I will describe the way the Pioneer anomaly is explained in scientific papers. As I have already pointed out the $v_{mod}$ velocity is derived from Newton’s law of universal gravitation, that is, from Einstein’s general theory of relativity. This velocity derivation included not only the aforementioned fundamental gravitational theories but some other non-gravitational effects influencing the trajectory as well. The outcomes of one such all-encompassing analysis are spacecraft’s trajectory and velocity. The velocity derived in this way is marked as $v_{mod}$ in scientific papers. It represents the spacecraft’s real velocity, that is, the fact that at a given point of the trajectory the spacecraft has got that velocity. It being the real velocity, I will equate it to $v_{real}$

$$v_{mod} = v_{real}$$

A radio signal of the $v_0$ frequency is emitted from the Earth, from the DSN system, to be more precise. After recoiling, the system registers a radio wave of a certain frequency. From the special theory of relativity, the (21.7) equation is applied and the DSN system should register the following frequency

$$v^e = v_0 \left(1 - 2 \frac{v_{mod}}{c}\right) \quad (21.12)$$

In scientific papers this equation (21.12) is written as following

$$v_{mod} = v_0 \left(1 - 2 \frac{v_{mod}}{c}\right) \quad (21.13)$$

In order to get the equation (21.13), the special theory of relativity has to be used.

It was expected that the DSN system would register the $v_{mod}$ frequency. However, the system registers the frequency different from the expected one. The registered frequency is usually marked as
The two frequencies differ, the occurrence which represents the Pioneer anomaly. The value of the $v_{obs}$ frequency is bigger than the value of the $v_{mod}$ frequency. This means that there is a blueshift with regard to the $v_{mod}$ frequency. This blueshift is constantly present in radio signals’ measurements. The presence of that small anomalous frequency shift was discovered by Anderson after a careful analysis of the radio data.

The difference between the two frequencies is formulated and marked as $\Delta v$

$$v_{obs} - v_{mod} = \Delta v$$  \hspace{1cm} (21.14)

This equation (21.14) can be differentiated in time, and then it becomes

$$\frac{d}{dt}(v_{obs} - v_{mod}) = \frac{d}{dt}(\Delta v)$$  \hspace{1cm} (21.15)

$t$ represents the time within the Earth system.

Another value $\dot{v}_p$ is introduced in the following way

$$\frac{d}{dt}(\Delta v) = 2\dot{v}_p$$  \hspace{1cm} (21.16)

, and its numerical value is

$$\dot{v}_p = (5.99 \pm 0.01) \cdot 10^{-9} \text{s}^{-2}$$  \hspace{1cm} (21.17)

This $\dot{v}_p$ value was not constant throughout the mission, but over long intervals of time it had a constant value. Its variations in time with both Pioneer 10 and Pioneer 11 spacecraft (when both spacecraft were at a significant distance from the Sun) were not big. With this in mind the (21.15) equation can be formulated as following

$$v_{obs} - v_{mod} \approx 2\dot{v}_p t$$  \hspace{1cm} (21.18)

In the analyses, the value $a_p$ is used, which is linked to the $\dot{v}_p$ in the following relation

$$a_p = c \frac{\dot{v}_p}{v_0}$$  \hspace{1cm} (21.19)

and its value is expressed as

$$a_p = -(8.74 \pm 1.33) \cdot 10^{-10} \text{m/s}^{-2}$$  \hspace{1cm} (21.20)

The value $a_p$ is interpreted as the real acceleration by some authors. Since it is a sunward acceleration, opposite from the spacecraft movement, it brought about the slowdown of the Pioneer spacecraft, which, in turn, led to its trajectory diverging from the mathematically derived one. The $a_p$ value was not constant throughout the but it had a constant value over long intervals of time.

I will now interpret the Pioneer anomaly from the grounds of the ether theory. As it has already been said, from the special theory of relativity the following equation can be derived

$$v_{mod} = v_0 \left(1 - \frac{v_{mod}}{c}\right)$$

It was expected that the DSN system would register the $v_{mod}$ frequency. However, the system registers the frequency different from the expected one. The registered frequency is usually marked as $v_{obs}$.

A spacecraft at a particular time has got the velocity $v_{mod}$. On the grounds of the ether theory (the 21.10 equation), it can be expected that the DSN system would register the following frequency

$$v^* = v_0 \left(1 - \frac{v_{mod}}{c}\right)^2$$  \hspace{1cm} (21.21)

that is

$$v^* = v_0 \left(1 - \frac{v_{mod}}{c} + \left(\frac{v_{mod}}{c}\right)^2\right)$$  \hspace{1cm} (21.22)
Since the equation is derived from the ether theory, I will be using \( v_{te} \) instead of \( v \), so the equation (21.22) has the following form

\[
v_{te} = v_0 \left(1 - 2 \frac{v_{mod}}{c} + \left( \frac{v_{mod}}{c} \right)^2 \right)
\]  

(21.23)

The obvious difference between the two equations (21.13) and (21.23) is in a small factor \( \left( \frac{v_{mod}}{c} \right)^2 \).

The two frequencies \( v_{mod} \) and \( v_{obs} \) differ, the occurrence which represents the Pioneer anomaly. An analogous issue whether there is a difference between the two frequencies \( v_{te} \) and \( v_{obs} \) can also be raised?

For the purpose of further analysis, I will assume that the ether theory provides correct equations for the Doppler effect, which implies that there is no difference between the \( v_{te} \) and \( v_{obs} \) frequencies, that is

\[
v_{obs} = v_{te}
\]

The equation (21.14) is formulated as

\[
v_{obs} - v_{mod} = \Delta v
\]

If \( v_{obs} = v_{te} \), with the equations (21.13) and (21.23), the equation (21.14) has the following form

\[
\Delta v = v_{obs} - v_{mod} = v_{te} - v_{mod}
\]

\[
\Delta v = v_{te} - v_{mod} = v_0 \left( \frac{v_{mod}}{c} \right)^2
\]  

(21.24)

Now we can calculate the difference between the \( v_{obs} \) (the \( v_{te} \) frequency) measured by the DSN system, and the frequency that comes as the result of the special theory of relativity (the \( v_{mod} \) frequency). As I have stated, I will be using the value of 12.2 km/s as the mean velocity of the spacecraft Pioneer. The \( v_0 \) frequency has got a constant value of 2.29 GHz. From these data we can calculate the value of \( \Delta v \)

\[
\Delta v = 3.787 \text{ Hz}
\]

The \( \Delta v \) value (frequency residuals) was not constant throughout the mission, because there were changes in the value of \( v_{mod} \), that is, there were changes in the real velocity of the spacecraft (\( v_{real} \)). The changes of \( \Delta v \) in time will be explained further on in the text.

The value \( v_p \) is introduced in the following way

\[
2v_p = \frac{d}{dt} (\Delta v)
\]

The right-hand and the left-hand sides of the equation (21.24) can be differentiated to get

\[
\frac{d}{dt} v_0 \left( \frac{v_{mod}}{c} \right)^2 = 2v_p
\]

which can further be organized as

\[
\left( \frac{v_0}{c^2} \right) \frac{d}{dt} (v_{mod})^2 = 2v_p
\]  

(21.25)

If

\[
v_{mod} = v_{real}
\]

then the (21.25) equation can be formed as
\[
\left(\frac{v_0}{c^2}\right)\frac{d}{dt}(v_{\text{real}})^2 = 2\dot{v}_p
\]  
(21.26)

This equation is very interesting for further analysis.

The (21.26) equation can be written as
\[
\left(\frac{v_0}{c^2}\right) \frac{d}{dt}(v_{\text{real}})^2 = 2\dot{v}_p \, dt
\]  
(21.27)

, and both sides of the equation integrated.

This \(\dot{v}_p\) value was not constant throughout the mission, but over long intervals of time it had a constant value. Its variations in time with both Pioneer 10 and Pioneer 11 spacecraft (when both spacecraft were at a significant distance from the Sun) were not big. This value is also a time function \(\dot{v}_p = \dot{v}_p(t)\), and this dependence enables us to integrate the right-hand side of the equation. The paper [3] provides some data on the frequency residuals. In 8 years of the spacecraft movement, the value of the frequency residuals \(\Delta v\) was 1.5Hz, that is
\[
\int 2\dot{v}_p \, dt = 1.5 \, \text{Hz}
\]  
(21.28)

The paper [3] does not precisely determine the starting point of the given 8-year-long interval. There are some data missing, without which the integration of the left-hand side of the equation is impossible. However, even with this lack of data, the value of the integral in question can be estimated.

The integral has the following form
\[
\left(\frac{v_0}{c^2}\right) \int_{v_1}^{v_2} d(v_{\text{real}})^2 = \left(\frac{v_0}{c^2}\right) \left( (v_2)^2 - (v_1)^2 \right)
\]  
(21.29)

In the (21.27) equation \(v_{\text{real}}\) is the Pioneer velocity relative to the Earth. Apart from the kinetic energy, the spacecraft also had gravitational potential energy. When the spacecraft was close to the Earth, the dominant energy was the gravitational potential energy of the Earth. However, with greater distances from the Earth, the dominant force became the gravitational potential energy of the Sun. During its mission, the Pioneer spacecraft was also in the gravitational fields of other planets, so their potential energies should also be accounted for. It would be best to solve the problem within the heliocentric system which is the referential system linked to the Sun. In a system like that, it would be possible to determine the velocity of the Earth and the velocity of the Pioneer spacecraft, that is its velocity relative to the Earth. Having in mind that the gravitational potential energy of the Sun in dominant even at greater distances, I will simplify the analysis by presuming that the spacecraft was moving in the gravitational field of the Sun, and that \(v_{\text{real}}\) was the Pioneer velocity relative to the Earth. Such approximation will influence the final result of the integral, but due to the lack of certain data I am forced to resort to this approach. From it the following law of the conservation of energy can be derived
\[
\frac{mv_1^2}{2} - \gamma M_s m \frac{v_1^2}{r_1} = \frac{mv_2^2}{2} - \gamma M_s m \frac{v_2^2}{r_2}
\]  
(21.30)

In the (21.30) equation \(m\) represents the mass of the spacecraft, and \(r_1\) nd \(r_2\) stand for the starting and the final distances of the spacecraft from the Sun, respectively. The \(\gamma\) is the universal gravitational constant, and \(M_s\) represents the mass of the Sun. In accordance with the (21.30) equation, the (21.29) equation can be rewritten as following
\[
\left(\frac{v_0}{c^2}\right) \int_{v_1}^{v_2} d(v_{\text{real}})^2 = \left(\frac{v_0}{c^2}\right) \int_{r_1}^{r_2} d\left(\frac{2\gamma M_s}{r}\right)
\]  
(21.31)

that is
\[
\left(\frac{v_0}{c^2}\right) \int_{v_1}^{v_2} d(v_{\text{real}})^2 = \left(\frac{v_0}{c^2}\right) 2 \left( \frac{\gamma M_s}{r_1} - \left( \frac{\gamma M_s}{r_2} \right) \right)
\]  
(21.32)
It is necessary to estimate the \( r_1 \) and \( r_2 \) distances. The value of the integral (21.28) is derived for the time interval of 8 years. During this period of time, the Pioneer 10 spacecraft covered the distance of 22 au, that is
\[
\Delta r = r_2 - r_1 = 22 \text{ au}
\]
I will arbitrarily take \( r_1 = 20 \text{ au} \), and from the equation then \( r_2 = 42 \text{ au} \). Now we have enough data to calculate the value of the integral given in the (21.32) equation. It is derived that
\[
\left(\frac{v_0}{c^2}\right) \int_{v_1}^{v_2} d(v_{\text{real}}) = -1.18 \text{ Hz}
\]
(21.33)

The result of the equation is the negative frequency value. That, nevertheless, is of not much importance for this analysis. What is important is that the same values are derived after the integration of both the right-hand and the left-hand sides of the equation (21.27). There is an ongoing discussion in scientific papers about the way the \( \Delta \nu \) value (the value of frequency residuals) is defined. The definition of the \( \Delta \nu \) value determines it having the plus or the minus sign.

From the (21.27) and (21.31) equations, the following equation can be derived
\[
\left(\frac{v_0}{c^2}\right) \int_{r_1}^{r_2} d\left(\frac{2yM_s}{r}\right) = 2v_p dt
\]
(21.34)

With this equation, I estimated the \( r_1 \) and \( r_2 \) distances, the consequence of which is a certain discrepancy in the values of the integrals, that is, in the values of the following equations (21.28) and (21.33). To check the validity of my explanation of the Pioneer anomaly (represented in the (21.34) equation) original mission data are needed.

We assume that the following function \( v_p = v_p(t) \) is known. If at the \( t_1 \) moment the spacecraft has the velocity \( v_1 \) and the distance from the Sun \( r_1 \), and after some time \( \Delta t = t_2 - t_1 \) it comes to the \( r_2 \) position and has the velocity of \( v_2 \), based on these data the equation (21.34) integration can be done and the validity of my explanation of the Pioneer anomaly can be checked
\[
\left(\frac{v_0}{c^2}\right) \int_{t_1}^{t_2} d\left(\frac{2yM_s}{r}\right) = 2v_p dt
\]
(21.35)

The integration can be done more than once for different values of the starting and final position of the spacecraft.

In the text I have pointed out several times that the \( \Delta \nu \) value was not constant throughout the mission, because the \( v_{\text{mod}} \) value (the real velocity of the spacecraft \( v_{\text{real}} \)) was changing during the course of mission. To illustrate this, I will use the law of the conservation of energy, represented in the (21.30) equation
\[
\frac{mv_1^2}{2} - \frac{\gamma M_s m}{r_1} = \frac{mv_2^2}{2} - \frac{\gamma M_s m}{r_2}
\]

Here I presume that the gravitational force of the Sun was the dominant one, even though during certain time intervals the gravitational forces of some other planets were also significant. The value of the frequency residuals \( \Delta \nu \) is defined in the following way
\[
\Delta \nu = \nu_{\text{obs}} - \nu_{\text{mod}} = \nu_0 \left(\frac{v_{\text{real}}}{c}\right)^2
\]

At the \( t_1 \) moment the spacecraft has the velocity \( v_1 \) and the distance from the Sun \( r_1 \). The \( \Delta \nu_1 \) value is
\[
\Delta \nu_1 = \nu_0 \left(\frac{v_1}{c}\right)^2
\]

After some time \( \Delta t = t_2 - t_1 \) it comes to the \( r_2 \) position \( (r_2 > r_1) \) and has the velocity of \( v_2 \), then the \( \Delta \nu_2 \) value is
\[
\Delta \nu_2 = \nu_0 \left(\frac{v_2}{c}\right)^2
\]
We can make a difference between these two values
\[ \Delta v_2 - \Delta v_1 = \left(\frac{v_0}{c^2}\right)(v_2^2 - v_1^2) \]
From the law of the conservation of energy (the (21.30) equation), the following equation can be derived
\[ \Delta v_2 = \Delta v_1 + 2 \frac{v_0}{c^2} t \left( - \frac{\gamma M_s m}{r_1} - \left( - \frac{\gamma M_s m}{r_2} \right) \right) \]
We can conclude that the greater the distance of the spacecraft from the Sun is, the lower the \( \Delta v \) value (frequency residuals) is.

**Part two**

Two equations ((21.8) and (21.9)) were used in the analysis of the Pioneer Anomaly. These are the Doppler effect relations. When equations for the Doppler effect are derived within the special theory of relativity, the equation for time dilatation is used. The ether theory draws on time dilatation as well. However, the explanation of this occurrence differs substantially from the one provided by the special theory of relativity.

The following example will serve the purpose of analyzing time dilatation. Let us imagine one part of the space which is void of all the matter that could cause the decrease in the speed of light. That part of the space would be at a great distance from all other astronomical bodies. It could be, for example, a small region in the intergalactic space with ultra high vacuum. Being distant from other galaxies, it would have no gravitational field. The space in that region would be Euclidean. When defined like this, the region would represent the ideal case. It goes without saying that this is not the only idealization in physics. On the contrary, there are a lot of occurrences that are examined and analyzed in the similar fashion. It happens quite often that we are unable to analyze the occurrence in all its complexity, so we can only discuss its most important influences.

In a region like the one previously described we will construct a Cartesian system which is stationary with respect to the ether. I will mark the system 0, and we arrive at such a system by choosing a point in the stationary ether and relate a Cartesian system to it. The axes will be arbitrarily defined and oriented. Then we will set an observer in such a system. The observer will be absolutely motionless. The axes do not change with time. If an observer is at the standstill within the 0 system, then the system enables the observer to be at the standstill with respect to the ether. Numerous systems similar to this one can be constructed in the same way.

The notion of absolute movement did not exist in the special theory of relativity. By introducing absolute systems of reference (the 0 system), movement becomes absolute. We will take two more systems into consideration, next to the 0 system (which, we presume, is stationary). The two new systems will be marked as 1 and 2. The systems 1 and 2 are inertial, which means that they have linear motion when compared to the 0 system, without changing the axes orientation with time. The movement is along a common axis \( x \) in the same direction. The velocities, with regard to the 0 system, are \( u_1 \) and \( u_2 \), for the systems 1 and 2 respectively. The intensities of the \( u_1 \) and \( u_2 \) absolute velocities need to be smaller than \( c \). The two systems, 1 and 2, move with respect to each other at the relative velocity \( v \). Let us presume that \( u_2 > u_1 \). Figure 1 represents the movement of the two systems 1 and 2 with respect to the 0 system. In this example, the two systems move away from the 0 system.
Let us presume that identical clocks are placed within the 0, 1, and 2 systems. The clocks are activated at the moment the coordinate beginnings coincide. From the ether theory, the relations between the time intervals in the 1 and 2 systems are defined by equation 1

$$\frac{\Delta t^{(2)}}{\Delta t^{(1)}} = \frac{\sqrt{1 - (u_2/c)^2}}{\sqrt{1 - (u_1/c)^2}}$$  \hspace{1cm} (1)$$

The absolute velocity of the system 2 is bigger than the absolute velocity of the system 1, so from equation 1 we can conclude that less time lapsed in the system 2 ($\Delta t_2 < \Delta t_1$). The difference $\Delta \bar{t}$ in the time measured by the clocks is calculated by equation 2

$$\Delta \bar{t} = \Delta t^{(1)} - \Delta t^{(2)} = \Delta t^{(1)} \left(1 - \frac{\sqrt{1 - (u_2/c)^2}}{\sqrt{1 - (u_1/c)^2}}\right)$$  \hspace{1cm} (2)$$

In the ether theory, there is no twin paradox, for time dilatation is related to the movement with respect to the ether, and not with respect to the relative movement, which is the case with the special theory of relativity.

The ether theory uses time dilatation for the Doppler effect equations derivation. Here I will not present the entire derivation of the Doppler effect equation. I will consider the 1 and 2 systems again. Let us presume that there is the source of light in the 1 system, the frequency of which is $\nu_0$. From the ether theory, the observer in the 2 system registers the following frequency

$$\nu^{(2)} = \nu_0 \left(1 - \frac{v}{c}\right) \frac{\sqrt{1 - (u_1/c)^2}}{\sqrt{1 - (u_2/c)^2}}$$  \hspace{1cm} (3)$$

$v$ marks the relative velocities of the 1 and 2 systems. This equation can be transformed in the following way

$$\nu^{(2)} = \nu_0 \frac{\left(1 - \frac{v}{c}\right)}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$  \hspace{1cm} (4)$$

If we compare this equation with the one from the special theory of relativity (equation 5), we can detect the difference between them of the value of the $p$ factor, which represents a slight correction.

$$\nu^{(2)} = \nu_0 \frac{\left(1 - \frac{v}{c}\right)}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$  \hspace{1cm} (5)$$
Let us analyze the following example. There is the source of light in the 2 system, the frequency of which is \( \nu_0 \). From the ether theory, the observer in the 1 system registers the following frequency

\[
\nu^{(1)} = \nu_0 \left( 1 - \frac{v}{c} \right) \frac{\sqrt{1 - (u_2/c)^2}}{\sqrt{1 - (u_1/c)^2}} \tag{6}
\]

This equation can be transformed in the form similar to the one from the special theory of relativity

\[
\nu^{(1)} = \nu_0 \left( 1 - \frac{v}{c} \right) \sqrt{1 - \left( \frac{v}{c} \right)^2 - p} \tag{7}
\]

References